Closed Form Expression for Signal Transmission via AF Relaying over Nakagami-m Fading Channels

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Abstract

In this paper, we analyze the performance of a cooperative communication wireless network over independent and identically distributed (IID) Nakagami-m fading channels. A simple transmission scheme is considered where the relay is operating in amplify-forward (AF) mode. A closed-form expression for symbol error rate (SER) is obtained using the moment generating function (MGF) of the total signal to noise ratio (SNR) of the transmitted signal with binary phase shift keying (BPSK).

I. Introduction

Transmit and receive diversity is a powerful fading mitigating technique experienced by signal due to multipath propagation. These forms of spatial diversity gains can be obtained by cooperation among in-cell users, also referred to as user cooperation diversity.

Nabar et al. [2] has introduced three different transmission protocols. We analyze protocol-II for AF-relay system with maximal ratio combining (MRC) at the destination over nakagami-m fading.

II. System Model

2.1 System and channel description

We adopt the same system and channel model as in [2],[3]. The equivalent SNR, γ_{eq}, at destination for protocol - II after 2 time-slots will be given as

$$\gamma_{eq}=\left(E_{sd}|f_{sd}|^2+\frac{E_{sr}E_{rd}}{E_{sr}+N_o}\frac{|f_{sr}|^2|f_{rd}|^2}{\frac{E_{rd}}{E_{sr}+N_o}|f_{rd}|^2+1}\right)\frac{1}{N_o} \qquad (1)$$

where, N_o is the noise variance, E_{xy} and f_{xy} is the energy of signal and the channel between node X and Y, respectively. The pdf of f_{xy} is given as [1]

$$p(|f_{xy}|)=\frac{2m^m|f_{xy}|^{2m-1}}{\Gamma(m)\rho^m}e^{-\frac{m}{\rho}|f_{xy}|^2} \qquad (2)$$

where, $\varepsilon(|f_{xy}|^2)=1$, $\varepsilon(\cdot)$ is the expectation operator, m is the fade parameter, ρ is the spread parameter, and $\Gamma(\cdot)$ is the gamma function defined in [5, eq.8.310.1]. Then $|f_{xy}|^2$ follows gamma distribution.

2.2 Protocol description

During time slot 1, source (S) transmits a signal to both the relay (R) and the destination (D). During time slot 2, S remains silent and R forwards the amplified version of the received signal of the 1st time slot, to D.

Time slot 1	S→R, D
Time slot 2	R→D

Ⅲ. SER Analysis

In the following we analyze the SER performance of above transmission scheme. The MGF of γ_{eq}, considering IID fading is

$$M_{\gamma_{eq}}(s)=E(e^{-s\gamma_{eq}})$$

$$M_{\gamma_{eq}}=\left(1+\frac{k_1k_2|f_{rd}|^2}{m(k_4|f_{rd}|^2+1)}s\right)^{-m}\times\left(1+\frac{k_1k_3}{m}s\right)^{-m} \quad (3)$$

where, $k_1=1/N_o$, $k_2=E_{sr}E_{rd}/(E_{sr}+N_o)$, $k_3=E_{sd}$ and $k_4=E_{rd}/(E_{sr}+N_o)$. The average error probability for MPSK can be written as

$$P_e=\frac{1}{\pi}\int_0^{(M-1)\pi/M}M_{\gamma_{eq}}\left(\frac{g_{psk}}{\sin^2\theta}\right)d\theta \quad (4)$$

where, $g_{psk}=\sin^2(\pi/M)$ and M is the constellation size. In the following,, we will discuss the SER evaluation for BPSK when $|h_{rd}|^2=1$.

When BPSK modulation is considered with $|f_{rd}|^2=1$, then substituting for $M_{\gamma_{eq}}(s)$ yields

$$P_{e,bpsk}=\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+c_1}\right)^m\left(\frac{\sin^2\theta}{\sin^2\theta+c_2}\right)^m d\theta \quad (5)$$

where, $c_1=\frac{k_1k_2}{m(k_4+1)}$ and $c_2=\frac{k_1k_3}{m}$. The closed form solution of above equation can be given from [4, eq.5A.58] as

$$P_{e,bpsk}=\frac{(c_1/c_2)^{(m-1)}}{2(1-c_1/c_2)^{(2m-1)}}\left[\sum_{k=0}^{m-1}\left(\frac{c_2}{c_1}-1\right)^kB_kI_k(c_2)-\frac{c_1}{c_2}\sum_{k=0}^{m-1}\left(1-\frac{c_1}{c_2}\right)^kC_kI_k(c_1)\right] \quad (6)$$

where,

$$B_k=\frac{A_k}{\binom{2m-1}{k}},\ C_k=\sum_{n=0}^{m-1}\frac{\binom{k}{n}}{\binom{2m-1}{n}}A_n,$$

$$A_k=(-1)^{m-1+k}\frac{\binom{m-1}{k}}{(2m-1)!}\prod_{\substack{n=1\\ n\neq k+1}}^{m}(2m-n),$$

$$I_k(c)=1-\sqrt{\frac{c}{1+c}}\left[1+\sum_{n=1}^{k}\frac{(2n-1)!!}{n!2^n(1+c)^n}\right]$$

This is our desired closed form expression of error probability for BPSK signaling. When we consider the case $m=1$, (i.e. rayleigh fading), eq.6 reduces to

$$P_{e,bpsk}=0.5\left(1-\frac{c_1}{c_1-c_2}\sqrt{\frac{c_1}{1+c_1}}+\frac{c_2}{c_1-c_2}\sqrt{\frac{c_2}{1+c_2}}\right) \quad (7)$$

Ⅳ. Simulation Results

Figure 1 shows the SER performance for BPSK modulation scheme with $m=1$. It can be seen that our derived analytical expression performs very closely with the simulation counterpart. The small performance difference could be reduced with proper power allocation of transmitted signals.

V. Conclusion

SER expression for simple cooperative system operating in AF-mode with BPSK modulation is discussed. We used the MGF of the exact SNR of received signal to derive the closed form SER expression for the static case ($|f_{rd}|=1$). Further more, we derived the simple closed form expression for the case $m=1$. Finally, the simulation results showed that our derived analytical equation performs almost similar to its simulation counterpart.

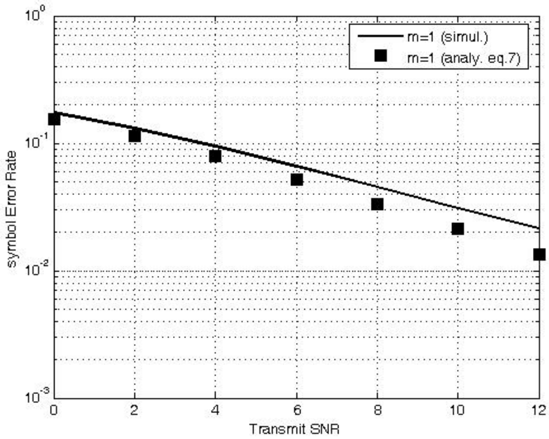

Figure 1. SER variation for $m=1$ and $|f_{rd}|^2=1$.

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