레일리 페이딩 채널에서 MRC 결합 기법 적용과 수학적 기법을 통한 Outage 확률 분석

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General Expression for Outage Probability with MRC Reception over Rayleigh Fading Channels

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Abstract

This paper provides a general and compact expression for the probability density function (pdf) and the moment-generating function (MGF) of the maximal ratio combiner output over Rayleigh fading channels. It is then used to derive closed form expression outage probability for repetition code problem. A variety of simulations is performed and shows that they match exactly with analytic ones.

I. INTRODUCTION

When evaluating the performance of repetition coding [1] on Rayleigh fading channels using Maximal Ratio Combining (MRC) technique, we usually deal with the problem of finding the expression for pdf of a sum of independent exponential random variables. For simplicity, limiting analysis was used in [1] instead of the exact one. In this paper, the most general case in which only some average channel conditions are distinct will be considered. Beside the general formula for pdf is established, the exact closed form expression of the outage probability was done in this paper

Ⅱ. DERIVATION & ANALYSIS

Let us define γ_i denotes the instantaneous SNR of each path received by the receiver with their expected values $\overline{\gamma_i}=(E_b/N_0)\lambda_i$ where λ_i is expected value of channel power of each Rayleigh fading path, E_b is the average energy per bit. For MRC, the output SNR of the combiner γ is defined as

$$\gamma=\sum_{i=1}^{N}\gamma_i \tag{1}$$

For ease of analysis, we sort and renumber the $\overline{\gamma_i}$ in ascending order as

$$\begin{aligned}\overline{\gamma}_1 = \cdots = \overline{\gamma}_{r_1} &= \beta_1\\ \overline{\gamma}_{r_1+1} = \cdots = \overline{\gamma}_{r_1+r_2} &= \beta_2\\ &\vdots\\ \overline{\gamma}_{r_1+r_2+\cdots+r_{K-1}+1} = \cdots = \overline{\gamma}_{r_1+r_2+\cdots+r_K} &= \beta_K\end{aligned} \tag{2}$$

where all β_k with k =*1,...,K* are distinct, $\sum_{k=1}^{N} r_k = N$ where r_k is a positive integer.

Because of all Rayleigh channels are independent; the Laplace transform of the Rayleigh pdf can be evaluated in closed form with the result [2]:

$$M_\gamma(s)=\prod_{i=1}^{N}M_{\gamma_i}(s)=\prod_{i=1}^{N}\left(\frac{1}{1-s\overline{\gamma}_i}\right)=\prod_{k=1}^{K}\left(\frac{1}{1-s\beta_k}\right)^{r_k} \tag{3}$$

where $M_{\gamma_i}(s)$ is the moment generating function (MGF) of the instantaneous fading γ_i given by

$$M_{\gamma_i}(s) = 1/(1 - s\overline{\gamma}_i) \tag{4}$$

Using the partial fraction expansion [3] for the MGF, it can be shown that

$$M_{\gamma}(s) = \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[\frac{A_{k,n}}{(1 - s\beta_k)^n}\right] \tag{5}$$

where

$$A_{k,n} = \frac{1}{(r_k - n)!}\left\{\frac{\partial^{(r_k - n)}}{\partial s^{(r_k - n)}}[(1 - s\beta_k)^{r_k} M_{\gamma}(s)]\right\}\Bigg|_{s=\frac{1}{\beta_k}} \tag{6}$$

Finally, the pdf of γ is determined by the inverse Laplace transform of $M_{\gamma}(s)$ as follows [3]:

$$f_{\gamma}(\gamma) = \left(\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}\frac{\gamma^{n-1}e^{-\gamma/\beta_k}}{\Gamma(n)\beta_k^n}\right]\right)U(\gamma) \tag{7}$$

where $U(.)$ is the unit-step function and $\Gamma(n) = \int_0^{\infty} t^{n-1}e^{-t}dt$.

P_{out} is defined as the probability that the MRC output SNR falls below a certain predetermined threshold SNR γ_{th} and hence can be obtained by integrating the pdf of γ:

$$P_{out} = \int_0^{\gamma_{th}} f_{\gamma}(\gamma)d\gamma = \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left\{A_{k,n}\left[1 - e^{-\frac{\gamma_{th}}{\beta_k}}\sum_{u=0}^{n-1}\frac{(\gamma_{th}/\beta_k)^u}{u!}\right]\right\} \tag{8}$$

III. Performance of Repetition Coding

The repetition coding problem is described in [1]. In here, by using our derived formula of pdf, the exact closed-form expression of outage probability instead of limiting analysis of repetition coding system over Rayleigh fading can be derived. In particular, we must calculate the following expression [1, eq. (9)]:

$$P_{out}(SNR, R) = \Pr\left[\sum_{i=1}^{N} SNR|h_i|^2 < 2^{NR} - 1\right] \tag{9}$$

where N denotes the number of blocks, h_i captures the effects of path-loss and multi-path fading; R is pre-specified transmission rate.

If we let $\gamma_i = SNR|h_i|^2$, $\gamma = \sum_{i=1}^{N}\gamma_i$ with $\overline{\gamma_i} = \lambda_i SNR$, $\gamma_{th} = 2^{NR} - 1$ and applying the result given by (8), we have

$$P_{out}(SNR, R) = \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left\{A_{k,n}\left[1 - e^{-\frac{\gamma_{th}}{\beta_k}}\sum_{u=0}^{n-1}\frac{(\gamma_{th}/\beta_k)^u}{u!}\right]\right\} \tag{10}$$

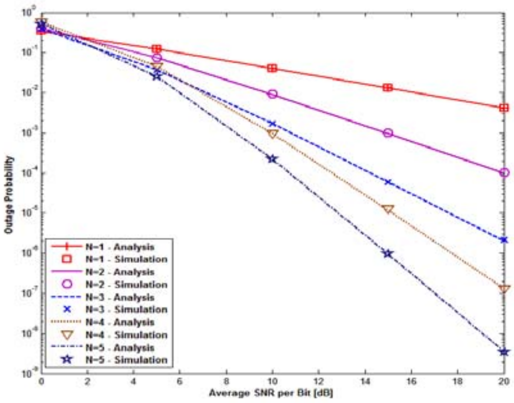

Fig. 1 Outage Probability of repetition coding for R =0.5 with different values of N.

IV. Conclusion

In this paper, the pdf & MGF of MRC reception over slow and frequency non-selective fading channels was analyzed. The expressions are general and offer a convenient way to evaluate any system which exploits MRC technique. Simulation results are in excellent agrement with the analytical formulas.

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