

# DETERMINATION OF GPS RECEIVER CLOCK ERRORS USING UNDIFFERENCE PHASE DATA

Ta-Kang Yeh<sup>1</sup>, Chen-Yu Chung<sup>2</sup>, Yu-Chung Chang<sup>2</sup>, Yu-Hsin Luo<sup>2</sup>

1. Institute of Geomatics and Disaster Prevention Technology, Ching Yun University, Taiwan, [bigsteel@cyu.edu.tw](mailto:bigsteel@cyu.edu.tw)

2. Institute of Geomatics and Disaster Prevention Technology, Ching Yun University, Taiwan

## ABSTRACT:

Enhancing the positioning precision is the primary pursuit of GPS users. To achieve this goal, most studies have focused on the relationship between GPS receiver clock errors and GPS positioning precision. This study utilizes undifferentiated phase data to calculate GPS clock errors and to compare with the frequency of cesium clock directly, thus verifying estimated clock errors by the method used in this paper. The relative frequency offsets from this paper and from National Standard Time and Frequency Laboratory of Taiwan match to  $1.5 \times 10^{-12}$  in the frequency instability, suggesting that the proposed technique has reached a certain level of quality. The built-in quartz clocks in the GPS receivers yield relative frequency offsets that are 3 to 4 orders higher than those of rubidium clocks. The frequency instability of the quartz clocks is on average two orders worse than that of the rubidium clock. Using the rubidium clock instead of the quartz clock, the horizontal and vertical positioning accuracies were improved by 26–78% (0.6–3.6 mm) and 20–34% (1.3–3.0 mm), respectively, for a short baseline. These improvements are 7–25% (0.3–1.7 mm) and 11% (1.7 mm) for a long baseline. Our experiments show that the frequency instability of clock, rather than relative frequency offset, is the governing factor of positioning accuracy.

**KEY WORDS:** GPS receiver, Positioning precision, Frequency instability, Relative frequency offset

## 1. INTRODUCTION

The development of high-accuracy geodetic methods using dual-frequency GPS carrier-phase observables has demonstrated positioning repeatability at the cm level for one day integrations (Zumberge *et al.*, 1997). However, the vertical precision of GPS is 2 to 3 times less accurate than its horizontal counterpart, due mainly to difficulties in correcting tropospheric delay, clock errors, multipath and antenna phase center variations (Rothacher and Beutler, 1998; Leick 2004). The International GNSS Service (IGS) has provided users with receiver clock errors and GPS satellite clock errors via internet to improve positioning accuracy (Ray and Senior, 2005). Timing of a GPS receiver is an important factor affecting positioning accuracy. In a study by Yeh *et al.* (2007), a 1-2 cm positioning error was found due to improperly modeled receiver clock errors. In GPS positioning, receiver clock errors are considered systematic errors that can be reduced by differencing GPS code and phase observables. However, for researchers interested in time and frequency standards, GPS observables can also be used to determine relative frequency offset and frequency instability. Time transfer by GPS is popular in many laboratories in the world, e.g., Bureau International des Poids et Mesures (BIPM). Using rapid orbits and final orbits of GPS, the Astronomical Institute of the University of Berne (AIUB) has cooperated with the Swiss Federal Office of Metrology and Accreditation (METAS) to provide time transfer service with a ns level accuracy (Dach *et al.*, 2003). Ray and Senior (2003) show that GPS time transfer can reach a precision of about 100 ps at each epoch in favorable cases. However, on average the absolute time transfer capability is limited to >1 ns, due to uncertainties in instrumental calibrations (Petit *et al.*, 2001).

The performance of a GPS receiver can be assessed by comparing the coordinates obtained with the receiver with given, sufficiently accurate coordinates at a calibration site. In Taiwan, The Taiwan National Measurement Laboratory (TNML) of the Industrial Technology Research Institute (ITRI) has established

a GPS calibration network, which is tied to the International Terrestrial Reference Frame 2000 (ITRF 2000; Altamimi *et al.*, 2002). In addition to coordinate comparison, the method of frequency calibration is also a feasible method for assessing the quality of a GPS receiver (Yeh *et al.*, 2006). Moreover, the GPS time in some of the receivers has been found to have some bias error. This demands the necessity of the calibration of the GPS receiver prior to its precise application (Banerjee *et al.*, 2007). Aiming at improving the precision and reliability of GPS relative positioning, this study utilizes undifferentiated phases collected mainly at TNML to determine receiver clock errors and to compare the performances of a quartz clock and a rubidium clock ready at TNML. The relationship between receiver clock error and GPS positioning accuracy is also investigated.

## 2. COLLECTION OF GPS DATA

The main GPS data used in this paper were collected at the TNML tracking station, which is also a station in the IGS network and receives continuous GPS data. This GPS station is located on the top floor of a building that houses the Center for Measurement Standards (CMS) of ITRI. The GPS receiver is an Allen Osborne Associates (AOA) BenchMark with an AOAD/M choke ring antenna. In a regular mode, the quartz clock and the clock steering function of this receiver are turned off, and the frequency source is based on a Datum 8040A rubidium clock. The frequency of this rubidium clock may change over time, so a regular frequency calibration is made. For the experiments in this paper, this rubidium clock was sent to the National Standard Time and Frequency Laboratory of Taiwan on May 5 (DOY 125), 2005 for calibration. The built-in quartz clock and its clock steering function were turned on again. The clock steering uses GPS observations to synchronize the time of the quartz clock with the times from the cesium and rubidium clocks of GPS satellites (Allen Osborne Associates, 1997). On May 13, 2005, the calibration of the rubidium clock

was completed, and it replaced the quartz clock again at TNML. The GPS data collected at TNML with respect to the quartz and rubidium clocks. Hereafter the three GPS observation periods (before, during, and after the calibration) are named Sessions 1, 2 and 3.

To determine the offset and instability of the frequency of a receiver clock, a reference (standard) frequency is needed. This reference frequency is assumed to be free from offset. In this study, the frequency of a HP 5071A cesium clock at the TWTF GPS station is adopted as the reference frequency. TWTF is located 25 km from TNML and is operated by the National Standard Time and Frequency Laboratory in Chungli, Taiwan. TWTF is a continuous GPS station with an Ashtech Z-XII3T receiver and an ASH701945C\_M choke ring antenna. This HP 5071A cesium clock serves as an external clock for the GPS receiver at TWTF. The coordinates of TWTF were held fixed when computing the coordinates of TNML by GPS relative positioning.

During the 21 days, the temperature, humidity, and pressure were recorded and the qualities of GPS observations were analyzed using the TEQC software. TEQC is developed by the University Navstar Consortium and can be used to convert the binary receiver format to the standard Receiver Independent Exchange (RINEX) format, edit existing RINEX files, and check the data quality (Estey and Meertens, 1999). Since there are no major changes in the environmental records and the quality indices of the observations, the positioning accuracies at TNML are governed by the receiver clocks used. That is, the relative frequency offset and frequency instability of clock will be the dominating factors for the coordinate variation.

### 3. ASSESSMENT OF RELATIVE FREQUENCY OFFSET AND FREQUENCY INSTABILITY

In this paper, undifferentiated GPS phase observables (no time differences, satellite differences and receiver differences) are used to calculate relative frequency offsets and frequency instabilities of the GPS receivers, based on the method of Dach *et al.* (2003). This is called the indirect method of frequency calibration, in comparison to the direct method of frequency calibration using a HP 5071A cesium clock (see below). Figure 1 shows the geometry and principle for determining relative frequency offset and frequency instability in a baseline involving two receivers (A and B). The difference between the clock error of a receiver and that of a GPS satellite is to be determined. The clock of receiver A (a rubidium clock) is regarded as a reference clock.

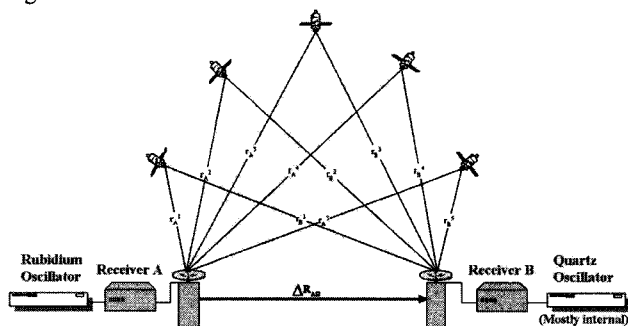


Figure 1. The geometry and principle of determination of the receiver clock errors

We used the Bernese version 4.2 software (Beutler *et al.*, 2001) to process GPS data and estimate the receiver clock errors. Since the use of differenced observables may eliminate

systematic errors and signals of interest, we decide to use undifferentiated observables to estimate clock errors. To use undifferentiated observables, data cleaning is necessary and was carried out before parameter estimations. The input to the program for data cleaning, which stands for RINEX smoothing, is a single RINEX file. The output is a RINEX file again, hopefully free from outliers and cycle slips. A summary of the actions taken by the program is contained in the output of Bernese 4.2. Each RINEX file is processed satellite by satellite.

Next we use the HP 5071A cesium clock of the National Standard Time and Frequency Laboratory to evaluate the rubidium clock. This is the direct method of frequency calibration. A counter was used to record the clock offsets between the rubidium clock and the HP 5071A cesium clock. We compared these measurements and concluded the rubidium clock has a relative frequency offset of  $6.6E-10$  and frequency instability of  $5.0E-12$ . The relative frequency offset and frequency instability from this comparison agree very well with our results, suggesting that our data processing method is effective and the method of modified Allan deviation for the determination of frequency instability is adequate. The relative frequency offset of the quartz clock is 4 orders of magnitude smaller (better) than that of the rubidium clock. However, the frequency instability of the rubidium clock is 2 orders of magnitude smaller (better) than the quartz clock.

We also analyzed the daily frequency changes of the quartz clock and the rubidium clock. Figure 2 shows the relative frequency offsets of the quartz clock relative to the rubidium clock in DOY 126. Since the AOA BenchMark GPS receiver has synchronized its time with the GPS time (clock steering), the relative frequency offsets oscillate around zero with a mean value of about  $7.75 \times 10^{-14}$ . Figure 3 shows the frequency instabilities of the quartz clock in DOY 126. The instability decreases with increasing time. Since clock steering was activated, the frequency instabilities of the quartz clock are mainly due to external noises. Figure 4 shows the relative frequency offsets of the rubidium clock in DOY 118. The offsets vary linearly with time, and this is typical of a rubidium clock. The mean offset is  $6.57 \times 10^{-10}$  and the rate is less than  $1 \times 10^{-10}$  /year. Because the offset of the quartz clock is compensated by clock steering, it is smaller than that of the rubidium clock. Figure 5 shows the frequency instabilities of the rubidium clock in DOY 118, which are on average about 1/100 smaller than those of the quartz clock. For example, at the 30<sup>th</sup> second, the frequency instabilities of the rubidium and quartz clocks are  $3.98 \times 10^{-12}$  and  $1.02 \times 10^{-10}$ , respectively.

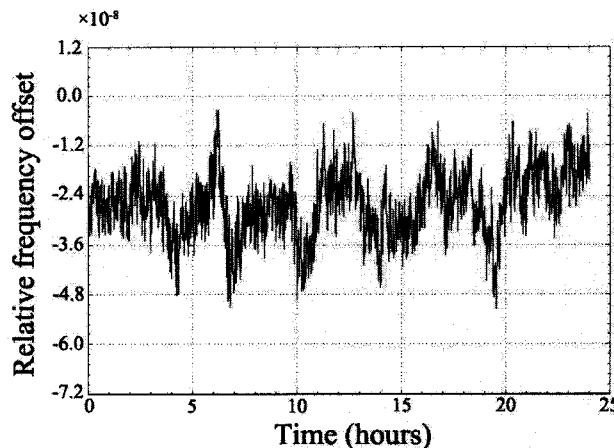


Figure 2. Relative frequency offsets of the quartz clock in DOY 126

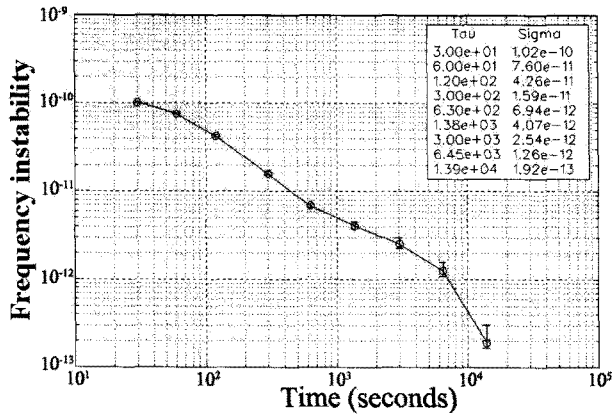


Figure 3. Frequency instabilities of the quartz clock in DOY 126

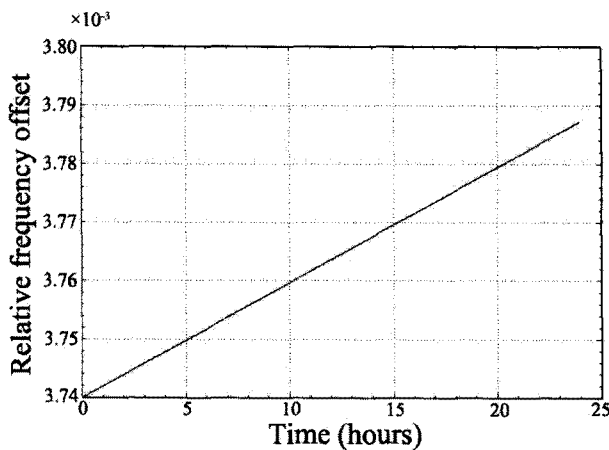


Figure 4. Relative frequency offsets of the rubidium clock in DOY 118

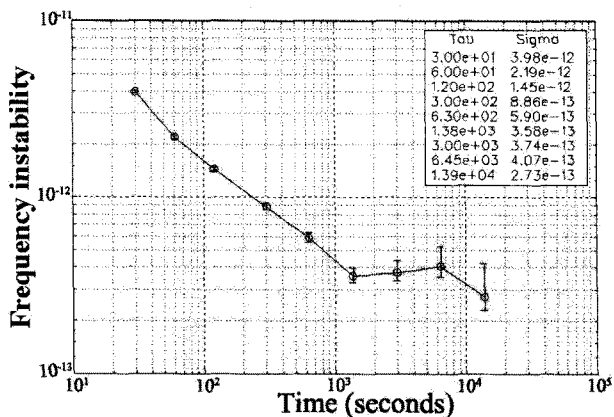


Figure 5. Frequency instabilities of the rubidium clock in DOY 118

#### 4. ACCURACY ASSESSMENT OF STATIC RELATIVE POSITIONING USING A SHORT BASELINE

To better understand the error characteristics of the quartz and rubidium clocks, we also assess the accuracies of GPS static relative positioning based on the two clocks. Again, we used the observations collected in three sessions to carry out static relative positioning between TNML and TWTF, which is considered a short baseline. We used Bernese 4.2 software to

process the double-differenced phases for the relative positioning. Phase ambiguities were solved and kept fixed to integer values. Precise IGS satellite orbits were used and a cut off angle of 15 degrees was adopted. Pole tide, solid earth tide and ocean tide loading corrections were applied to the GPS data. Different strategies for reducing tropospheric delays have been tested, and an optimal strategy is adopted as follows. With the ionosphere-free combination of phase, we first estimated zenith delay parameters for each day using the Saastamoinen a priori delay and the  $1/\cos(Z)$  mapping function, where  $Z$  is the zenith angle. Then, the site coordinates were corrected for the effect of tropospheric delay by applying the estimated zenith delay parameters (Yeh *et al.*, 2008). When calculating the coordinates of TNML, the coordinates of TWTF were held fixed. The averages of all coordinate components were determined and removed from the original coordinates to compute the coordinate variations in the three sessions. Figure 6 shows the variations of the three coordinate components.

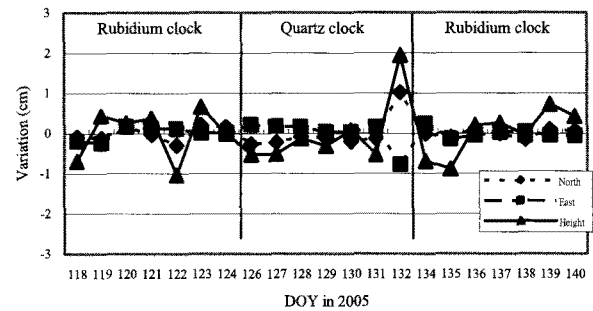


Figure 6. Variations of three-dimensional coordinates at TNML obtained from the static relative positioning before, during and after the calibration

The standard deviation of coordinate variations is now considered a descriptor of positioning accuracy. The standard deviations corresponding to the three sessions are given in Table 1. The performance of the rubidium clock remains the same when re-installing it to the GPS receiver, as suggested by the consistency between the standard deviations from Sessions 1 and 3 in Table 1. Use of the rubidium clock (both before and after the calibration) has led to a better coordinate repeatability (i.e., small standard deviation) than the quartz clock in the relative static positioning. The result of this experiment suggests that the frequency instability governs the accuracy of GPS relative positioning. Specifically, compared to the quartz clock, the rubidium clock has a smaller frequency instability, which yields smaller coordinate variations. This can be explained as follows. A clock error,  $\delta$ , can be expressed as a function of time,  $t$ , as

$$\delta = a_0 + a_1(t - t_{oe}) + a_2(t - t_{oe})^2$$

where  $t_{oe}$  is a reference epoch, and  $a_0$ ,  $a_1$  and  $a_2$  represent bias, drift, and drift-rate, respectively. The clock bias is responsible for the frequency offset and can be reduced or removed using GPS data processing, e.g., single differencing and double differencing. Thus the frequency offset has little impact on positioning accuracy. The frequency instability is due to the drift and the drift-rate terms, which cannot be reduced by differencing GPS phases or codes. Thus the frequency instability is a key factor for positioning accuracy.

Table 1. Standard deviations of coordinate variation (in mm) at three sessions

	Session 1 (rubidium)	Session 2 (quartz)	Session 3 (rubidium)
North	1.9	4.6	1.0
East	1.6	3.5	1.2
Height	6.3	8.9	5.9

## 5. CONCLUSIONS

The frequency offsets of the rubidium and quartz clocks as computed by the method in this paper (the indirect method, see Section 3) are consistent with those from a direct comparison of these two clocks with the HP 5071A cesium clock (the direct method), despite a mean difference of  $1.5E-12$  in the frequency instability between the two methods. The quartz clock's relative frequency offset is small due to clock steering that synchronizes receiver clock time and GPS time. However, the frequency instability of the quartz clock is degraded progressively due to time resetting. In one experiment, we turned off clock steering of the rubidium clock, and the result is that the rubidium clock's relative frequency offset became increasingly large but its frequency instability remained small. When changing the frequency source from the quartz clock to the rubidium clock in the GPS baseline solutions between TNML and TWTF (25 km), the positioning accuracy is improved by 26–78% (0.6–3.6 mm) in the horizontal coordinate components, and 20–34% (1.3–3.0 mm) in the vertical component. For the long baseline (YNML-WUHN, 920 km), the maximum improvements in the horizontal and vertical components are 7–25% (0.3–1.7 mm) and 11% (1.7 mm), respectively.

The quality of the frequency source (in terms of frequency stability) has a greater impact on GPS positioning accuracy for a short baseline than for a long baseline. The likely reason is that the error sources in the short baseline positioning are mostly reduced or removed due to differencing of GPS observables, so that the frequency source becomes the dominant factor on positioning accuracy. In the case of long baseline, differencing of GPS observables cannot remove systematic errors such as tropospheric and ionospheric delays and these remaining errors still dominate positioning accuracy, leaving the impact of changing frequency source less dominant but still influential. In summary, a better receiver clock (in terms of frequency stability) leads to a better GPS positioning accuracy. As future work, we recommend a scenario where two receivers of the same type share the same antenna but use two different clocks (e.g., a quartz clock and a rubidium clock). This would make the result insensitive to GPS satellite constellations and environmental factors such as temperature and pressure.

## 6. ACKNOWLEDGMENTS

The authors would like to thank the National Science Council of the Republic of China, Taiwan, for financially supporting this research, under Contract No. NSC94-2218-E-231-001.

## 7. REFERENCES

Allan D, Weiss M (1980) Accurate time and frequency transfer during common-view of a GPS satellite. *Proceedings of 1980 IEEE Frequency Control Symposium*, Philadelphia: 334-356.

Allen Osborne Associates (1997) *User Manual – The TurboRogue Family of GPS Receivers*. ITT Aerospace/Communications Division, Westlake Village, CA.

Altamimi Z, Sillard P, Boucher C (2002) ITRF2000: A new release of the International Terrestrial Reference Frame for earth science applications. *Journal of Geophysical Research*, 107(10): ETG 2-1-2-19.

Beutler G, Brockmann E, Dach R, Fridez P, Gurtner W, Hugentobler U, Johnson J, Mervart L, Rothacher M, Schaer S, Springer T, Weber R (2001) *Bernese GPS Software Version 4.2*. Astronomical Institute, University of Bern.

Banerjee P, Suman, Suri A K, Chatterjee A, Bose A (2007) A study on the potentiality of the GPS timing receiver for real time applications. *Measurement Science and Technology*, 18(12): 3811-3815.

Dach R, Beutler G, Hugentobler U, Schaer S, Schildknecht T, Springer T, Dudle G, Prost L (2003) Time transfer using GPS carrier phase: error propagation and results. *Journal of Geodesy*, 77: 1-14.

Estey L H, Meertens C M (1999) The multi-purpose toolkit for GPS/GLONASS data. *GPS Solutions*, 3(1): 42-49.

Leick A (2004) *GPS Satellite Surveying*, John Wiley and Sons, Inc., Hoboken, New Jersey.

Lesage P, Ayi T (1984) Characterization of Frequency Stability: Analysis of the Modified Allan Variance and Properties of Its Estimate. *IEEE Transaction Instrument Measure*, IM-33(4): 332-336.

Petit G, Jiang Z, White J, Beard R, Powers E (2001) Absolute calibration of an Ashtech Z12-T GPS receiver. *GPS Solutions*, 4: 41-46.

Ray J, Senior K (2003) IGS/BIPM pilot project: GPS carrier phase for time/frequency transfer and time scale formation. *Metrologia*, 40(4): 270-288.

Ray J, Senior K (2005) Geodetic techniques for time and frequency comparisons using GPS phase and code measurements. *Metrologia*, 42(3): 215-232.

Rothacher M, Beutler G (1998) The role of GPS in the study of global change, *Physics and Chemistry of the Earth*. 23(9-10): 1029-1040.

Yeh T K, Wang C S, Lee C W, Liou Y A (2006) Construction and uncertainty evaluation of a calibration system for GPS receivers. *Metrologia*, 43(5): 451-460.

Yeh T K, Wang C S, Chao B F, Chen C S, Lee C W (2007) Automatic data-quality monitoring for continuous GPS tracking stations in Taiwan. *Metrologia*, 44(5): 393-401.

Yeh T K, Liou Y A, Wang C S, Chen C S (2008) Identifying the degraded environment and bad receivers setting by using the GPS data quality indices. *Metrologia*, 45(5): 562-570.

Zumberge J F, Heflin M B, Jefferson D C, Watkins M M, Webb F H (1997) Precise point positioning for the efficient and robust analysis of GPS data from large networks. *Journal of Geophysical Research*, 102: 5005-5017.