확률집합의 구간치 용적 범함수에 관한 연구

A note on interval-valued functionals of random sets.

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Abstract

In this paper, we consider interval probability as a unifying concept for uncertainty and Choquet integrals with resect to a capacity functional. By using interval probability, we will define an interval-valued capacity functional and Choquet integral with respect to an interval-valued capacity functional. Furthermore, we investigate Choquet Choquet weak convergence of interval-valued capacity functionals of random sets.

Key words : random sets, interval probability, interval-valued capacity functional, Choquet integrals, Choquet weak convergence.

1. Interval probabilities and Choquet integrals

Throughout the paper, R is the set of real numbers and

$$I(R)=\{[a,b]|a,b\in R \text{ and } a\le b\}.$$

Then a element in $I(R)$ is called an interval number. On the interval number set, we define; for each pair $[a,b],[c,d]\in I(R)$ and $k\in R$,

$$[a,b]+[c,d]=[a+c,b+d]$$

$$[a,b]\cdot[c,d]=[a\cdot c\wedge a\cdot d\wedge b\cdot c\wedge b\cdot d,\ a\cdot c\vee a\cdot d\vee b\cdot c\vee b\cdot d]$$

$$k[a,b]=\begin{cases}[ka,kb], & k\ge 0\\ [kb,ka], & k<0\end{cases}$$

$$[a,b]\le[c,d] \quad \text{if and only if} \quad a\le c \text{ and } b\le d$$

We note that $(I(R),d_H)$ is a metric space, where d_H is the Hausdorff metric defined by

$$d_H(A,B)=\max\{\sup_{a\in A}\inf_{b\in B}|a-b|,\ \sup_{b\in B}\inf_{a\in A}|a-b|\}$$

for all $A,B\in I(R)$. Then it is easy to see that for $[a,b],[c,d]\in I(R)$,

$$d_H([a,b],[c,d])=\max\{|a-c|,|b-d|\}.$$

Let X be a locally, compact, second-countable, Hausdorff (LCSCH) space. Let Φ be the class of all closed sets in X, Ψ the class of all compact sets in X, O the class of all open sets in X, $\Psi_0=\Psi-\{\varnothing\}$, and $B(X)$ the class of all Borel sets in X. We note that $B(\Phi)$ is the σ-algebra classes a generated by Φ^K, $K\in\Psi$ and Φ_G, $G\in O$, where $\Phi^K=\{F\in\Phi|F\cap K=\varnothing\}$ and $\Phi_G=\{F\in\Phi|F\cap G=\varnothing\}$.

Definition 2.1 ([6]) Let (Ω,Σ,P) be a probability space and $(\Phi,B(\Phi))$ ameasurable space.

(1) A measurable mapping $S:(\Omega,\Sigma,P)\to(\Phi,B(\Phi))$ is called a random closed set.

(2) The probability measure Q induced on $B(\Phi)$ is defined by for each $B\in B(\Phi)$, $Q(B)=P(S^{-1}(B))$.

We note that the distribution of a

random set S is uniquely determined by its hitting functional T_S on Ψ such that

$$T_S(K) = P(\{w|S(w) \cap K \neq \varnothing\}), K \in \Psi.$$

From this, it is easy to see that T_S satisfies the following properties(see[5]):

(T_1) T_S is upper semi-continuous on Ψ, i.e., $K_n \searrow K$ in $\Psi \Rightarrow T_S(K_n) \searrow T_S(K)$;

(T_2) $T_S(\varnothing) = 0$ and $0 \le T_S \le 1$;

(T_3) T_S is monotone increasing on Ψ and for $K_1, K_2, \cdots, K_n \in \Psi, n \ge 2$,

$$T_S(\cap_{j=1}^n K_j) \le \sum_{\varnothing \neq J \subset \{1,2,\cdots,n\}} (-1)^{|J|+1} T_S(\cup_{j \in J} K_j).$$

Definition 2.2 ([6]) An interval-valued set function $\overline{P}(\cdot)$ on Σ is called an interval probability if

(i) $\overline{P}(A) = [L(A), U(A)]$,

$$0 \le L(A) \le U(A) \le 1, \forall A \in \Sigma.$$

(ii) $\{A \in \Sigma | L(A) \le P(A) \le U(A)\} \neq \varnothing$.

Definition 2.3 The interval probability measure $\overline{Q}$ induced on $B(\Phi)$ is defined by for each $B(\Phi)$, $\overline{Q}(B) = \overline{P}(S^{-1}(B))$.

Definition 2.4 (1) For every random closed set S, an nterval-valued mapping $\overline{T}_S$ is said to be an interval-valued capacity functional if there exist two capacity functionals T_S^1 and T_S^2 such that

$$\overline{T}_S = [T_S^1, T_S^2].$$

(2) Let $f \in C_b(X)$. The Choquet integral of f with respect to $\overline{T} = [T^1, T^2]$ is defined by

$$(C)\int f d\overline{T} = [(C)\int f dT^1, (C)\int f dT^2].$$

(3) The sequence of interval-valued capacity functionals $\overline{T}_n$ d_H-converges in the Choquet weak sense to the interval-valued capacity functional $\overline{T}$ in X if

$$(C)\int f d\overline{T}_n \rightarrow_{d_H} (C)\int f d\overline{T}, \forall f \in C_b(X)$$

whenever $\overline{T}_n \rightarrow_{C-W} \overline{T}$ as $n \rightarrow \infty$.

Theorem 2.5 Let $\{\overline{T}_n = [T_n^1, T_n^2]\}$ be a sequence of interval-valued capacity functionals and $T = [T^1, T^2]$ an interval-valued capacity functional. Then $\overline{T}_n \rightarrow_{d_H - C - W} \overline{T}$ as $n \rightarrow \infty$ if and only if $T_n^i \rightarrow_{C-W} T^i$ for $i = 1, 2$ as $n \rightarrow \infty$.

Theorem 2.6 Let $\{\overline{T}_n = [T_n^1, T_n^2]\}$ be a sequence of interval-valued capacity functionals. If $\overline{T}_n \rightarrow_{d_H - C - W} \overline{T}$ as $n \rightarrow \infty$, then we have

$$d_H - \operatorname{Lim}_{n \rightarrow \infty} \sup \overline{T}_n(B) \le \overline{T}(B), \forall B \in \Phi..$$

3. References

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