Generalized fuzzy (r, s)-continuous mappings
일반화된 퍼지 (r, s)-연속함수

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Abstract

In this paper, we introduce the concept of generalized fuzzy (r, s)-closed sets on intuitionistic fuzzy topological spaces in Šostak's sense. Using this concept, we introduce the notions of generalized fuzzy (r, s)-continuous mappings, and then we investigate some of their properties.

Keywords : generalized fuzzy (r, s)-closed set, generalized fuzzy (r, s)-continuous

1. Introduction

Chang [3] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [11], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [10].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. G. Balasubramanian and P. Sundaram [2] introduced the notion of generalized fuzzy continuous mappings and M. E. El-Shafei and A. Zakari [7] introduced the concept of semi-generalized continuous mappings on Chang's fuzzy topological spaces, and established some of their properties.

In this paper, we introduce the concept of generalized fuzzy (r, s)-closed sets on intuitionistic fuzzy topological spaces in Šostak's sense. Using this concept, we introduce the notions of generalized fuzzy (r, s)-continuous mappings, and then we investigate some of their properties.

2. Preliminaries

Definition 2.1. ([2]) A fuzzy set μ of a fuzzy topological space (X, τ) is said to be generalized closed fuzzy set if and only if $\mathrm{cl}(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is open fuzzy set.

3. Generalized fuzzy (r, s)-closed sets

We define the notion of generalized fuzzy (r, s)-closed sets on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

Definition 3.1. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be *generalized fuzzy* (r, s)*-closed* if $\mathrm{cl}(A, r, s) \subseteq B$ whenever $A \subseteq B$ and B is fuzzy (r, s)-open. The complement of a generalized fuzzy (r, s)-closed set is called *generalized fuzzy* (r, s)*-open*.

Theorem 3.2. Let A_1 and A_2 be intuitionistic fuzzy sets in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. If A_1 and A_2 are generalized fuzzy (r, s)-closed sets, then $A_1 \cup A_2$ is a generalized fuzzy (r, s)-closed set.

Theorem 3.3. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. If A is a generalized fuzzy (r, s)-closed set and $A \subseteq B \subseteq \mathrm{cl}(A, r, s)$, then B is generalized fuzzy (r, s)-closed.

Theorem 3.4. Let A be intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then A is generalized fuzzy (r,s)-open if and only if $B \subseteq \text{int}(A,r,s)$ whenever $B \subseteq A$ and B is fuzzy (r,s)-closed.

Theorem 3.5. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. If $\text{int}(A,r,s) \subseteq B \subseteq A$ and A is generalized fuzzy (r,s)-open, then B is generalized fuzzy (r,s)-open.

Theorem 3.6. Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be SoIFTSs and $(r,s) \in I \otimes I$. Let A be a generalized fuzzy (r,s)-closed set in X. If $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is fuzzy (r,s)-continuous and fuzzy (r,s)-closed, then $f(A)$ is generalized fuzzy (r,s)-closed in Y.

Definition 3.7. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the *generalized fuzzy (r,s)-closure* is defined by

$$\text{gcl}(A,r,s) = \bigcup\{B \in I(X) \mid A \subseteq B, \ B \text{ is generalized fuzzy } (r,s)\text{-closed}\}$$

If A is generalized fuzzy (r,s)-closed, then $\text{gcl}(A,r,s) = A$. The converse is not true, because the intersection of generalized fuzzy (r,s)-closed sets need not be generalized fuzzy (r,s)-closed.

Theorem 3.8. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then $A \subseteq \text{gcl}(A,r,s) \subseteq \text{cl}(A,r,s)$.

4. Generalized fuzzy (r,s)-continuous mappings

Now, we define the notion of generalized fuzzy (r,s)-continuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and establish some of their properties.

Definition 4.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called a *generalized fuzzy (r,s)-continuous* mapping if $f^{-1}(B)$ is a generalized fuzzy (r,s)-closed set in X for each fuzzy (r,s)-closed set B in Y.

Theorem 4.2. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is generalized fuzzy (r,s)-continuous if and only if $f^{-1}(B)$ is generalized fuzzy (r,s)-open in X for each fuzzy (r,s)-open set B in Y.

Theorem 4.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. If f is generalized fuzzy (r,s)-continuous, then $f(\text{gcl}(A,r,s)) \subseteq \text{cl}(f(A),r,s)$ where A is any intuitionistic fuzzy set in X.

Definition 4.4. Let $(X, \mathcal{T})$ be a SoIFTS and $(r,s) \in I \otimes I$. Then $(X, \mathcal{T})$ is said to be *fuzzy (r,s)-$T_{\frac{1}{2}}$* if every generalized fuzzy (r,s)-closed set in X is fuzzy (r,s)-closed in X.

Theorem 4.5. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ and $g : (Y, \mathcal{U}) \to (Z, \mathcal{S})$ be mappings, Y be fuzzy (r,s)-$T_{\frac{1}{2}}$ and $(r,s) \in I \otimes I$. If f and g are generalized fuzzy (r,s)-continuous, then $g \circ f$ is generalized fuzzy (r,s)-continuous.

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