Multi-person multi-attribute decision making problems based on interval-valued intuitionistic fuzzy information

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Abstract

Based on the interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and the interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator, we investigate the group decision making problems in which all the information provided by the decision-makers is presented as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by interval-valued intuitionistic fuzzy numbers, and the information about attribute weights is partially known. A numerical example is used to illustrate the applicability of the proposed approach.

Keywords : Multi-person multi-attribute decision making, IIFWG operator, IIFHG operator.

1. Introduction

Interval-valued intuitionistic fuzzy sets, introduced by Atanassov and Gargov [1], each of which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers, are a very useful means to describe the decision information in the process of decision making. Some researcher have applied the interval-valued intuitionistic fuzzy set theory to the field of decision making. Xu and Chen [8] developed some arithmetic aggregation operators, such as the interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator, the interval-valued intuitionistic fuzzy ordered weighted averaging (IIFOWA) operator and the interval-valued intuitionistic fuzzy hybrid averaging (IIFHA) operator for aggregating interval-valued intuitionistic fuzzy information, and gave an application of IIFHA operator to multi-attribute group decision making with interval-valued intuitionistic fuzzy information. Xu [6] developed some geometric aggregation operator, such as the interval-valued intuitionistic fuzzy geometric (IIFG) operator and interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator and applied them to multi-attribute group decision making with interval-valued intuitionistic fuzzy information. Xu and Chen [7] and Wei and Wang [4], respectively, developed some geometric aggregation operator, such as the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator and interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and applied them to multi-attribute group decision making with interval-valued intuitionistic fuzzy information. However, they used the IIF-WOG operator and the IIFWG operator in the situation where the information about attribute weights is completely known. In this paper, we investigate the group decision making problems in which all the information provided by the decision-makers is presented as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by interval-valued intuitionistic fuzzy number, and the information about attribute weights is partially known. First, we use the IIFHG operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices provided by the decision-makers into the collective interval-valued intuitionistic fuzzy decision matrix, and then we use the score function to calculate the score of each attribute value and construct the score matrix of the collective interval-valued intuitionistic fuzzy decision matrix. From the score matrix and the given attribute weight information, we establish some optimization models to determine the weights of attributes, and then we use the obtained attribute weights and the IIFWG operator to fuse the interval-valued intuitionistic fuzzy information in the collective interval-valued intuitionistic fuzzy decision matrix to get the overall interval-valued intuitionistic fuzzy values of alternatives by which the raking of all the given alternatives can be found. Finally, a numerical example is used to illustrate the applicability of the proposed method.

2. Basic concepts and relations

Let a set X be fixed and $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. An interval-valued intuitionistic fuzzy set (IVIFS) A in X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{1}$$

where $\mu_A : X \to D[0,1]$, $\nu_A : X \to D[0,1]$ with the condition $\sup \mu_A(x) + \sup \nu_A(x) \leq 1$ for any $x \in X$.

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element x to A. Then for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals and their lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\nu_{AL}(x)$ and $\nu_{AU}(x)$, resprctively, and thus we can replace (1) with

$$A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]\rangle : x \in X\},$$

where $0 \leq \mu_{AL}(x)$, $0 \leq \nu_{AL}(x)$ and $0 \leq \mu_{AU}(x) + \nu_{AU}(x) \leq 1$ for any $x \in X$.

For each IVIFS A in X, Ye [10] called

$$\begin{aligned}\pi_A(x) &= 1 - \mu_A(x) - \nu_A(x) \qquad (2)\\ &= [1 - \mu_{AU}(x) - \nu_{AU}(x), 1 - \mu_{AL}(x) - \nu_{AL}(x)]\end{aligned}$$

an intuitionistic fuzzy interval of X in A.

For convenience, we call $\tilde{a} = \langle a, b], [c, d]\rangle$ an interval-valued intuitionistic fuzzy number (IVIFN) [6], where $[a, b] \subset [0,1]$, $[c, d] \subset [0,1]$ and $b + d \leq 1$.

Xu [6] defined a score function s to measure a IVIFN $\tilde{a}$ as follows:

$$s(\tilde{a}) = \frac{1}{2}(a - c + b - d), \tag{3}$$

where $s(\tilde{a}) \in [-1, 1]$. The larger the value of $s(\tilde{a})$, the higher the IVIFN $\tilde{a}$. Especially, if $s(\tilde{a}) = 1$, then $\tilde{a} = \langle [1,1], [0,0]\rangle$, which is the largest IVIFN; if $s(\tilde{a}) = -1$, then $\tilde{a} = \langle [0,0], [1,1]\rangle$, which is the smallest IVIFN.

Wei nd Wang [4] defined an accuracy function h to evaluate the accuracy degree of a IVIFN $\tilde{a}$ as follows:

$$h(\tilde{a}) = \frac{1}{2}(a + b + c + d), \tag{4}$$

where $h(\tilde{a}) \in [0,1]$. The larger the value of $h(\tilde{a})$, the higher the degree of accuracy of the IVIFN $\tilde{a}$.

From (2), we define the hesitancy degree of the IVIFN $\tilde{a} = \langle [a, b], [c, d]\rangle$ as the midpoint of intuitionistic fuzzy interval of $\tilde{a}$, i.e.,

$$\pi(\tilde{a}) = \frac{1}{2}((1 - a - c) + (1 - b - d)). \tag{5}$$

Then we get the relation between the hesitancy degree and the accuracy degree of the IVIFN $\tilde{a}$

$$\pi(\tilde{a}) = \frac{1}{2}((1 - a - c) + (1 - b - d)) = 1 - h(\tilde{a}),$$

i.e.,

$$\pi(\tilde{a}) + h(\tilde{a}) = 1. \tag{6}$$

From (6), we know that the higher the accuracy degree $h(\tilde{a})$, the lower the hesitancy degree $\pi(\tilde{a})$.

Let $\tilde{a}_1 = \langle [a_1, b_1], [c_1, d_1]\rangle$ and $\tilde{a}_2 = \langle [a_2, b_2], [c_2, d_2]\rangle$ be two IVIFNs, Xu [6] defined two operational laws of IVIFNs as follows:

1. $\tilde{a}_1 \otimes \tilde{a}_2 = \langle [a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2]\rangle$;
2. $\tilde{a}_1^\lambda = \langle a_1^\lambda, b_1^\lambda], [1-(1-c_1)^\lambda, 1-(1-d_1)^\lambda]\rangle$, $\lambda \geq 0$,

which can ensure the operational results are also IVIFNs. Moreover, Xu [6] defined a method to compare two IVIFNs, which is based on the score function and the accuracy function:

Let $s(\tilde{a}_1) = \frac{1}{2}(a_1 - c_1 + b_1 - d_1)$ and $s(\tilde{a}_2) = \frac{1}{2}(a_2 - c_2 + b_2 - d_2)$ be the score of $\tilde{a}_1$ and $\tilde{a}_2$, respectively, and let $h(\tilde{a}_1) = \frac{1}{2}(a_1 + b_1 + c_1 + d_1)$ and $h(\tilde{a}_2) = \frac{1}{2}(a_2 + b_2 + c_2 + d_2)$ be the score of $\tilde{a}_1$ and $\tilde{a}_2$, respectively, then

- if $s(\tilde{a}_1) < s(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1 \prec \tilde{a}_2$;
- if $s(\tilde{a}_1) = s(\tilde{a}_2)$, then
  1) if $h(\tilde{a}_1) = H(\tilde{a}_2)$, then $\tilde{a}_1$ and $\tilde{a}_2$ represent the same information, i.e., $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$ and $d_1 = d_2$, denoted by $\tilde{a}_1 = \tilde{a}_2$;
  2) if $h(\tilde{a}_1) < h(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1 \prec \tilde{a}_2$.

3. Multi-person multi-attribute decision making

For multi-person multi-attribute decision making problem, let $O = \{O_1, O_2, \ldots, O_n\}$ be the set of n alternatives, $D = \{d_1, d_2, \ldots, d_l\}$ be the set of l decision-makers, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)^T$ be the weight vector of decision-makers, where $\lambda_k \geq 0$, $k = 1, 2, \ldots, l$, and $\sum_{k=1}^{l} \lambda_k = 1$. Let $U = \{u_1, u_2, \ldots, u_m\}$ be the set of m attributes, and suppose that the decision-makers provide the attribute weight information presented in the forms of weak ranking or strict ranking [3, 2]. For convenience, we denote by H the set of the known information about attribute weights provided by the decision-makers. Let $R^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ be an interval-valued intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij}^{(k)} = \langle [a_{ij}^{(k)}, b_{ij}^{(k)}], [c_{ij}^{(k)}, d_{ij}^{(k)}] \rangle$ is an IVIFN, provided by the decision-maker $d_k \in D$ for the alternative O_j with respect to the attribute $u_i \in U$, $[a_{ij}^{(k)}, b_{ij}^{(k)}]$ indicates the degree that the alternative $O_j \in O$ satisfy the attribute u_i, expressed by the decision-maker d_k, while $[c_{ij}^{(k)}, d_{ij}^{(k)}]$ indicates the degree that the alternative $O_j \in O$ does not satisfy the attribute u_i, expressed by the decision-maker d_k, and

$$[a_{ij}^{(k)}, b_{ij}^{(k)}] \subset [0,1], [c_{ij}^{(k)}, d_{ij}^{(k)}] \subset [0,1], \qquad (7)$$
$$b_{ij}^{(k)} + d_{ij}^{(k)} \leq 1,\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$$

To make a final decision in the process of group decision making, we need to fuse all individual decision opinion into group opinion. To do this, we use the IIFHG operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = (r_{ij}^{(k)})_{m\times n}$ $(k = 1, 2, \ldots, l)$ into the collective interval-valued intuitionistic fuzzy decision matrix $R = (\tilde{r}_{ij})_{m\times n}$, where

$$\begin{aligned} r_{ij} &= \mathrm{IIFHG}_{\omega,\lambda}(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(l)}) \qquad (8)\\ &= (\dot{r}_{ij}^{(\sigma(1))})^{\omega_1} \otimes (\dot{r}_{ij}^{(\sigma(2))})^{\omega_2} \otimes \cdots (\dot{r}_{ij}^{(\sigma(l))})^{\omega_l} \\ &= \Bigg\langle [\prod_{k=1}^{n} (\dot{a}_{ij}^{(\sigma(k))})^{\omega_k}, \prod_{k=1}^{n} (\dot{b}_{ij}^{(\sigma(k))})^{\omega_k}], \\ &\quad [1 - \prod_{k=1}^{n} (1 - \dot{c}_{ij}^{(\sigma(k))})^{\omega_k}, 1 - \prod_{k=1}^{n} (1 - \dot{d}_{ij}^{(\sigma(k))})^{\omega_k}] \Bigg\rangle, \end{aligned}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_l)^T$ is weight vector of IIFHG operator with $\omega_k > 0$ $(k = 1, 2, \ldots, l)$ and $\sum_{k=1}^{l} \omega_k = 1$, and $r_{ij} = \langle [\dot{a}_{ij}, \dot{b}_{ij}], [\dot{c}_{ij}, \dot{d}_{ij}] \rangle$, $\dot{r}_{ij}^{(\sigma(k))} = \langle [\dot{a}_{ij}^{\sigma((k))}, \dot{b}_{ij}^{\sigma((k))}], [\dot{c}_{ij}^{\sigma((k))}, \dot{d}_{ij}^{\sigma((k))}] \rangle$ is the kth largest of the weighted IVIFNs $\dot{r}_{ij}^{(k)} = (r_{ij}^{(k)})^{l\lambda_k}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

In the cases that the information about attribute weights is completely known, that is, the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attributes u_k $(k = 1, 2 \ldots, m)$ can be completely determine in advance, then based on the collective interval-valued intuitionistic fuzzy decision matrix $R = (\tilde{r}_{ij})_{m\times n}$, we can use the IIFWG operator [6]:

$$\begin{aligned} r_j &= \mathrm{IIFWG}_w(r_{1j}, r_{2j}, \ldots, r_{mj}) \qquad (9)\\ &= r_{ij}^{w_1} \otimes r_{2j}^{w_2} \otimes \cdots \otimes r_{mj}^{w_m} \\ &= \Bigg\langle [\prod_{i=1}^{m} a_{ij}^{w_i}, \prod_{i=1}^{m} b_{ij}^{w_1}], [1 - \prod_{i=1}^{m} (1 - c_{ij})^{w_i}, \\ &\quad 1 - \prod_{i=1}^{m} (1 - d_{ij})^{w_i}] \Bigg\rangle, j = 1, 2, \ldots, n \end{aligned}$$

to obtain the overall the alternative O_j. The greater the value of r_j, the better the alternative O_j will be.

However, the information about attribute weights provided by the decision-makers is usually incomplete (see, [2, 3]). So an interesting and important issue is how to utilize the collective interval-valued intuitionistic fuzzy decision matrix and the known weight information to find the most desirable alternative(s).

In the following, we present an approach to determining the weight of attributes.

Definition. Let $R = (r_{ij})_{m\times n}$ be the collective interval-valued intuitionistic fuzzy decision matrix. Then we call $S = (s_{ij})_{m\times n}$ the score matrix of $R = (r_{ij})_{m\times n}$, where

$$s_{ij} = s(r_{ij}) = \frac{1}{2}(a_{ij} - c_{ij} + b_{ij} - d_{ij}), \qquad (10)$$
$$i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n$$

and $s(r_{ij})$ is called the score of r_{ij}.

Based on the score matrix , we present the overall score values of each alternatives O_j $(j = 1, 2, \ldots, m)$:

$$s_j(w) = \sum_{i=1}^{m} w_i s_{ij},\ j = 1, 2, \ldots, n. \qquad (11)$$

Obviously, the greater the value $s_j(w)$, the better the alternative O_j. When we only consider the alternative O_j, then a reasonable vector of attribute weights $w = (w_1, w_2, \ldots, w_m)^T$ should be determined. Thus, we establish the following optimization model to maximize $s_j(w)$:

$$\textbf{(M-1)} \qquad \text{Maximize } s_j(w) = \sum_{i=1}^{m} w_i s_{ij}$$
$$\text{Subject to}: w = (w_1, \ldots, w_m)^T \in H,$$
$$w_i \geq 0, i = 1, \ldots, m, \sum_{i=1}^{m} w_i = 1.$$

By solving the model (M-1), we obtain the optimal solution $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, \ldots, w_m^{(j)})^T$ corresponding to the alternative O_j. However, in the process of determining the weight vector $w = (w_1, w_2, \ldots, w_m)^T$, we need to consider all the alternatives O_j ($j = 1, 2, \ldots, n$) as a whole. Thus, we construct weight matrix $W = (w_i^{(j)})_{m\times n}$ of the optimal solutions $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, \ldots, w_m^{(j)})^T$ ($j = 1, 2, \ldots, n$) as:

$$W = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix}$$

and we calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of the matrix $(S^T W)^T (S^T W)$, and then we construct a combined weight vector as follows:

$$w = W\omega = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix} \quad (12)$$
$$= \omega_1 w^{(1)} + \omega_2 w^{(2)} + \cdots + \omega_n w^{(n)},$$

and thus we derive the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attributes u_k ($k = 1, 2 \ldots, m$).

Based on the analysis above, in the following we present an approach to multi-person multi-attribute interval-valued intuitionistic fuzzy decision making with incomplete attribute weight information:

Step 1. Utilize the IIFHG operator (8) to aggregate all individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = (r_{ij}^{(k)})_{m\times n}$ ($k = 1, 2, \ldots, l$) into a collective interval-valued intuitionistic fuzzy decision matrix $R = (r_{ij})_{m\times n}$.

Step 2. Calculate the score matrix $S = (s_{ij})_{m\times n}$ of the collective interval-valued intuitionistic fuzzy decision matrix R.

Step 3. Utilize the model (M-1) to obtain the optimal weight vectors $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, \ldots, w_m^{(j)})^T$ ($j = 1, 2, \ldots, n$) corresponding to the alternatives O_j ($j = 1, 2, \ldots, n$), and then construct the weight matrix W.

Step 4. Calculate the normalized eigenvector $\omega = \omega_1, \omega_2, \ldots, \omega_n)^T$ of the matrix $(S^T W)^T (S^T W)$.

Step 5. Utilize (12) to derive the weight vector $w = (w_1, w_2, \ldots, w_m)^T$.

Step 6. Use the IIFWG operator (9) to get the overall values r_j of the alternatives O_j ($j = 1, 2, \ldots, n$).

Step 7. Use the score function to calculate the scores $s(r_j)$ of the overall values r_j of the alternatives O_j ($j = 1, 2, \ldots, n$).

Step 8. Utilize the scores $s(r_j)$ to rank the alternatives O_j ($j = 1, 2, \ldots, n$), and then select the most desirable one(s) (if two scores $s(r_i)$ and $s(r_j)$ are identical, then we calculate the accuracy degrees $h(r_i)$ and $h(r_j)$ of the overall values r_i and r_j, respectively, and then rank the alternatives O_i and O_j according to the accuracy degrees $h(r_i)$ and $h(r_j)$).

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