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Fuzzy Entropy Construction for Non-Convex Fuzzy Membership Function

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Abstract

Fuzzy entropy is designed for non-convex fuzzy membership function using well known Hamming distance measure. Design procedure of convex fuzzy membership function is represented through distance measure, furthermore characteristic analysis for non-convex function are also illustrated. Proof of proposed fuzzy entropy is discussed, and entropy computation is illustrated.

Key Words : Fuzzy entropy, non-convex fuzzy membership function, distance measure

1. Introduction

Characterization and quantification of fuzziness are important issues that affect the management of uncertainty in many system model and designing. The results about the fuzzy set entropy have been well known by the previous researchers [1-6]. Liu had proposed the axiomatic definitions of entropy, distance measure and similarity measure, and discussed the relations between these three concepts. Kosko viewed the relation between distance measure and fuzzy entropy. Bhandari and Pal gave a fuzzy information measure for discrimination of a fuzzy set relative to some other fuzzy set. Pal and Pal analyzed the classical Shannon information entropy. Also Ghosh used this entropy to neural network. However, all these results are based on the convex fuzzy membership functions.

For fuzzy set, there is an uncertainty knowledge in fuzzy set itself. Hence information of the data can be obtained from analyzing the fuzzy membership function. Thus most studies about fuzzy set are focussed on considering membership function. At this point we have an interest for non-convex fuzzy membership. In this paper we design fuzzy membership function for non-convex fuzzy membership function based on distance measure. Applying fuzzy entropy to non-convex fuzzy membership function, first we analyze the characteristics for fuzzy sets.

In the next chapter, the axiomatic definitions of entropy, previous fuzzy entropy for convex membership function are introduced. Also fuzzy entropy of non-convex membership function is proposed and proved. Notations of Liu's are used in this paper [4].

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2. Fuzzy entropy for Non-Convex membership function

We introduce some preliminary results about axiomatic definitions of fuzzy entropy and fuzzy entropy of convex membership function.

Definition 2.1 (Liu, 1992) A real function e: $F(X) \to R^+$ or $e : P(X) \to R^+$ is called an entropy on $F(X)$, or $P(X)$ if e has the following properties:

(E1) $e(D) = 0, \forall D \in P(X)$

(E2) $e([1/2]) = max_{A \in F(X)} e(A)$

(E3) $e(A^*) \le e(A)$, for any sharpening A^* of A

(E4) $e(A) = e(A^c), \forall A \in F(X)$.

where $[1/2]$ is the fuzzy set in which the value of the membership function is $1/2$.

Definition 2.1 is not restricted to convex membership function. Next we introduce fuzzy entropy.

Theorem 2.1 Let d be a σ-distance measure on $F(X)$; if d satisfies

$$d(A^c, B^c) = d(A, B), A, B \in F(X),$$

then

$$e(A) = 2d((A \cap A_{far}), [0]) + 2d((A \cup A_{far}), [1]) \quad (1)$$

is a fuzzy entropy.

Fuzzy entropy (1) satisfies Definition 2.1. However we does not check whether those satisfy for non-convex or not.

Next, we introduce non-convex fuzzy membership function. Definition of non-convex fuzzy membership function can be found in reference [7]. Non-convex fuzzy sets are not common fuzzy membership function. Definition of non-convexity derived from convexity definitely.

Definition 2.2 [7] A fuzzy set A is convex if and only if for any x_1, $x_2 \in X$ and any $\lambda \in [0,1]$,

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \ge min\{\mu_A(x_1), \mu_A(x_2)\} \quad (2)$$

Non-convexity fuzzy set is said if it is not convex. Furthermore non-convex membership functions can be notified naturally 3 sub classes. Elementary, Time related, and Consequent non-convex membership functions. Discrete fuzzy set express elementary non-convex fuzzy membership functions. Time related non-convex membership functions can be found in energy supply by time of day or year, mealtime by time of day. Finally, Mamdani fuzzy inferencing is a typical example of consequent non-convex sets. In a rule based fuzzy system the result of Mamdani fuzzy inferencing is a non-convex fuzzy set where the antecedent and consequent fuzzy sets are triangular and/or trapezoidal. Jang *et.al* also insisted that the definition of convexity of a fuzzy set is not as strict as the common definition of onvexity of a function [7].

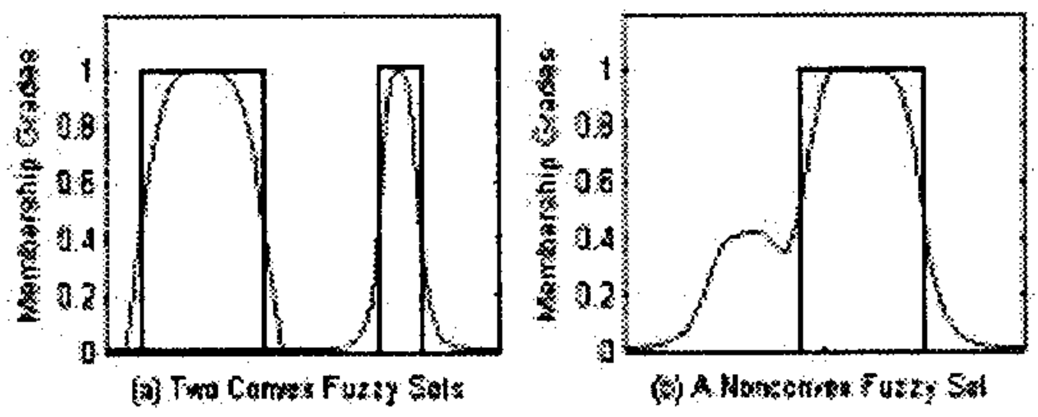

Fig.1 Non-convex MF

By the definition of Jang *et.al,* fuzzy entropy of Fig. 1 (a) are satisfied. However if two fuzzy set are considered as one fuzzy, then it has to be consider as non-convex fuzzy set. Hence by the computation of (1), we can obtain the fuzzy entropy value. Fig. 1 (b) is typical non-convex membership function. If the crisp set is applied as rectangular, we also compute fuzzy entropy.

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