Quantification of rock behavior around shallow depth tunnel by Analytic Hierarchy Process and Fuzzy Delphi Method

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1. INTRODUCTION

Quantitatively identifying rock behaviors expected in excavating tunnel could assist engineers not only to select tunneling method and support pattern but also to evaluate tunnel stability by numerical analysis adjusted to rock behavior. Parameters influencing rock behavior and their relative importance should be determined to quantify rock behavior.

Many researchers have studied how to identify rock behavior from parameters influencing rock behavior. Poschl and Kleberger (2004) described that parameters influencing rock behavior was divided into rock mass properties and circumstantial factors and that rock behavior was identified by combination between them in unsupported stage of tunneling. Martin et al. (2003) illustrated two common modes of failure: stress-driven and gravity-driven based on geological strength index (GSI) value, rock mass strength, and in-situ stress. Goricki et al. (2004) described that rock behavior types were developed from rock mass type and influencing factors such as groundwater, joint orientation, and primary stress conditions and classified the rock behavior into eleven types in the pre-construction stage. Stille and Palmstrom (2007, 2008) have shown that three main ground behaviors, namely gravity driven, stress driven, and water influenced behavior, could be classified by the composition and structure of rock mass, the effect of stresses, ground water, and excavation features and that they might be expressed by a qualitative chart. Goel (2001) reported nine rock behavior types from Himalayan tunneling. Many researchers have studied how to assess rock behavior and to predict instability. Martin et al. (2003) described the instability of spalling behavior in brittle hard rocks through uncertainty analysis using the Monte Carlo Simulation (MCS) method. Sakurai (1997) described the tunnel instability by using critical strain defined as a ratio between rock mass strength and modulus of deformation. Singh et al. (2007) demonstrated that squeezing behavior was quantified by the ratio of the expected strain and the critical strain for overstressed soft rock. Cai et al. (2004) illustrated brittle failure zone on quantified GSI chart determined by block volume and joint surface condition.

Rock Engineering Systems (RES) and Analytic Hierarchy Process (AHP) could be applied to identify rock behavior by combination among parameters and to compute their weighting. Benardos and Kaliampakos (2004) reported about the method to assess the hazards by using a vulnerability index, which is computed based on the principles of the RES, and determine identify the weighting of the eight parameters on tunnel boring machine tunneling. The AHP has recently been applied to make optimal decision on rock mechanics problems under complexity and uncertainty environment of tunneling. Liu and Chen (2007) described how to assess slope rock mass quality estimates and rock mass rating by combining the AHP and the Fuzzy Delphi Method (FDM). Yavuz et al. (2008) described how to determine optimal support pattern among different alternatives by the AHP. Ayalew et al. (2005) described how to produce landslide susceptibility maps from weighting of parameters influencing landslide by the AHP.

Most of studies to identify rock behavior were qualitative except from some quantitative studies on brittle rock behavior such as spalling and squeezing. These quantitative studies didn't consider relative importance and weighting among parameters affecting rock behavior. The purpose of this study is to quantify rock behavior of shallow depth tunnel by the AHP and the FDM. The FDM is applied to minimize uncertainty of expert's judgment. Rock behaviors such as cave-in, rock fall, and plastic deformation are identified from rock mass intrinsic parameters (uniaxial compressive strength (UCS), RQD, joint surface condition), rock mass extrinsic parameters (stress, ground water, earthquake), and a design parameter (excavation span). All seven parameters are mutually independent and easily able to be evaluated. We applied the proposed method to the basic design of Seoul Metro Line 9 and quantified rock behavior by three rock behavior index (RBI) on fall, cave-in, and plastic deformation.

2. Identifying rock behaviors and their influencing parameters

Rock behavior could be identified by combination among parameters influencing rock behavior. Cai et al. (2004) suggested more than a dozen parameters that should be considered when describing a rock mass and using the results for certain design purposes. Based on these parameters, we classified parameters influencing rock behavior into three categories: rock mass intrinsic parameter, rock mass extrinsic parameter and design parameter (Table 1).

Table 1. Group of Parameters affecting rock behaviors

Group of parameters		Individual parameters
Rock mass inherent parameters	1. Intact rock parameters	*Strength of intact rock*
		Rock modulus
	Joint parameters	Number of joint sets
		Joint frequency
		RQD
		Joint condition (roughness, infilling, weathering)
		Joint size/length, persistency
		Joint orientation
Rock mass external parameters		*In situ stress*
		Ground water
		Dynamic condition (earthquake, blasting)
Design (excavation) parameters		*Excavation size*
		Excavation shape
		Construction method
		Blasting damage

All parameters affecting rock behaviors could not be considered to identify rock behavior due to limitation of geology survey and difficulty in modifying tunnel alignment and tunnel construction method. Because the strength and elastic modulus of intact rock were highly correlated, the strength of intact rock was only considered as intact rock parameter. As it was difficult to obtain information on joint size, joint orientation and joint number of joint set without exposed rock face sampling such as scanline sampling and window sampling, joint condition and RQD obtained from boring were considered as joint parameters. Extrinsic parameters were considered as groundwater, stress, and dynamic condition such as earthquake and blasting. Excavation size of four design parameters was only regarded, as another three design parameters were generally determined in the feasibility study stage. Eventually, seven paramete9rs influencing on rock behavior were determined: uniaxial compressive strength, RQD, joint surface condition, stress, ground water, earthquake, and tunnel span.

Many researchers have studied rock behavior type and main parameters affecting rock behavior (Table 2). In deep tunneling, rock mass strength, ground stress, and RMR were key parameters to identify brittle failure; In shallow depth tunneling, joint condition, ground water, ground stress, and tunnel size could be important parameters to identify gravity driven rock behavior and stress driven rock behavior.

From experiences gained by the new Austrian tunneling method (NATM) tunneling in shallow depths, the ground behavior could be identified by the ground conditions. For heavily joint rock condition, plastic deformation behavior or cave-in behavior was expected.

Table 2. Comparison on main parameters on rock behaviors

Researchers	Main parameters	Rock behavior type
Martin et al. (1999)	RMR UCS Ground stress	10 types: stable, rock falls, cave-in, buckling, rupturing, spalling/slabbing, rockburst, plastic behavior, squeezing or swelling, swelling clay
Kaiser et al. (2000)	RMR UCS Ground stress Induced stress	Low mining-induced stress Intermediate mining-induced stress High mining-induced stress
Martin et al. (2003)	GSI UCS Ground stress	Stress-induced plastic yielding Gravity-induced structurally controlled block movement Stress-induced brittle spalling
Goricki et al. (2004)	Rock type Ground water Joint orientation Ground stress Tunnel size, shape	11 types: stable, stable with the potential of discontinuity controlled block fall, shallow shear failure, deep-seated shear failure, rockburst, buckling, shear failure under low confining pressure, ravelling ground, flowing ground, swelling, Heterogeneous rock mass with frequently changing deformation characteristics
Palmstrom & Stille (2007)	Rock type Ground water Joint orientation Ground stress Tunnel size, shape	Gravity-induced (4 types): stable, block falls, cave-in, running ground Stress-induced (6 types): buckling, rupturing from stresses, slabbing, rockburst, plastic behavior, squeezing Groundwater influenced (4 types): raveling from slaking, swelling, flowing ground, water ingress

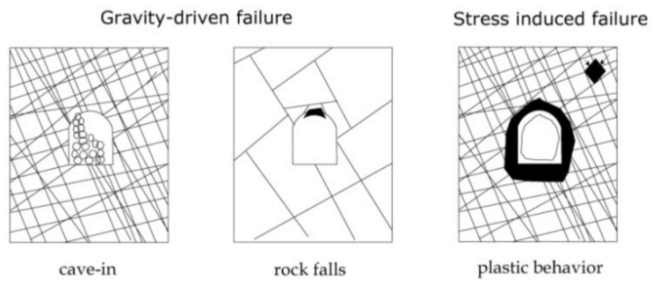

Fig. 1. Rock behavior types in shallow depth rock tunnel

Plastic deformation indicates that under low confining pressure, shear failure progressively increases and causes plastic displacement. It could also be defined as the shear displacement of the ground which causes the tunnel periphery to move inward and is initially caused by redistributed stress after excavation. Cave-in indicates an inward, quick movement of a large volume of rock fragments or pieces. For blocky rock condition, rock fall was expected, indicating that the rock blocks move down driven by gravity.

3. Quantifying rock behavior by AHP and FDM

3.1. Procedure of AHP and FDM

The Analytic Hierarchy Process (AHP, Saaty, 1980) has been one of the most widely used Multiple Criterion Decision Making (MCDM) tools. It has been applied to solve complex and unstructured geotechnical problem. In the field of tunnel engineering, it has been applied to select the optimum support design and to evaluate rock mass quality (Chen and Liu, 2007). We applied it to obtain weighting of parameters affecting rock behavior and to qualify rock behavior. It required group decision making to accelerate the consensus of various opinions from experts on tunneling. The Fuzzy Delphi Method (FDM, Kaufmann and Gupta, 1988) was applied to transform subjective data of experts into quasi-objective data. Figure 2 shows the procedure of AHP and FDM.

In the first stage, goal was decided. The goal was to determine weighting of parameters affecting rock behavior and priority on rock behaviors such as rock fall, cave-in, and plastic deformation in shallow depth rock tunnel.

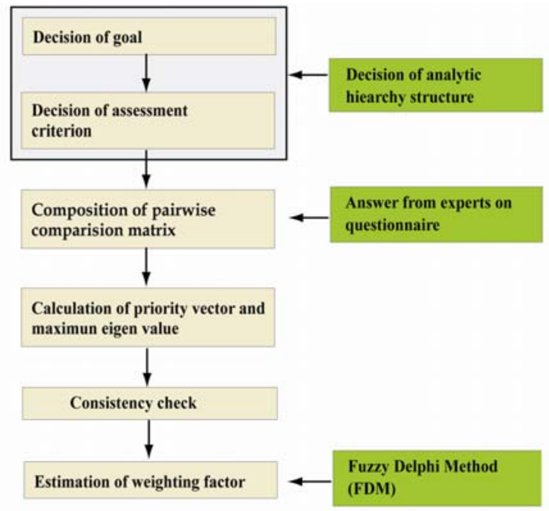

Fig. 2. A flow chart of AHP and FDM

In the second stage, assessment criterion on goal was decided. It was divided into primary criterion and secondary criterion. Primary criterion was related to rock mass intrinsic parameter, rock mass extrinsic parameter, and excavation parameter (Fig. 3). As secondary criterion, RQD, joint surface condition, and intact rock strength was related to rock mass intrinsic parameter; ground stress, groundwater, and earthquake was related to rock mass extrinsic parameter; excavation size was related to design parameter. In the third stage, pairwise comparison matrix was constructed based on answer from experts through questionnaire. Satty (1980) suggested a scale of 1-9 when comparing two component. The score of a_{ij} in the pairwise comparison matrix represents the relative importance of the component on row (i) over the component on column (j). The reciprocal value of the expression $(1/a_{ij})$ means that the component j is more important than the component i. In the fourth stage, priority vector and maximum eigen value were calculated. In the fifth stage, consistence check was performed to verify the confidence on experts' answer. AHP method is well documented in a standard text (Saaty, 1980).

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix} \quad (1)$$

Finally, the FDM was applied to compute fuzzy weights of parameters. To reflect particular degrees of uncertainty regarding the decision-making process, the α-cut concept was applied to compute the triangular fuzzy numbers (TFNs) $\tilde{a}_{ij}$ (Fig. 4). In the practical application, $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$ were used to indicate the decision-making conditions that had pessimistic, moderate, and optimistic view.

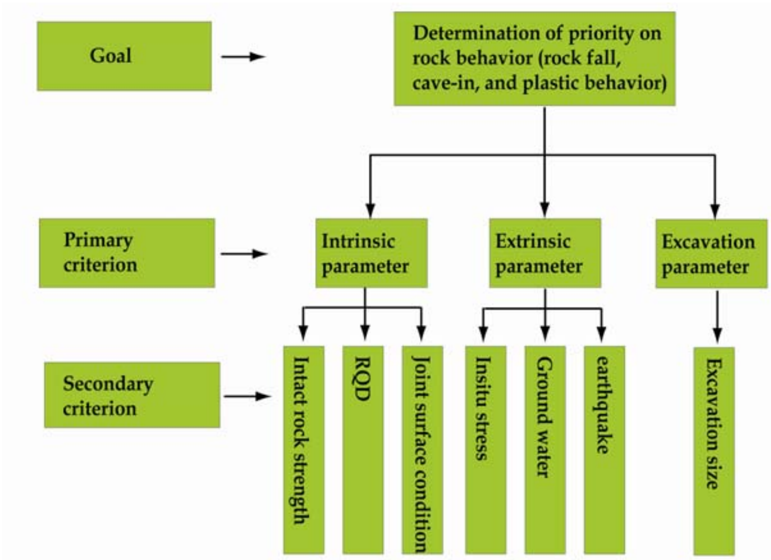

Fig. 3. Analytic Hierarchy Structure to decide the weighting factor of parameters on rock behavior

$$\tilde{a}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}) = \left(p_{ij} + \alpha(m_{ij} - p_{ij}), m_{ij}, o_{ij} - \alpha(o_{ij} - m_{ij})\right) \quad (2)$$
$$p_{ij} = Min(\delta_{ijk}), k = 1, \cdots, n \quad (3)$$
$$m_{ij} = \left(\prod_{k=1}^{n} \delta_{ijk}\right)^{1/n}, k = 1, \cdots, n \quad (4)$$
$$o_{ij} = Max(\delta_{ijk}), k = 1, \cdots, n \quad (5)$$

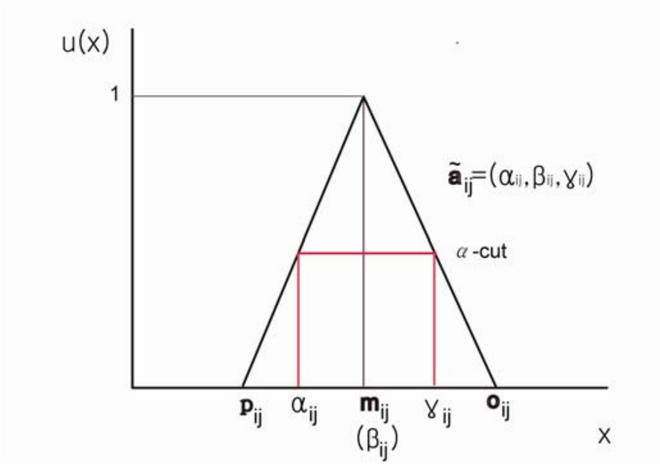

Fig. 4. Membership function of Fuzzy Delphi Method

Where, $p_{ij} \leq m_{ij} \leq o_{ij}$, $p_{ij}, m_{ij}, o_{ij} \in [1/9,1] \cup [1,9]$ and p_{ij} indicate the lower bound and o_{ij}

indicates the upper bound. δ_{ijk} indicates the relative intensity of importance of expert k between activities i and j. n is number of experts in consisting of a group.
Fuzzy positive reciprocal matrix $\widetilde{A}$ is obtained.

$$\widetilde{A} = [\tilde{a}_{ij}],\ \tilde{a}_{ij} \times \tilde{a}_{ji} \approx 1, \forall i, j = 1,2,\cdots,n \tag{6}$$

The relative fuzzy weights of the evaluation factors are calculated.

$$\tilde{Z}_i = [\tilde{a}_{ij} \otimes \cdots \otimes \tilde{a}_{in}]^{1/n},\ \widetilde{W}_i = \tilde{Z}_i \otimes (\tilde{Z}_i \oplus \cdots \oplus \tilde{Z}_n)^{-1} \tag{7}$$

$$\tilde{a}_1 \otimes \tilde{a}_2 \cong (\alpha_1 \times \alpha_2, \delta_1 \times \delta_2, \gamma_1 \times \gamma_2) \tag{8}$$

The symbol $\otimes$ here denotes the multiplication of fuzzy numbers and the symbol $\oplus$ here denotes the addition of a fuzzy numbers. $\widetilde{W}_i$ is row vector in consist of a fuzzy weight of ith factor. $\widetilde{W}_i = (\omega_1, \omega_2, \ldots, \omega_n), i = 1,2,\ldots,n$ and W_{ir} (r=1: rock fall, r=2:cave-in, r=3: plastic deformation) is a fuzzy weight of ith factor. The defuzzification is based on geometric average method.

$$W_{ir} = \left(\prod_{i=1}^{3} \omega_j\right)^{1/3} \tag{9}$$

3.2. Determining weights of parameters affecting rock behavior

We asked twenty-five experts to answer questionnaire. They composed of engineers who have worked in engineering consultant companies or construction companies and university professors in rock mechanics. Based on the survey responses, the weighting of parameters affecting rock behaviors was determined by the AHP and the FDM in Section 3.1 (Table 3-4). In the practical application, $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$ were used to indicate the decision-making conditions that had pessimistic, moderate, and optimistic view.

Table 3. Weighing of the primary parameters according to rock behaviors

Primary Parameters	Rock fall			Cave-in			Plastic deformation		
	α=0.0	α=0.5	α=1.0	α=0.0	α=0.5	α=1.0	α=0.0	α=0.5	α=1.0
Rock mass intrinsic parameter	0.47	0.48	0.63	0.44	0.45	0.60	0.28	0.24	0.28
Rock mass extrinsic parameter	0.22	0.19	0.15	0.24	0.22	0.17	0.50	0.54	0.53
Excavation parameter	0.31	0.33	0.22	0.32	0.33	0.23	0.21	0.22	0.19

Table 3 shows that rock mass intrinsic parameter dominates rock fall and cave-in behavior, whereas rock mass extrinsic parameter dominates plastic deformation. Table 4 shows that tunnel span, joint surface condition, and RQD in the order dominate rock fall and cave-in behavior, whereas groundwater, tunnel span, and ground stress dominate plastic deformation.

Table 4. Weighting of the secondary parameters according to rock behaviors

Secondary Parameters	Rock fall			Cave-in			Plastic deformation		
	α=0.0	α=0.5	α=1.0	α=0.0	α=0.5	α=1.0	α=0.0	α=0.5	α=1.0
UCS	0.08	0.05	0.07	0.06	0.05	0.06	0.04	0.03	0.03
RQD	**0.18**	**0.18**	**0.25**	**0.16**	**0.17**	**0.24**	0.11	0.09	0.11
Joint condition	**0.22**	**0.24**	**0.32**	**0.22**	**0.24**	**0.30**	0.14	0.13	0.14
Stress condition	0.07	0.06	0.05	0.08	0.06	0.06	**0.17**	**0.15**	**0.18**
Ground water condition	0.09	0.09	0.07	0.10	0.10	0.08	**0.21**	**0.25**	**0.27**
Earthquake	0.05	0.05	0.02	0.06	0.05	0.03	0.12	0.13	0.08
tunnel span	**0.31**	**0.33**	**0.22**	**0.32**	**0.33**	**0.23**	**0.21**	**0.22**	**0.19**

3.3. Calculating rock behavior index (RBI)

Rock behavior index (RBI) could be used as a potential indicator on rock behavior expected in shallow depth rock tunnel. It was expressed as linear combination of the parameter's weighting (w_{ir}) and its respective rating (P_i) as shown in Table 5.

$$RBI_r = (100 - \sum_{i=1}^{n} W_{ir} \frac{P_i}{P_{max}}) \quad (10)$$

Where, P_{max} is the maximum value a parameter can take as normalization factor.

The RBI ranges from 0 to 100 (Table 6). In a more explicit manner, five major categories could be classified. If RBI value is more than 60, the possibility of rock behavior can be high.

Table 5. Suggested rating of parameters affecting rock behavior in shallow depth tunnel

Parameters	Description	Classes	Rating 1	2	3	4	5
UCS (MPa)	-	P_1	< 25	25-50	50-100	100-250	> 250
RQD (%)	-	P_2	< 20	20-40	40-60	60 - 80	> 80
Joint surface	weathering+ infilling+JRC	P_3	< 3	3-8	8-12	12 - 15	> 15
Stress	Overburden /Tunnel Span	P_4	< 1.0	1.0-1.5	1.5-2.5	2.5-3.5	> 3.5
Groundwater	Ground water level - tunnel crown level (m)	P_5	> 15	10-15	5-10	0-5	< 0
Earthquake	Earthquake intensity	P_6	>0.25g	0.20-0.25g	0.15-0.20g	0.1-0.15g	< 0.10g
Excavation span	Joint spacing/Tunnel span	P_7	< 1/200	1/200-1/100	1/100-1/50	1/50-1/5	> 1/5

Table 6. Rock behavior index (RBI) catagories

Rock Behavior Index (RBI)	0~20	20~40	40~60	60~80	80~100
Linguistic terms	Very Low Probable	Low Probable	Moderately Probable	High Probable	Very High Probable

4. Case Study

Seoul metro Line 9 will be located in Kangnam-Gu, Seoul. The total length of the case study site of Seoul metro Line 9 is 1.77km with a tunnel length of 1.41km. The tunnel will be excavated by conventional drill and blasting method. Gneiss was mostly distributed in this site and four fracture zones were estimated from boring, geophysical prospecting and geological survey (Fig. 5). Most of the tunnel sections were double-lane track sections (PD). Sections connected to the tunnel station were enlarged sections (PW) and 2-arch sections. The overburden height ranged from 16 to 38m. The depth of tunnel excavation was mostly composed of rock masses.

The AHP and the FDM was applied to qualify rock behavior. Expected rock behaviors were considered as rock fall, cave-in, and plastic deformation. The seven parameters influencing rock behaviors were regarded as three rock mass intrinsic parameters (intact rock strength, RQD, and joint surface condition), three rock mass extrinsic parameters (ground stress, groundwater, and earthquake), and one excavation parameter (excavation span). In the FDM, α-cut was regarded as 0.5 to indicate the moderate decision-making conditions. Table 8 shows suggested rating of parameters affecting rock behavior as shown in Table 6 and RBIs according to support pattern. From estimated RBIs, possibility on rock fall and cave-in was higher than that on plastic deformation in all sections. More detailed analysis should be performed in the sections that have more than 60 RBI value.

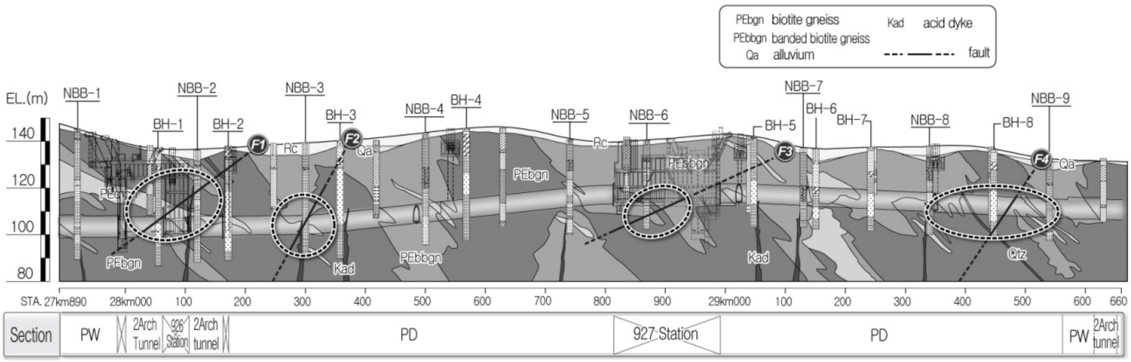

Fig. 5. Geology and tunnel longitudinal section (PD: double lane track section, PW: enlarged section)

Table 7. Suggested rating of parameters and rock behavior index (RBIs) according to support pattern of PD section

support pattern	Site position	P_1	P_2	P_3	P_4	P_5	P_6	P_7	RBI_1	RBI_2	RBI_3
PD-4	28,170	2	0	2	3	2	2	1	66.25	65.75	56.75
PD-5A	28,200	3	2	2	3	3	2	1	53.25	53.00	45.00
PD-4	28,300	2	0	1	3	4	2	0	76.00	74.75	53.75
PD-5B	28,350	1	1	1	3	4	2	1	64.25	63.25	46.25
PD-5A	28,430	2	0	1	3	3	2	1	70.00	69.00	54.00
PD-4	28,500	2	2	2	3	4	2	1	52.25	51.75	39.75
PD-5A	28,550	1	1	2	2	3	2	1	62.00	61.50	53.00
PD-5A	29,000	2	0	1	2	3	2	1	71.50	70.50	58.00
PD-5A	29,200	3	2	2	2	3	2	2	46.50	46.25	43.25
PD-3B	29,540	2	1	2	2	2	2	1	63.00	62.75	58.25

5. Conclusion

Quantitatively to identify rock behavior in shallow depth tunnel, rock behavior index (RBI) was suggested by combining the Analytic Hierarchy Process (AHP) and the Fuzzy Delphi Method (FDM). The weighting of parameters affecting rock behaviors could be computed through this method. Pairwise comparison matrix, the basis of AHP, was constructed from the questionnaire responses of 25 experts. The FDM was applied to overcome uncertainty of expert's judgment. Rock behavior types were regarded as rock fall, cave-in, and plastic deformation. Seven parameters influencing on rock behavior were determined: three rock mass intrinsic parameters (UCS, RQD, joint surface condition), three rock mass extrinsic parameters (ground stress, ground water, earthquake), and one excavation parameter (excavation span). From the results of estimating weighting of parameters, rock mass intrinsic parameter and design parameter dominates rock fall and cave-in behavior, whereas rock mass extrinsic parameter dominates plastic deformation. We applied this proposed method to basic design of Seoul Metro Line 9 and quantified rock behavior into RBI on fall, cave-in, and plastic deformation. From estimated RBIs, possibility on rock fall and cave-in was higher than that on plastic deformation in all sections.

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Reference

1. Poschl I., and Kleberger J., 2004, Geotechnical risk in rock mass characterization-a concept, Geotechnical risks in rock tunnels, 145-154.
2. Cai M., Kaiser P. K., Uno H., Tasaka Y., and Minami M., 2004, Estimation of rock mass deformation modulus and strength of jointed hard rock masses using the GSI system, International Journal of Rock Mechanics & Mining Sciences 41, 3-19.
3. Martin C. D., Kaiser P. K., Christiansson R., 2003, Stress, instability and design of underground excavations, International Journal of Rock Mechanics & Mining Sciences 40, 1027-1047.
4. Goricki A., Schubert W., Riedmueller G., 2004, New developments for the design and construction of tunnels in complex rock masses, International Journal of Rock Mechanics and Mining Sciences Volume 41, Supplement 1, 720-725
5. Palmstrom A. & Stille H., 2007, Ground behavior and rock engineering tools for underground excavations, Tunnelling and Underground Space Technology 22, 363-376.
6. Stille H. & Palmstrom A., 2008, Ground behavior and rock mass composition in underground excavations, Tunnelling and Underground Space Technology 23, 46-64.
7. Goel R. K., 2001, Status of tunnelling and underground construction activities and technologies in India, Tunnelling and Underground Space Technology 16, 63-75.
9. Kaiser P. K., Diederichs M. S., Martin C. D., Sharp J., and Steiner W., 2000, Underground works in hard rock tunneling and mining, An international conference on geotechnical and geological engineering.
10. Sakuri S., 1997, Lessons learned from field measurements in tunnelling, Tunnelling and Underground Space Technology 12, 453-460.
11. Singh M., Singh B.,Choudhari J., 2007. Critical strain and squeezing of rock mass in tunnels, Tunnelling and Underground Space Technology 22, 343-350.
12. Benardos A. G. and Kaliampakos D. C., 2004, A methodology for assessing geotechnical hazards for TBM tunneling-illustrated by Athens Metro, Greece, International Journal of Rock Mechanics & Mining Sciences 41, 987-999.
13. Yavuz M., Iphar M., and Once G., 2008, The optimum support design selection by using AHP method for the main haulage road in WLC Tuncbilek colliery, Tunnelling and Underground Space Technology 23, 111-119.
14. Chen C. S. and Liu Y. C., 2007, A methodology for evaluation and classification of rock mass quality on tunnel engineering, Tunnelling and Underground Space Technology 23, 377-387.
15. Ayalew L., Yamagishi H., Marui H., and Kanno T., 2005, Landslides in Sado Island of Japan: Part Ⅱ. GIS-based susceptibility mapping with comparisons of results from two methods and verifications, Engineering Geology 81, 432-445.
16. Liu Y. C. and Chen C. S., 2007, A new approach for application of rock mass classification on rock slope stability assessment, Engineering Geology 89, 128-143.
17. Saaty T., 1980, The Analytic Hierarchy Process. McGraw-Hill International Book Company, New York.
18. Kaufmann A. and Gupta M.M, 1998, Fuzzy Mathematical Models in Engineering and Management Science. North-Holland, Amsterdam.