The effect of advertising on sales
-Considering aggregated data bias-

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Abstract

“How does advertising affect sales?” is the fundamental issue of modern advertising research. There is an interesting issue for estimating carry over effects of advertising on sales, and the aggregated data biases exist in the duration of advertising effect.

This research suggests a modified model at micro-data using Koyck model (Koyck 1954) by estimated model the aggregate data, and empirically shows the aggregated data bias.

Our modified model with the aggregated level of actual data is more appropriate than the base model for micro-data. The result shows that it is very important to consider the disaggregated data level in the analysis of dynamic effects of advertising such as lagged effects.

Introduction

One of the commonly raised questions in business world is “how much should we spend on advertising?” and in order to answer this question adequately, we need to consider a fundamental issue like “How does advertising affect sales?”. Therefore, the study of relationship between promotional efforts and sales response has been one of the cornerstones of advertising research.

There are many theory-based normative models (Sethi 1979), however, only a few models contain the dynamic process of carryover effect. As early as the 1950s and 1960s, significant amount of research began to focus on the carryover or ‘lagged” effects of advertising in one period on sales in subsequent period (Jastram 1955; Palda 1964). However, until recently carryover effect was not precisely specified in most models and the coefficient of determination was not always reported (Assmus, Farley and Lehmann1984).

It is well known in marketing and econometrics that temporal data aggregated data bias has estimated in the duration of advertising effect researches (e.g., Clarke 1976, 1982). The bias is due to the specification error (Bass and Lone 1983, Rao 1986, Weinberg and Weiss 1982). In this research, we try to find a better solution of aggregate bias by modifying the original model applicable to micro-data. The purpose of research is 1) to develop a optimal model for aggregated advertising data by modifying the classical ‘Koyck model’ (Koyck 1954), 2) comparing the fit of the modified models and 3) empirically revealing aggregated biases with Yogurt advertising expenditure Data.

Literature review

Advertising-sales effect (Carryover Effect)

“This is clearly an important managerial issue given the human and dollar resources spent on advertising” (Leone, 1995, p 141), since measuring effectiveness of advertisement expenditures on sales are the biggest concerns in practice, many researchers pay attention to the shape of response function of advertisements on sales. There was a variety of econometric models identified how advertising affects sales which estimated parameters of general demand functions from early 1960s (Assums, Farley and Lehmann, 1984; Bendixen, 1993).

Morarty (1983) argues that current expenditures on marketing activities often do not have entire impact on sales in the same period in which they are implemented. Kluyver and Brodie(1987) indicate that lagged advertising is more significant than other marketing activities. Assmus, Farley and Lehmann (1984) assessed the effect of advertising on sales through Meta-Analysis and indicated that an appropriate model in these situations generally should incorporate carryover effects.

A number of technical issues related to the measurement of these duration intervals have been studied in many previous literatures (Bass and Clarke 1972, Clarke 1973, Palda 1964, Weiss and Windal 1980) and they try to measure the cumulative effects of advertising expenditures with some form of a lagged-variable response model e.g., (Weiss, Weinberg, and Windal 1983). Some studies consider advertisement’s “carry-over effect” as the delayed response effect and customer holdover effect. (Lilien, Kotler1983; Simon and Arndt 1980; Little 1979, Hahn
et al. 1992). “How long the carryover effect of advertising on sales persists?” become the research issue. Clarke (1976) points out for low priced products, the carryover effect of advertising lasts a matter of months rather than years. Moriarty (1983) provides preliminary evidence that for some durables, advertising effects may have a duration interval that exceeds a year. Umoshe et.al. (1990) indicate the carryover effect with several product categories for monthly and bimonthly measurement periods. Thereby, the parameter estimation with different level of aggregation (annually, monthly, weekly or daily) becomes an important issue in the advertising carryover effect studies.

Aggregated Bias Estimation

Previous research focus has primarily been on (a) the influence of disturbance structures and their characteristics on estimated carryover effects (Houston and Weiss 1975, Clarke and McCann 1977, Weiss and Windal 1980), and (b) the testing and evaluation of various distribution lag functions (Bass and Clarke 1972, Weiss and Windal 1980).

Clarke (1976) firstly observed the systematic relationship between the estimates of the duration interval and the level of aggregation. He seeded a notion that the purchase interval was the true data interval and using these small disaggregated data are not good at measuring carryover effects. Seven years later, Weiss et al. (1983, p.279) said, “An emerging convention seems to consider...[the true micro interval]...to be the interval between purchases.” Some research have investigated temporal aggregation and suggested approach to minimize the data interval bias either through a continuous (Blattberg and Jeuland, 1981; Rao, 1986) or discrete time framework (Bass and Leone, 1983, 1986; Weiss et al, 1983; Srinivasan and Weir, 1988; Vanhonacker, 1983, 1987; Russell, 1988; Givons and Horsky 1990).

Before 1980s, research had attempted to develop the micro period aggregate specification with partial adjustment model (Theil, 1954; Mundlak, 1961; Moriguchi 1970), but most of these research focused on the annual data estimation, Weiss, Weinberg and Windal (1983) developed a microlevel brand loyal model to estimate the narrow aggregation (monthly) parameter, and they indicated their model dealt well with aggregated bias. Vanhonacker (1983) among others pointed out that neither mathematical proof nor any theory can support this premise. The econometrics literature has also not reveal aggregated bias empirically.

Model specification

1) Basic Koyck Model

\[ S_t = \alpha + \beta_0 \cdot A_t + \beta_1 \cdot A_{t-1} + \cdots + \beta_{i-1} \cdot A_i \]  (1)

Equation (1) is the underlying structure of the basic (finite) Koyck Model.

The most important assumption of Koyck Model is Equation (2) which has the constant decay parameter ($\lambda$).

\[ \beta_{it} = \lambda \cdot \beta_i \]  (2)

Based on Equation (1) and (2), we can get the original Koyck Equation (3) which is sales response model for the specific advertising budget.

\[ \hat{S}_t = \alpha \cdot (1 - \lambda) + \beta \cdot A_t + \lambda S_{t-1} \]  ($\beta = \beta_0$)  (3)

2) Modified Koyck Model 1

The basic Koyck Model is useful to understand the relationship between advertising rates and sales response rates; however, actual advertising expenditure data cannot directly apply to the basic model. Since, for the most part, the exact initial advertising budget is difficult to capture precisely. (e.g., our data are time series data from Jan. 2004 to Dec.2006; it is left censored like most of actual data). We assume initial advertising budget of our data is the budge of Jan. 2004.

Then we can set up Equation (4) for the initial state and Equation (5) for the next state. Using Equation (4) and (5), we can get modified Koyck Model, Equation (6).

\[ \hat{S}_1 = \alpha + \beta_0 \cdot A_1 \]  (4)

\[ \hat{S}_2 = \alpha + \beta_0 \cdot A_2 + \beta_0 \cdot \lambda \cdot A_1 \]

\[ = \alpha + \beta_0 \cdot A_2 + \lambda (\hat{S}_1 - \alpha) \]  (5)
\[ \hat{S}_t = \alpha + \beta \cdot A_t + \lambda (\hat{S}_{t-1} - \alpha) \]

\[ = \alpha (1 - \lambda) + \beta \cdot A_t + \lambda \cdot \hat{S}_{t-1} \quad (6) \]

Equation (6) is very similar to Equation (3) except having different variable \( \hat{S}_{t-1} \) in the equation. That means Equation (3) uses the actual data \( S_{t-1} \) for the parameter estimation, whereas Equation (6) uses the estimated result \( \hat{S}_{t-1} \).

3) Modified Koyck Model 2

There are some limitations with Equation (6). Firstly, basic Koyck Model represents the carry-over effects of advertising rates, but can’t represent the lag-effects. Since lag-effects describe the situation where one variable is correlated with the values of another lagging variable at later times, we cannot expect to get the biggest advertising effects at that expected time of periods but only after having time lags. Secondly, general temporal aggregate data just include monthly advertising budgets and sales, however, the actual advertising didn’t launch at the last day of month. We try to resolve this problem with the Modified Koyck Model 2.

[Figure 1] shows the relationship between advertisement budgets and advertisement rates. \( T_1, T_2, T_3, \ldots \) represent the time intervals and they contain several advertisement rates in each interval. If this company runs daily advertisements, then \( T \) is 30. We emphasize both the aggregated advertising budgets and disaggregated advertising rates.

[Figure 1]

![Disaggregated Advertising rates](image1)

Suppose each interval has the same number of advertisement. (e.g. daily = 30, weekly = 4). Then, advertisement budget can be aggregated by Equation (7).

\[ A_{T_k} = A_{T_{k,1}} + A_{T_{k,2}} + \cdots + A_{T_{k,k}} \quad (7) \]

\( T_s \) : The actual data interval

\( k \) : The number of advertising in the \( T_s \)

\( A_{T_s} \) : Aggregated advertising budgets for \( T_s \)

\[ A_{T,s,k} : k^{th} \text{Disaggregated advertising in the } T_s \]

Therefore, based on Equation (7), we can modify the Koyck model to Equation (8).

\[ \hat{S}_{T_s} = \alpha + \beta \cdot A_{T,s,1} + \beta \cdot \lambda \cdot A_{T,s,2} + \cdots + \beta \cdot \lambda^{k-1} \cdot A_{T,s,k} \quad (8) \]

For example,

\[ \hat{S}_{T_s} = \alpha + \beta \cdot \frac{A_{T,s}}{k} (1 + \lambda + \cdots + \lambda^{k-1}) = \alpha + \beta \cdot \frac{A_{T,s}}{k} \frac{(1 - \lambda^k)}{1 - \lambda} \quad (9) \]

Assume \( A_{T,s,k} = \frac{A_{T,s}}{k} \), then equation (9) can be simply modified.

\[ \hat{S}_{T_s} = \alpha + \beta \cdot \frac{A_{T,s}}{k} (1 + \lambda + \cdots + \lambda^{k-1}) = \alpha + \beta \cdot \frac{A_{T,s}}{k} \frac{(1 - \lambda^k)}{1 - \lambda} \quad (10) \]

From the method of modified Koyck model 1, we can get the equation (11).

\[ \hat{S}_{T_s} = \alpha (1 - \lambda^k) + \lambda^k \cdot \hat{S}_{T,s} + \beta \cdot \frac{A_{T,s}}{k} \frac{(1 - \lambda^k)}{1 - \lambda} \quad (11) \]

In the final proposed model (Equation 11), we used disaggregated advertising rates in our equation whereas sales were not measured by disaggregated rates, even though we can get the disaggregated level of sales, it would make the model too complicated. Thus, this will be the major limitation in our research. Basically the Koyck model cannot capture delayed-response effects which are a slightly different from carry-over effect. [Figure 2] shows the delayed-response effect means that the highest response rate can be achieved after having marketing activities in previous periods.

[Figure 2]

![Response after marketing](image2)

We can analyses and verify our model by using empirical data, even though our proposed model has some technical limitations, since we believe our model can capture the effectiveness of advertising on sales with good theoretically supporting backgrounds.

5. Data collection

Data was provided by a major dairy product company in Korea. The data contains actual sales &
advertising expenditures of yogurt product from January 2004 to December 2006.

**Table 1**

<table>
<thead>
<tr>
<th>Content</th>
<th>Period</th>
<th>Analysis Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>-product sales</td>
<td>JAN,04–DEC,06</td>
</tr>
<tr>
<td>Advertising</td>
<td>-advertising</td>
<td>JAN,04–DEC,06</td>
</tr>
</tbody>
</table>

<Table 1> is a summary of data collection (Actual Advertising and sales), since yogurt is frequent purchased product category and relatively stable. In this research, for simple model estimation we only use the product sales and advertising expenditure.

Despite the fact that salespeople play a very important role in marketing activity, since yogurt product’s door-to-door sales are dominant in Korea. There has not been a change in the number of salespeople. In addition, interestingly, there is no price promotion in our data what so ever. Thus, we can conclude that advertising is the main factor that affects sales in our data and we can safely say that in our study other marketing mix is controlled appropriately.

**Analysis & Result**

We test 4 models (basic kooyck model, modified kooyck model 1, modified kooyck model 2 (k=4 weekly), modified kooyck model 2 (k=30, daily)) which are already explained in the Model specification Section. In this test, we only estimated the parameter by using LSE method with EXCEL Solver and did not test the significance of parameters. [Table 2] is the summary of parameters estimation and model fits.

From [Table 2], we realize that modified model 2 (k=30, daily) is the best model (in fact it is slightly better). This indicates that means analyzing the aggregated data by using the modified model which considered the aggregated level of actual data is more appropriate than using the basic model for micro-data.

![Figure 3] Basic Model

![Figure 4] Modified Model 2 K=30

[Figure 3] and [Figure 4] show the prediction performance for basic and modified model 2 (k=30). From [Figure 2] and [Figure 3], we can observe that modified model 2 perform slightly better than basic model, although both models are not good at predicting future performances. Here are some supporting reasons. First of all, as we previously mentioned in this paper, advertising has lagged effect on sales. And, unfortunately, the kooyck model is not proper to capture this effect.

Another explanation is that the media effect and prints effect of advertising. Even though we controlled marketing mix like price cut and change of sales members, we could not control the media (TV, prints, etc), prints effects and content of advertisements in our model. To improve our model, we need to consider these factors in our model in the future.

**Conclusion**

From the section of model specification and analysis & result, we show that the modified model which considers the aggregated level of actual data is more appropriate than the basic model using micro-data. Additionally, we can confirm that aggregated bias is due to specification error caused by assuming that the original model at the micro-data applies as well to the aggregate data. Furthermore, our model finds the carry-over effect during the inter-aggregated interval. This implies that considering data at the disaggregated level is very important in the analysis of dynamic effects of advertising like carry-over effects.
Reference
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Russel, GJ (1988),"Recovering measures of advertising carryover from aggregate data: The role of the firm’s decision behavior,” Marketing Science, 7(3) 252-270.