# 마이크로믹서의 혼합해석을 위한 매핑법 적용

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Application of a mapping method for mixing analysis of micromixers

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### Abstract

Under typical operating conditions, flows in microfluidic devices are laminar and molecular diffusion across the channels is slow, which makes an efficient mixing in microfluidic devices difficult to achieve. The mechanism to achieve effective mixing in laminar flows is that of repetitive stretching and folding. Essential is to generate spatially periodic flows with crossing cross sectional streamlines. A mapping method is employed to analyze mixing in micromixers, enabling us to investigate the progress of mixing both qualitatively and quantitatively. The progress of mixing is characterized by a measure of mixing, called the discrete intensity of segregation. The mapping method is applied to mixing in such micromixers as the staggered herringbone mixer, the barrier embedded micromixer, and the three-dimensional serpentine channel to demonstrate the capability of the numerical scheme to tackle general mixing problems in microfluidic devices.

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## 1. Introduction

Most of the efficient chaotic micromixers proposed in the last decade are based on a mechanism called the baker's transformation as known from the working principle of static mixers frequently applied in different polymer processing operations. The mechanism to achieve effective mixing in laminar flows is that of repetitive stretching and folding, and flows with low Reynolds and high Péclet numbers, which are typically for micromixers, require application of this mechanism to obtain sufficient mixing. Essential in micromixers is to generate a spatially periodic three-dimensional flow with

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transversal velocity components. Using a mapping method, mixing in three well-known mixers, the staggered herringbone mixer (SHM), the barrierembedded mixer (BEM), and the three-dimensional serpentine channel (TSC), will be reviewed.

## 2. Mapping method

The optimum design of any mixer represents the situation where a required mixing is achieved using a minimum amount of energy or pressure drop. The question is how to find the optimum geometry and CFD analyses for a large number of different threedimensional geometries are usually not an attractive and efficient technique. The mapping method, which is based on an original suggestion by Spencer and Wiley [1], is an elegant engineering approach to find this optimum. Anderson and Meijer [2], Galaktionov et al. [3], and Kruijt et al. [4] successfully developed and utilized the mapping method to optimize macro-mixing devices like the lid-driven cavity flow and the Kenics static mixer. The original mapping method has been modified such that the coefficients of the mapping matrices can be computed in a simpler way (see Singh et al. [5]). The idea is as follows: a distribution matrix  $\varphi$  is formed to

store information about the distribution of fluid from one cross-section to the next due to a specified flow. To define the coefficients of the distribution matrix, the initial cross section of the flow domain is subdivided into a large number N of discrete cells of identical size. During flow, the material from a donor cell is transferred to different recipient cells and the fraction of material that is transferred from the donor cell to a recipient cell gives the distribution coefficient of the donor cell with respect to that recipient cell. To approximate the coefficients of the mapping matrix a number of passive markers inside all cells are tracked. To determine the final distribution of markers, the markers are advected during the flow from z=0 to z=L, (see Fig. 1). Given a concentration distribution at z=0, the concentration at z=L is computed by a matrix-vector multiplication, i.e.,  $C_{z=L} = \Phi C_{z=0}$ .

## 3. Results and discussion

#### 3.1 Staggered herringbone mixer (SHM)

One of well-known passive micromixers is the SHM, the staggered herringbone mixer [6]. The mixer consists of two different basic units which both create a transversal flow consisting of two counter-rotating vortices. Material elements travelling down the mixer periodically go from one unit to the next, thus from one transversal flow to the next, and hence periodically stretch and fold. A complete analysis of mixing in the SHM is presented in Singh et al. [5] and here some main results are highlighted. For validation of the method, results from mapping are compared with the experimental and other numerical results (see Fig. 2).



Fig. 1 Illustration of the computation of the coefficient  $\Phi_{ij}$  in the mapping matrix  $\Phi$ .



Fig. 2 Evolution of the concentration in the SHM during 5 cycles; (a) mapping results, (b) experimental results from Stroock et al. [6] and (c) numerical results from Kang and Kwon [7]



Fig. 3 The barrier embedded micromixer (BEM). (a) Typical periodic unit of the BEM, (b) Two mixing protocols constituting the BEM.



Fig. 4 Evolution of mixing in micromixers consisting of the mixing protocols listed in Fig. 3. (a) A mixer consisting on only P1, (b) the BEM, a periodic repetition of P1 and P2, and (c) a variation of the BEM composed of an aperiodic sequence of P1 and P2 in a recursive way (see [7] for more details).

#### 3.2 Barrier embedded micromixer (BEM)

Next introduced is the barrier embedded micromixer (BEM), which consists of two repeating mixing protocols (see Fig. 3). The channel has grooves on the bottom surface and a barrier at the top [8]. Similar as for the SHM design, periodic units can be defined yielding alternating transversal flow, leading to chaotic mixing. Various combinations of two or more mixing sequences (functional modules) of the BEM provide numerous competitive designs. Again the mapping method can be used as an efficient tool to analyze various layouts of these micromixers, see for a detailed analysis of mixing in different BEM geometries Kang et al. [9]. The resulting mixing patterns are plotted in Fig. 4.

#### 3.3 Three-dimensional serpentine channel

As a last example of a passive microfluidic mixer we consider the three-dimensional serpentine channel [10]. The basic building block is a "C-shaped" channel (see Fig. 5). Figure 6 depicts the progress of mixing along the down-channel positions after 1, 2, 3, 4, and 10 mixing cycles for Re = 0.01, 10, 50, and 70 (see Singh et al. [5]). As the Reynolds number increases, stretching and folding of interfaces becomes more vigorous and it is evident that the flow above a critical Reynolds number (in this case Re<sub>c</sub>=50) is capable of producing the globally chaotic mixing, while in Stokes flow regime, the flow can not mix the two fluids due to the symmetry of the channel.



Fig. 5 Periodic unit ("C-shaped" structure) of the threedimensional serpentine channel [10].



**Fig. 6** Effect of increasing Reynolds number on the evolution of mixing patterns in the serpentine channel at several down-channel positions. (a) Re=0.01, (b) Re=10, (c) Re=50, (d) Re=70.

### 4. Summary

The principles of efficient mixing in laminar flows, which are repetitive stretching and folding (and followed by bringing back to the original position), lead to chaotic advection also in microchannels. Of course limitations exist in the design choice of geometrical complexity, and alternative routes to induce asymmetric transverse flows that alternate and therefore lead to crossing streamlines portraits, a prerequisite for exponential stretching in periodic hyperbolic points, had to be, and are, found. Both the SHM and BEM represent elegant solutions for improving mixing in low Reynolds number flows. For intermediate Reynolds number flows, secondary flows can generate transverse velocities, e.g. the Dean vortex in curved channels, resulting in efficient mixing. A prime example of such geometry is the three-dimensional serpentine channel. An engineering tool, called the mapping method, allows us to perform a large number of mixing computations in a very fast way, thus giving the tool to optimize mixing geometries or mixing protocols. Here, this method is used to analyze and optimize the most widely used passive mixer designs.

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