TRACKING FOR HIGH-ORDER DAMPING OF THIN BEAM OSCILLATION

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Abstract

An estimation of high-order damping in flexible multibody dynamic simulation is introduced in this paper. The suggested damping model based on the experimental modal analysis leads to more accurate correlation results comparing to the traditional linear damping model because it directly uses the modal parameters of each mode achieved from experiment even high frequency modes. The modal parameters until the 5th mode are extracted from the experimental modal testing of the flexible beam using a high speed camera. And using the measured damping ratio and natural frequency until the 5th mode, the generic damping model is constructed. Then, the ANCF (absolute Nodal Coordinate Formulation) simulation results are compared to experimental results until the 5th mode.

INTRODUCTION

The ANCF (Absolute Nodal Coordinate Formulation) which was introduced recently [1,2] can represent arbitrary large deformations easily because it use finite slopes as nodal variables from the inertial frame. Thus, the ANCF is well known as proper formulation to simulate large deformable structures. And there are numerous papers to verify their large deformable models with experiments [3,4]. But, All researchers of these papers used the linear damping model for the simplicity. And none of all have focused on high mode behaviors of their flexible structures. Because it is impossible for the linear damping model to predict high order behaviors, they had only focused first mode behavior. And Garcia-Vallejo suggested the internal damping model based on linear visco-elasticity for the ANCF [8]. He made an effort to make a damping model which can express no energy dissipation under rigid body motion. But because of their thin structure in large deformation problem, the external damping is more important than the internal damping in most mechanical application except hyperelastic or viscoelastic materials.

So, in this paper, a high-order damping in multibody dynamic simulation is introduced to correlate the high frequency behaviors. The suggested damping model based on the experimental modal analysis leads to more accurate correlation results comparing to the traditional linear damping model because it directly uses the modal parameters of each mode achieved from experiment even high frequency modes.

MODAL TESTING AND PARAMETERS IDENTIFICATION

Experimental setup

In this research, a thin spring-steel beam heat-treated to increase its strength and durability is selected. And the standard sectional properties of beam are shown in Table 1. In this experiment, the fundamental frequency of test beam is only 1.85[Hz]. Therefore, a high-speed camera is used to capture the motion instead of accelerometers. The high-speed camera, REDLAKE Motion Scope 1000s shown in Fig. 1, is used in this study. Vibration signal made by LabVIEW is transferred to the Exciter using PXI 4461 board. The large deformation experimental setup for the beam is constructed as shown in Fig. 1. The target point is necessary to track the beam position by using the high speed camera.



Fig. 1 Experimental Equipment Table 1 Properties of the beam

	Length	Width	Thickness	Density	Elastic Modulus
	[mm]	[mm]	[mm]	$[kg/m^3]$	[MPa]
Standard	500	5.0	0.50	8000.0	210,000
Optimum	500	5.0	0.49	8108.0	205,130

Modal testing and parameter identification

Due to the flexibility of the thin beam, the traditional modal testing is difficult to use. So, in this study, the damped decay test of free vibrating beam is used as a proper test procedure. The beam is located on the vertical direction and is excited along the horizontal direction with their natural frequencies until a steady states behavior occurs. When the beam reaches to a steady-state, a sudden drop of the excitation makes the beam vibrate with their natural properties. And then the natural frequency and damping ratio can be found with Eq. (1), which is a 1 D.O.F. free vibration equation.

$$x(t) = e^{-\zeta\omega_n t} \left(x_0 \cos(\omega_d t) + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d} \sin(\omega_d t) \right)$$
(1)

$$\min \Re\left(\omega_r, \zeta_r\right) = \sqrt{\sum_{i=0}^n \left(x_{\exp} - x_{ide}\right)^2}$$
(2)

where $\omega_d = \omega_n \sqrt{1 - \xi^2}$ is damped natural frequency. The modal parameters to be calculated are the values which the residue between Eq. (1) and test data is minimized such as Eq. (2). Table 2 shows the resultant modal parameters identified from the experiment results of beam.

	natural frequency	damping ratio
	[Hz]	[%]
1st mode	1.85	1.00
2nd mode	10.19	0.28
3rd mode	26.84	0.28
4th mode	53.20	0.23
5th mode	88.60	0.30

 Table 2. Identified modal parameters

MATCHING OF HIGH FREQUENCY MODES

Optimization to match natural frequency

Because the spring-steel used in this study has been heat treated, there is no guarantee that the natural properties of simulation coincide with those of experiment. Table 3 shows the difference of natural frequencies in the case of using standard properties of steel.

So, first of all, the optimization process had been carried out to find proper material properties. The beam used in this study has 20 finite elements, 84 coordinates and 4 constraint equations. Design variables are selected as the width and height of each beam elements and elastic modulus and density. And the variance of those design variables is limited within 10[%]. To correlate with natural characteristic, the objective function is chosen to minimize the residue of natural frequencies and modes from the 1st to the 5th mode. The resultant design variables are listed in Table 1, and Table 3 shows the optimized results.

Table 3. Optimized results of frequency analysis						
Notural Energ	Test [Hz]	Simulation [Hz]				
Natural Fleq.		Standard	Optimum			
1st mode	1.85	1.67	1.84			
2nd mode	10.19	10.47	10.18			
3rd mode	26.84	29.32	26.81			
4th mode	53.20	57.46	53.15			
5th mode	88.60	94.97	88.79			

Linear viscous damping model

Considering effects of internal and external dissipation, linear damping model is widely used in structural dynamic environment for the sake of simplicity. In this model, a particular form of proportional Rayleigh damping is implemented such as Eq. (3).

$$\mathbf{C}_{l} = \boldsymbol{\alpha} \mathbf{M} + \boldsymbol{\beta} \mathbf{K} \tag{3}$$

$$\alpha = \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)}{\omega_2^2 - \omega_1^2}, \ \beta = \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2}$$
(4)

where **M** is a mass matrix and **K** is a stiffness matrix. And values, α and β , depend on frequencies $\omega 1$ and $\omega 2$ as well as on damping ratios $\zeta 1$ and $\zeta 2$ for the first two modes of the system.

Generic viscous damping model

In the previous section, one can see that it is impossible to correlate the high frequency behavior of the structure using the linear damping model. The linear damping model can express the motion of only maximum two frequencies. So, a new damping model which can have frequency dependent properties and express the independent high frequency damping characteristic is necessary. So, in this study, frequency dependent generic damping model based on the experimental modal analysis is also introduced into flexible multibody dynamics [5,6].

From the identified modal parameters, the generalized damping matrix can be derived using the modal transformation. Modal transformation is the transformation from identified damping matrix in modal domain to general damping matrix in physical reference domain using mode vectors. With the undamped natural modal vector $\mathbf{\Phi}$, one can calculate the generic viscous damping matrix using orthogonal properties, as shown in Eq. (5). Eq. (6) shows the resultant generic damping model matrix.

$$\mathbf{C}_r = \boldsymbol{\Phi}^T \mathbf{C}_g \boldsymbol{\Phi}, \quad r = 1, \cdots, n \tag{5}$$

$$\mathbf{C}_{g} = \mathbf{\Phi}^{\mathsf{T}} \mathbf{C}_{r} \mathbf{\Phi}^{\mathsf{T}} = \mathbf{M} \mathbf{\Phi} [2\zeta_{r} \omega_{r}] [\mathbf{\Phi}]^{\mathsf{T}} \mathbf{M}$$
(6)

where ζ_r, ω_r are the critical damping ratio and natural frequency extracted from the experiment, respectively.

Results from ANCF simulation

Experiment and simulation with base excitation are carried out from the 1st mode to the 5th mode. Using marker tracking method, displacement of the beam is measured. In Fig. $2 \sim 4$, 'Experiment' means the measurement of modal

testing, 'Simulation 01', blue thin dashed line, means the results of the generic damping model and 'Simulation 02', red thick dashed line, means those of the linear damping model. Fig. 2 shows damped behaviors of experiment and simulation at the 1st and 2nd mode frequency. Both results are almost same. So, one can know that the identified parameters and generic damping model give good results. Due to two constraint parameters using the first two modes, the linear damping model also gives good results until the 2nd mode frequency. Fig. $3 \sim 4$ show the damped decay response of the 3rd, 4th and 5th mode frequency, respectively.



Fig. 2 Damped behavior of Exp. and Sim. : 1st mode(left), 2nd mode(right)



Fig. 3 Damped behavior of Exp. and Sim. : 3rd mode(left), 4th mode(right)



Fig. 4 Damped behavior of Exp. and Sim. : 5th mode

Those results show that the linear damping model can overestimate the damping characteristics comparing to those of the experiment. Because there is no any information of high mode frequency in linear damping model, the estimated results of linear damping model are the arbitrary values. Contrary to linear damping model, the generic damping model can estimate the damping characteristics well enough in spite of high frequency region because it directly uses the modal parameters of each mode achieved from experiment.

CONCLUSION

It is well-popular that it is difficult to measure and identify the frequency behaviors even the 2nd mode in the large deformation problem. That's the reason why most researchers have focused on the 1st mode. But, in this study, using the concept of global parameters, the proper modal testing and identification method are presented. And using proposed methods, the measuring of high frequency until the 5th mode, 89[Hz], has done. And this study also presents the proper correlation method with introducing the generic damping model. And this research shows the linear damping model which is used traditionally for the simplicity has not good performance in high frequencies. Through comparisons between numerical simulation and experiment, this study shows that the presented generic damping matrix can estimate the high frequency response well enough in flexible multibody dynamics undergoing large displacement.

REFERENCES

[1] Shabana A. A., "Flexible Multibody Dynamics: Review of Past and Recent Developments", Multibody System Dynamics, 1, 189-222 (1997).

[2] Shabana A. A., "Dynamics of Multibody Systems, 3rd Edition", Cambridge University Press, Cambridge, (2005).

[3] Yoo W. S., Lee J. H., Sohn J. H., Park S. J., Dmitrochenko O., Pogorelov D., "Large Oscillation of a thin cantilever beam: physical experiments and simulation", Nonlinear Dynamics, 34, 3-29 (2003).

[4] Yoo W. S., Park S. J., Dmitrochenko O., Pogorelov D., "Verification of Absolute Nodal Coordinate Formulation in Flexible Multibody Dynamics via Physical Experiments of Large Deformation Problems", Journal of Computational and Nonlinear Dynamics, 1, 81-93 (2006).

[5] Ewins, D. J., "Modal Testing: theory, practice and application, 2nd Edition", (Research studies press Ltd., 2000)

[6] McConnell, K. G., "Vibration Testing : Theory and Practice", (John Wiley & Sons, Inc., 1995)

[7] Takahashi Y., Shimizu N., Suzuki K., "Introduction of Damping Matrix into Absolute Nodal Coordinate Formulation", W-1-1-4, Proceeding of ACMD'02, (2002)

[8] Garcia-Vallejo D., Valverde J., Dominguez J., "An Internal Damping Model for the Absolute Nodal Coordinate Formulation", Nonlinear Dynamics, 42, 347-369 (2005).

[9] BerzeriM., Shabana A. A., "Development of simple models for the elastic forces in the absolute nodal coordinate formulation", Journal of Sound and Vibration, 235, 4, 539-565 (2000).