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Integrated Structural and PD-Control Optimization of Flexible Rotor Supported by Active Magnetic Bearings

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Key words: Optimal control, vibration control, flexible rotor, magnetic bearing

ABSTRACT

This paper proposes new searching algorithm for the optimal PD gains of flexible rotor supported by active magnetic bearings. Under the assumption of linearized bearing parameters with respect to PD gains, the performance index in quadratic form is defined and steepest descent method is adopted for determining local minimum. Moreover, the eigenpair sensitivity concept is utilized to evaluate the sensitivity of performance index. To evaluate the effectiveness of suggested algorithm, the finite element model is constructed and its reduced model is retained in modal domain. Given starting gains, the optimal gains are successfully found and the control performance is demonstrated by simulation to show the efficiency of the proposed method.

1. INTRODUCTION

During last few decades, active magnetic bearing (AMB) system has been drawing much attention for its broad applicability to high speed rotating machinery with non-contact support of a rotor. However, AMB has inherent instability due to negative position stiffness so that it needs feedback control, to render the controlled system stable, based on augmented model of both rotor's structural dynamics and AMB's electromagnetic dynamics which includes levitation controller [1].

Even though many modern control methods, such as H_∞[2], μ-synthesis[3] and so on, have been developed, typical PD control method is largely accepted for stable levitation controller of the rotor due to its simplicity and easy physical interpretation. When we control the rigid rotor-AMB system with PD controller, linear quadratic regulation (LQR) approach has been already developed well and can be applied successfully to determine optimal PD gains [1]. However, in case of flexible rotor of which some natural frequencies of flexible modes are below a bandwidth of the closed loop system, the LQ approach for PD gains determination has not been accomplished yet.

The rotor-AMB system is usually supported by each pair of upper and lower AMB in radial direction. It means that boundary bearing condition of the rotor is completely dependent on equivalent parameters as stiffness and damping; they are analytically derived as approximate linear functions of both P and D gains. Given bearing parameters and structural properties of the rotor, finite element model [4] can be constructed into exact description of whole system. However, the large dimension of this FE model generally limit direct use to control model, consequently reduced modal model can be used alternatively for controller design.

With varying reduced modal model according to PD gains, a given reduced model may not be valid anymore after changing PD gains. It implies that a procedure to obtain optimal PD gains in flexible rotor-AMB system should consider the interaction between rotor shaft's dynamics and bearing parameters, equivalently PD gains.

In this paper, to begin with, Simple modal model reduction method is suggested by using H2 norm evaluation. Next, sensitivity equation of eigenpair such as eigenvalue and eigenvector with respect to PD gains will be derived out which is used to approximate closed-loop system dynamics in first order [5]. Performance index will be subsequently defined similarly as that of conventional optimal control, and used to search suboptimal PD gains satisfying necessary condition for local minimum utilizing steepest descent search method.

The proposed optimal control method is applied to the flywheel system supported by AMBs in order to simulate the control performance subjected to levitation operation and impulse disturbances.

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2. Reduced modal model of rotor-bearing system and its eigenpair sensitivities

2.1 Finite element model and its reduced modal model by use of H2 norm evaluation

Typical rotor-AMB system can be modeled by finite element analysis as the equation of motion of isotropic rotor-AMB system can be described as [4]

$$\mathbf{M}_c\ddot{\mathbf{p}}+(\mathbf{C}_c+\mathbf{C}_b-j\Omega\mathbf{G})\dot{\mathbf{p}}+(\mathbf{K}_c+\mathbf{K}_b)\mathbf{p}=\mathbf{g} \qquad (2\text{-}1)$$

Where j and Ω are the imaginary number and rotational speed. $\mathbf{M}_c, \mathbf{C}_c, \mathbf{G}$ and $\mathbf{K}_c \left(\in \mathbb{C}^{2N\times 2N}\right)$ are the mass, structure damping, gyroscopic and structure stiffness matrices, respectively. $\mathbf{p}=\mathbf{y}+j\mathbf{z}\in\mathbb{C}^{2N\times 1}$ is the complex coordinate vector of each nodes and $\mathbf{g}\in\mathbb{C}^{2\times 1}$ is external forcing vectors. Finally, $\mathbf{C}_b$ and $\mathbf{K}_b\left(\in\mathbb{C}^{2N\times 2N}\right)$ represent equivalent bearing parameters as linear functions of each P and D control gains.

The state space form of Eq. (2-1) is

$$\mathbf{A}_0\dot{\mathbf{w}}=\mathbf{B}_0\mathbf{w}+\mathbf{f} \qquad (2\text{-}2)$$

where

$$\mathbf{A}_0=\begin{bmatrix}\mathbf{0} & \mathbf{M}_c\\ \mathbf{M}_c & \mathbf{C}_c+\mathbf{C}_b-\mathrm{j}\Omega\mathbf{G}\end{bmatrix}$$

$$\mathbf{B}_0=\begin{bmatrix}\mathbf{M}_c & \mathbf{0}\\ \mathbf{0} & -\mathbf{K}_c-\mathbf{K}_b\end{bmatrix},\ \mathbf{w}=\begin{bmatrix}\dot{\mathbf{p}}\\ \mathbf{p}\end{bmatrix},\ \mathbf{f}=\begin{bmatrix}\mathbf{0}\\ \mathbf{g}\end{bmatrix}$$

By solving self-adjoint eigenvalue problem of Eq. (2-2) and its resulting modal transformation, modal equation having 4N dof can be obtained as

$$\dot{\zeta}=\mathbf{A}\zeta+\mathbf{B}\mathbf{g},\ \ \mathbf{y}=\mathbf{C}\zeta \qquad (2\text{-}3)$$

where

$$\mathbf{A}=diag\left\{\lambda_1^B\ \ \lambda_1^F\ \ \lambda_2^B\ \cdots\ \lambda_{2N}^F\right\}$$

$$\mathbf{B}=\begin{bmatrix}{}_{b1}u_1^B & {}_{b1}u_1^F & {}_{b1}u_2^B & \cdots & {}_{b1}u_{2N}^F\\ {}_{b2}u_1^B & {}_{b2}u_1^F & {}_{b2}u_2^B & \cdots & {}_{b2}u_{2N}^F\end{bmatrix}^T \qquad (2\text{-}4)$$

and $\mathbf{C}=\mathbf{B}^T\left(\in\mathbb{C}^{2\times 2N}\right)$

and $\zeta\left(=\left\{\zeta_1^B\ \ \zeta_1^F\ \ \zeta_2^B\ \cdots\ \zeta_{2N}^F\right\}^T\right)$ is a complex modal state vector. Lower left subscripts denote nodes' number at bearing positions. λ and u are eigenvalue and modal input coefficient, i.e. eigenvector, respectively. Upper right superscripts B and F mean backward and forward direction of modes. Lower right subscripts mean corresponding mode number.

When the full modal model of the flexible rotor-AMB system is reduced by the modal truncation, 'cutting frequency' for mode selection is normally determined by considering a bandwidth of controlled closed loop system, physical disturbance condition and so on. Let the reduced modal model be given by

$$\dot{\zeta}_R=\mathbf{A}_R\zeta_R+\mathbf{B}_R\mathbf{g}\ ,\ \ \mathbf{y}=\mathbf{C}_R\zeta_R \qquad (2\text{-}5)$$

where right subscript R denotes reduced state or matrix.

Here, the error between full model and reduced model may be the measure how close the reduction is conducted from original model. In this study, H_2-norm concept are adopted for error evaluation.

For model error evaluation, at first, given system matrices of Eq. (2-3) and (2-5), state space realizations of both transfer functions $\mathbf{H}(s)\left(=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}\right)$ and $\mathbf{H}_R(s)\left(=\mathbf{C}_R(s\mathbf{I}-\mathbf{A}_R)^{-1}\mathbf{B}_R\right)$ are as follows:

$$\mathbf{H}(s)=\left[\begin{array}{c|c}\mathbf{A} & \mathbf{B}\\ \hline \mathbf{C} & \mathbf{0}\end{array}\right] \qquad (2\text{-}6)$$

$$\mathbf{H}_R(s)=\left[\begin{array}{c|c}\mathbf{A}_R & \mathbf{B}_R\\ \hline \mathbf{C}_R & \mathbf{0}\end{array}\right] \qquad (2\text{-}7)$$

A state space realization of error system is

$$\begin{aligned}\mathbf{E}(s)&=\mathbf{H}(s)-\mathbf{H}_R(s)\\ &=\left[\begin{array}{cc|c}\mathbf{A} & \mathbf{0} & \mathbf{B}\\ \mathbf{0} & \mathbf{A}_R & \mathbf{B}_R\\ \hline \mathbf{C} & -\mathbf{C}_R & \mathbf{0}\end{array}\right]\\ &=\left[\begin{array}{c|c}\mathbf{A}_E & \mathbf{B}_E\\ \hline \mathbf{C}_E & \mathbf{0}\end{array}\right]\end{aligned} \qquad (2\text{-}9)$$

Then, H_2-norm reduction error can be expressed as:

$$\begin{aligned}\|\mathbf{E}(s)\|_2&=\|\mathbf{H}(s)-\mathbf{H}_R(s)\|_2\\ &=\mathrm{tr}\left[\mathbf{C}_E\mathbf{P}_E\mathbf{C}_E^*\right]\end{aligned} \qquad (2\text{-}10)$$

$$\text{subject to } \mathbf{A}_E\mathbf{P}+\mathbf{P}\mathbf{A}_E^*+\mathbf{B}_E\mathbf{B}_E^*=0$$

where $\mathbf{P}$, the solution of Lyapunov matrix equation, is called as controllability grammian.

H_2 norm is commonly interpreted as the 2-norm of output resulting from applying unit impulses to each input channel. Therefore, this H_2 norm error, between the original system and the reduced system, may become an efficient index explaining how good the approximation is.

2.2 Eigenpair sensitivity with respect to PD control gains

The equation of motion of Eq. (2-1) can be re-written like

$$\mathbf{M}\ddot{\mathbf{p}}+\mathbf{D}\dot{\mathbf{p}}+\mathbf{K}\mathbf{p}=\mathbf{g} \qquad (2\text{-}11)$$

here $\mathbf{M}=\mathbf{M}_c$, $\mathbf{D}=\mathbf{C}_c+\mathbf{C}_b-j\Omega\mathbf{G}$ and $\mathbf{K}=\mathbf{K}_c+\mathbf{K}_b$.

Self-adjoint eigenvalue problem of Eq. (2.11) is described in complex domain as

$$\left|\lambda^2\mathbf{M}+\lambda\mathbf{D}+\mathbf{K}\right|=0$$
$$\left(\lambda^2\mathbf{M}+\lambda\mathbf{D}+\mathbf{K}\right)\mathbf{u}=0 \tag{2-12}$$

where $\lambda\left(=\lambda_1^B,\lambda_1^F,\ldots,\lambda_{2N}^B\right)$, $\mathbf{u}\left(=\mathbf{u}_1^B,\mathbf{u}_1^F,\ldots,\mathbf{u}_{2N}^B\right)$ are eigenvalue set and modal vector set respectively. Orthonormality condition may be satisfied as

$$\mathbf{u}^T\left(2\lambda\mathbf{M}+\mathbf{D}\right)\mathbf{u}=1$$
$$\mathbf{u}^T\left(\lambda^2\mathbf{M}-\mathbf{K}\right)\mathbf{u}=\lambda \tag{2-13}$$

In order to determine eigenvalue sensitivity, Eq. (2-13) is differentiated with respect to all PD control gains k_m, $m=p1,p2,d1,d2$, in which p and d mean P and D gain and 1 and 2 denote the first and the second bearing. Subscript $,k_m$ means derivative with respect to k_m.

$$\left(2\lambda\mathbf{M}+\mathbf{C}\right)\mathbf{u}\lambda_{,k_m}+\left(\lambda^2\mathbf{M}_{,k_m}+\lambda\mathbf{D}_{,k_m}+\mathbf{K}_{,k_m}\right)\mathbf{u}$$
$$+\left(\lambda^2\mathbf{M}+\lambda\mathbf{D}+\mathbf{K}\right)\mathbf{u}_{,k_m}=0 \tag{2-14}$$

Equation (2-14) is pre-multiplied by $\mathbf{u}^T$, and then it results in

$$\lambda_{,k_m}=\mathbf{u}_{,k_m}\left(\lambda^2\mathbf{M}_{,k_m}+\lambda\mathbf{D}_{,k_m}+\mathbf{K}_{,k_m}\right)\mathbf{u} \tag{2-15}$$

However eigenvector sensitivity cannot be attained by solving Eq. (2-15) due to singularity. Therefore, there already exist many algorithms developed to get eigenvector sensitivity, among which an algebraic approach is utilized.

Rearranging Eq. (2-14), then we have

$$\left(\lambda^2\mathbf{M}+\lambda\mathbf{D}+\mathbf{K}\right)\mathbf{u}_{,k_m}+\left(2\lambda\mathbf{M}+\mathbf{D}\right)\mathbf{u}\lambda_{,k_m}$$
$$=-\left(\lambda^2\mathbf{M}_{,k_m}+\lambda\mathbf{D}_{,k_m}+\mathbf{K}_{,k_m}\right)\mathbf{u} \tag{2-16}$$

Differentiating normalization condition Eq. (2-13) with respect to control gain k_m gives

$$\mathbf{u}^T\left(2\lambda\mathbf{M}+\mathbf{D}\right)\mathbf{u}_{,k_m}+\mathbf{u}^T\mathbf{M}\mathbf{u}\lambda_{,k_m}$$
$$=-0.5\mathbf{u}^T\left(2\lambda\mathbf{M}_{,k_m}+\mathbf{D}_{,k_m}\right)\mathbf{u} \tag{2-17}$$

Algebraic equation in two unknowns $\lambda_{,k_m}$ and $\mathbf{u}_{,k_m}$ can be formulated by combining Eq. (2-16) and (2-17) as following [5]

$$\begin{bmatrix}\lambda^2\mathbf{M}+\lambda\mathbf{D}+\mathbf{K} & \left(2\lambda\mathbf{M}+\mathbf{D}\right)\mathbf{u}\\ \mathbf{u}^T\left(2\lambda\mathbf{M}+\mathbf{D}\right) & \mathbf{u}^T\mathbf{M}\mathbf{u}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_{,k_m}\\ \lambda_{,k_m}\end{Bmatrix}$$
$$=-\begin{Bmatrix}\left(\lambda^2\mathbf{M}_{,k_m}+\lambda\mathbf{D}_{,k_m}+\mathbf{K}_{,k_m}\right)\mathbf{u}\\ 0.5\mathbf{u}^T\left(2\lambda\mathbf{M}_{,k_m}+\mathbf{D}_{,k_m}\right)\mathbf{u}\end{Bmatrix} \tag{2-18}$$

By solving this simple algebraic equation, the sensitivity of both eigenvalue and eigenvector can be obtained simultaneously. Finally, eigenpair about altered control gains may be approximated in first order form like

$$\lambda_r^i\left(k_0+\Delta k\right)\cong\lambda_r^i\left(k_0\right)+\sum_m\left.\frac{\partial\lambda_r^i}{\partial k_m}\right|_{\mathbf{k}_0}\cdot k_m \tag{2-19}$$

$$\mathbf{u}_r^i\left(k_0+\Delta k\right)\cong\mathbf{u}_r^i\left(k_0\right)+\sum_m\left.\frac{\partial\mathbf{u}_r^i}{\partial k_m}\right|_{\mathbf{k}_0}\cdot k_m \tag{2-20}$$

where $k_0\left(=k_{p10},k_{p20},k_{d10},k_{d20}\right)$ is nominal PD gains, and $\Delta k\left(=\Delta k_{p1},\Delta k_{p2},\Delta k_{d1},\Delta k_{d2}\right)$ perturbed.

Considering equivalent bearing parameters with respect to PD gains [2], Eq. (2-1) may be transformed to

$$\mathbf{M}\ddot{\mathbf{p}}+\left(\mathbf{C}_c-j\Omega\mathbf{G}+k_ik_sk_c\mathbf{K}_d\right)\dot{\mathbf{p}}$$
$$+\left(\mathbf{K}+k_ik_sk_c\mathbf{K}_p\right)\mathbf{p}=\mathbf{g} \tag{2-21}$$

Here, k_i,k_s,k_c and $\mathbf{K}_p,\mathbf{K}_d$ are current stiffness, proximity sensor sensitivity, current amp gain and PD gain matrix, respectively [2]. Matrix derivatives of Eq. (2.21) with respect to gains are easily attained as

$$\mathbf{M}_{,k_m}=0,\ \mathbf{C}_{,k_m}=k_ik_sk_c\mathbf{K}_{d,k_m},\ \mathbf{K}_{,k_m}=k_ik_sk_c\mathbf{K}_{p,k_m} \tag{2-22}$$

Accordingly, Eq. (2-18) of eigenpair sensitivity is reproduced like

$$\begin{bmatrix}\lambda^2\mathbf{M}+\lambda\mathbf{D}+\mathbf{K} & \left(2\lambda\mathbf{M}+\mathbf{D}\right)\mathbf{u}\\ \mathbf{u}^T\left(2\lambda\mathbf{M}+\mathbf{D}\right) & \mathbf{u}^T\mathbf{M}\mathbf{u}\end{bmatrix}\begin{Bmatrix}\mathbf{u}_{,k_m}\\ \lambda_{,k_m}\end{Bmatrix}$$
$$=-\begin{Bmatrix}\left(\lambda\mathbf{D}_{,k_m}+\mathbf{K}_{,k_m}\right)\mathbf{u}\\ 0.5\mathbf{u}^T\mathbf{D}_{,k_m}\mathbf{u}\end{Bmatrix} \tag{2-23}$$

3. Integrated structural and PD-control optimization

3.1 Performance index for free vibration

To define a quadratic performance index for integrated optimization, the PD control input $\mathbf{u}$ has to be defined in modal domain such that

$$\mathbf{u}(t)=\kappa\mathbf{k}_p\mathbf{p}_b(t)+\kappa\mathbf{k}_d\dot{\mathbf{p}}_b(t)$$
$$=\kappa\left(\mathbf{k}_p\mathbf{C}_R\zeta(t)+\mathbf{k}_d\mathbf{C}_R\mathbf{A}_R\zeta(t)\right) \tag{3-1}$$

where

$$\kappa=k_ik_sk_c$$
$$\mathbf{k}_p=\mathrm{diag}\left\{k_{p1},k_{p2}\right\},\ \mathbf{k}_d=\mathrm{diag}\left\{k_{d1},k_{d2}\right\}$$

and $\mathbf{p}_b$ is the displacement state at bearing nodes. Next, the performance index J_f is defined in typical quadratic form as

$$\begin{aligned} J_f &= \int_0^\infty \left(\mathbf{y}^*\mathbf{S}\mathbf{y} + \mathbf{u}^*\mathbf{T}\mathbf{u}\right) dt \\ &= \int_0^\infty \left(\mathbf{p}_b^*\mathbf{S}_1\mathbf{p}_b + \dot{\mathbf{p}}_b^*\mathbf{S}_2\dot{\mathbf{p}}_b + \mathbf{u}^*\mathbf{T}\mathbf{u}\right) dt \end{aligned} \quad (3\text{-}2)$$

where $\mathbf{S}$ and $\mathbf{T}$ are positive semi definite weighting matrix and positive definite weighing matrix respectively and the asterisk denotes a complex conjugate. By inserting control input Eq. (3-1), above equation becomes

$$\begin{aligned} J_f &= \int_0^\infty \left\{\left(\mathbf{p}_b^*\mathbf{S}_1\mathbf{p}_b + \dot{\mathbf{p}}_b^*\mathbf{S}_2\dot{\mathbf{p}}_b\right) \right. \\ &\qquad \left. + \left(\mathbf{k}_p\mathbf{p}_b + \mathbf{k}_d\dot{\mathbf{p}}_b\right)^* \mathbf{T}\left(\mathbf{k}_p\mathbf{p}_b + \mathbf{k}_d\dot{\mathbf{p}}_b\right)\right\} dt \\ &= \int_0^\infty \zeta^*\mathbf{Q}_f\zeta dt \end{aligned} \quad (3\text{-}3)$$

where

$$\mathbf{Q}_f = \mathbf{Q}_{f1} + \mathbf{Q}_{f2} + \mathbf{Q}_{f3}$$

and $\mathbf{Q}_{f1} = \mathbf{C}_R^*\left(\mathbf{S}_1 + \mathbf{K}_p^*\mathbf{T}\mathbf{K}_p\right)\mathbf{C}_R$,

$$\mathbf{Q}_{f2} = \mathbf{A}_R^*\mathbf{C}_R^*\left(\mathbf{S}_2 + \mathbf{K}_d^*\mathbf{T}\mathbf{K}_d\right)\mathbf{C}_R\mathbf{A}_R,$$

$$\mathbf{Q}_{f3} = \mathbf{C}_R^*\mathbf{K}_p^*\mathbf{T}\mathbf{K}_d\mathbf{C}_R\mathbf{A}_R + \mathbf{A}_R^*\mathbf{C}_R^*\mathbf{K}_d^*\mathbf{T}\mathbf{K}_p\mathbf{C}_R$$

Given the homogeneous system dynamics from Eq. (2-5), optimal solution can be obtained by minimizing performance index J_f. If all eigenvalues in this deterministic system are located in the left-half plane, then the above performance index, using trace identity, can be converted as

$$J_f = \mathrm{tr}\left[\mathbf{P}_f\mathbf{Z}_0\right] \quad (3\text{-}4)$$

where $\mathbf{Z}_0 = E\left\{\zeta(0)\zeta^*(0)\right\}$ and $E\left\{\zeta(0)\right\} = 0$ and $\mathbf{P}_f$ satisfies following Lyapunov matrix equation

$$\mathbf{A}_R^*\mathbf{P}_f + \mathbf{P}_f\mathbf{A}_R + \mathbf{Q}_f = 0 \quad (3\text{-}5)$$

Note that $\mathbf{P}_f$ and $\mathbf{Q}_f$ are also dependent on control gain k_m.

The sign of sensitivity of the performance index indicates the direction from which the controller gain should be altered from current one to render the above index smaller. Making use of the formulas

$$\frac{\partial}{\partial \mathbf{Z}}\mathrm{tr}\left[\mathbf{N}\mathbf{Z}^T\right] = \mathbf{N},\ \mathrm{tr}\left[\mathbf{N}\mathbf{Z}\right] = \mathrm{tr}\left[\mathbf{Z}^T\mathbf{N}^T\right] \quad (3\text{-}6)$$

the variation of performance index Eq. (3-4) can be derived as

$$\frac{\partial J_f}{\partial k_m} = \mathrm{tr}\left(\frac{\partial \mathbf{P}_f}{\partial k_m}\mathbf{Z}_0 + \mathbf{P}_f\frac{\partial \mathbf{Z}_0}{\partial k_m}\right) \quad (3\text{-}7)$$

From the sensitivity Eq. (3-7), it is noted that the covariance matrix $\mathbf{Z}_0$ is already given but the sensitivity of $\mathbf{P}_f$ is not. Therefore, this unknown sensitivity of $\mathbf{P}_f$ should be given by differentiating Lyapunov Eq. (3-5) like

$$\frac{\partial \mathbf{A}_R^*}{\partial k_m}\mathbf{P}_f + \mathbf{A}_R^*\frac{\partial \mathbf{P}_f}{\partial k_m} + \frac{\partial \mathbf{P}_f}{\partial k_m}\mathbf{A}_R + \mathbf{P}_f\frac{\partial \mathbf{A}_R}{\partial k_m} + \frac{\partial \mathbf{Q}_f}{\partial k_m} = 0 \quad (3\text{-}8)$$

Rearranging above Eq. (3-8) makes new Lyapunov equation in which a solution is the sensitivity of $\mathbf{P}_f$

$$\begin{aligned} &\mathbf{A}_R^*\frac{\partial \mathbf{P}_f}{\partial k_m} + \frac{\partial \mathbf{P}_f}{\partial k_m}\mathbf{A}_R \\ &+ \left\{\frac{\partial \mathbf{A}_R^*}{\partial k_m}\mathbf{P}_f + \mathbf{P}_f\frac{\partial \mathbf{A}_R}{\partial k_m} + \frac{\partial \mathbf{Q}_f}{\partial k_m}\right\} = 0 \end{aligned} \quad (3\text{-}9)$$

Performance index Eq. (3-4) and its sensitivity Eq. (3.9) is used later to find optimal control gains in next section.

3.2 Optimal PD gain determination utilizing steepest decent method

The AMB system is commonly controlled and stabilized by PD controller; therefore it is important to find optimal control gains. Two characteristic features of flexible rotor-AMB system, i.e. varying dynamics due to gyroscopic effect and unavoidable unbalance response, may yield the optimal gains with respect to rotational speed. Although gain-scheduling approach, 'try and error' in many cases, is commonly adopted, it lacks physical and analytical ground.

For the flexible rotor-AMB system, optimal gains should consider dynamics of flexible modes. At a certain rotational speed, constant reduced modal-state equation obtained by the truncation can be constructed for control model; however this model does not consider gain-dependency of the system dynamics. Accordingly, even though the PD gains are determined by proper optimal control algorithm, those may be not optimal solution.

In this section, I will present new way to determine PD gains optimally. Proposed optimization procedure reflects the gain-dependency of the dynamics by first order approximation derived at last sections.

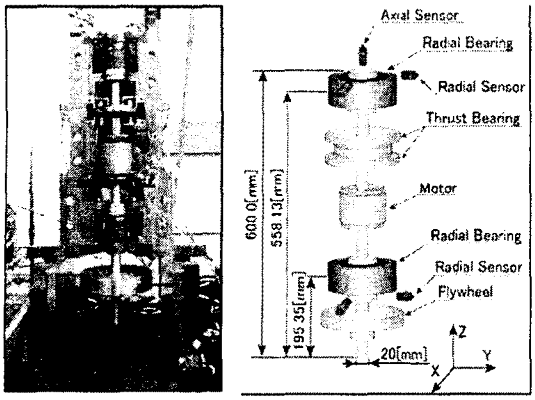

Fig. 1 Perspective view of the AMB system

If there is a constraint equation like homogenous part of Eq. (2-5) with performance index, it is common to introduce Lagrange multiplier and use matrix minimum principle for necessary condition of local minimum. However, due to gain-dependent dynamics, such approach cannot be applied so that direct search method by evaluating performance index and its sensitivities, i.e. steepest decent method, is chosen.

The gradients about P and D gains are expressed as

$$\nabla J_f(\mathbf{k}) = \begin{bmatrix} \dfrac{\partial J_f}{\partial \mathbf{k}_p} & \dfrac{\partial J_f}{\partial \mathbf{k}_d} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial J_f}{\partial k_{p1}} & \dfrac{\partial J_f}{\partial k_{p2}} & \dfrac{\partial J_f}{\partial k_{d1}} & \dfrac{\partial J_f}{\partial k_{d2}} \end{bmatrix} \quad (3\text{-}10)$$

where $\mathbf{k} = \begin{bmatrix} k_{p1} & k_{p2} & k_{d1} & k_{d2} \end{bmatrix}$

The necessary condition for local minimum gain is

$$\nabla J_f(\mathbf{k}) = 0 \quad (3\text{-}10)$$

Finally, optimal gains are given as

$$\mathbf{k}_{op} = \min_{\mathbf{k}} J_f \quad (3\text{-}10)$$

The optimization procedure can be stated as:

1. Select a starting PD gains and weightings $\mathbf{S}_1$,$\mathbf{S}_2$ and $\mathbf{T}$. Set iteration index i=0.
2. Compute $J_f(i)$ and its gradients of $\nabla J_f(\mathbf{k}_p(i))$ and $\nabla J_f(\mathbf{k}_d(i))$.
   Stop if $\left\| J_f(i) - J_f(i-1) \right\| / \left\| J_f(i-1) \right\| \le \varepsilon_R$
   Otherwise, define a direction vector of
   $\mathbf{d}_p(i) = -\nabla J(\mathbf{k}_p(i)), \mathbf{d}_d(i) = -\nabla J(\mathbf{k}_d(i))$
3. Determine 'small enough' step size α_p and α_d.
   Update $\mathbf{k}_p(i+1) = \mathbf{k}_p(i) + \alpha_p \mathbf{d}_p(i)$
   and $\mathbf{k}_d(i+1) = \mathbf{k}_d(i) + \alpha_d \mathbf{d}_d(i)$
4. Set i=i+1, $\mathbf{k}_p(i) = \mathbf{k}_p(i+1), \mathbf{k}_d(i) = \mathbf{k}_d(i+1)$ and go to step 2.

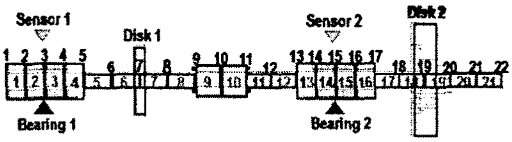

Fig. 2 Finite element model of the system

4. Simulation Results and Discussion

To investigate effectiveness of the proposed control algorithm by simulation, the rotor-AMB system shown in Fig.1 is established, which consists of a flexible rotor, two magnetic bearings, four proximity probes, a digital controller and current power amplifiers. Maximum Operational speed is designed to reach 20000rpm. For simulation, the sensor, the power amplifier is described as constant sensitivities $k_s(=5000\,{}^{V}\!/_{m})$ and $k_c(2\,{}^{A}\!/_{V})$.

Linearizing the magnetic force $f(t)$ w.r.t the neutral position, the net magnetic force due to small perturbation $y(t)$ in air gap can be expressed by

$$f(t) = k_i i(t) + k_y y(t) \quad (4\text{-}1)$$

where the current and position stiffnesses $k_i(=52.3\,{}^{N}\!/_{A})$ and $k_y(=-0.13\,{}^{N}\!/_{\mu m})$ imply the sensitivities of magnetic force relative to the control current $i(t)$ and the shaft displacement $y(t)$, respectively. The position stiffness has a negative value which becomes a cause of instability in magnetically suspended system.

For the finite element analysis, the rotor is divided into 21 one-axis Rayleigh's beam elements and lumped disk elements like Fig. 2; furthermore the magnetic bearings are assumed as pin-point stiffness and damping elements located at nodes. In spite of non-collocated sensors' position with the actuators, it is ignored and collocation assumption actually holds instead. The PD controller is designed to be as decentralized so that diagonal bearing damping and stiffness matrices in

Table 1 Specifications of the AMB system

Shaft			
Length(m)	0.6		
Radius(m)	0.02		
Density(kg/m^3)	7800		
Young's modulus(N/m^2)	2.07×10^{11}		
Disks			
No.	Mass (kg)	Polar moment (kg-m^2)	Diametral moment (kg-m^2)
For axial AMB	0.74	9.2×10^{-4}	4.7×10^{-4}
Flywheel	4.71	1.5×10^{-2}	7.9×10^{-3}
Magnet Journal			
Length(m)	0.048		
Radius(m)	0.025		

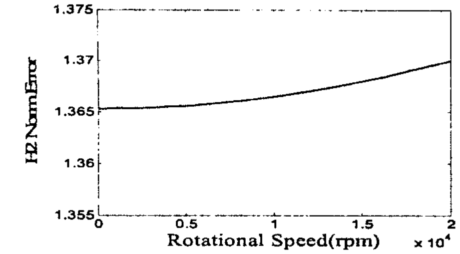

Fig. 3 H_2 norm model reduction error

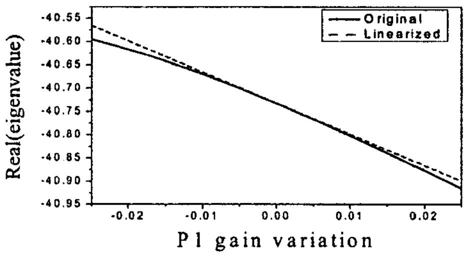

Fig. 4 Eigenvalue approximation of 1B mode when k_p=0.4, k_d=0.001

FEM model are used. The specifications of the system are listed in Table 1.

The rigid rotor-AMB system is designed to place rotor's mass center possibly in the middle of two magnetic bearings. With rigidity of the rotor, it enables respective control gains for two bearings to be regulated equally. However, since the mass center of the system deviates from the middle of the bearings as shown in Fig. 2, the control gains at two positions may not be equal if we try to optimize them. Before optimization work, the starting PD gains are determined equally for two bearings as $k_{p1} = k_{p2} = 0.4$ and $k_{d1} = k_{d2} = 0.001$ whereby finite element model is established and 10 modes from 5[th] backward to 5[th] forward are truncated after modal analysis. Resulting H_2 norm error, Eq. (2-10), is shown by Fig. 3 from which we find it to be around 1 % relative to that of full model. Moreover, Figure 4 denotes a representative result of eigenpair approximation, Eq. (2-23), about upper bearing's P gain k_{p1} of which condition is given by starting gains at rotational speed 5000rpm. A series of tests based on other conditions are performed, which conform that the error is negligibly small like Fig. 4.

Before carrying out optimization, the requiring parameters are listed as follows: $\mathbf{S}_1 = \text{diag}\{2,10\}\times10^3$, $\mathbf{S}_2 = \text{diag}\{10,50\}$, $\mathbf{T} = \text{diag}\{0.2,1\}\times10^{-4}$, $\varepsilon_R = 0.001$, $\alpha_p = 7.0\times10^{-1}$ and $\alpha_d = 5.0\times10^{-6}$. The performance index converges after 22 iterations and Fig. 5 shows its converging feature. Optimal P gains corresponding to 22 iterations are determined by

$$\mathbf{k}_{p,op} = \begin{bmatrix} k_{p1,op} & k_{p2,op} \end{bmatrix} = \begin{bmatrix} 0.55 & 0.62 \end{bmatrix} \qquad (4\text{-}2)$$

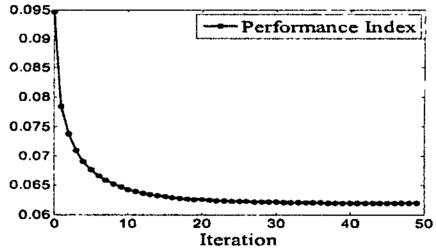

Fig. 5 Performance index convergence

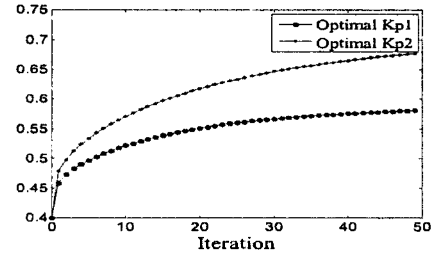

Fig. 6 P-gain convergence

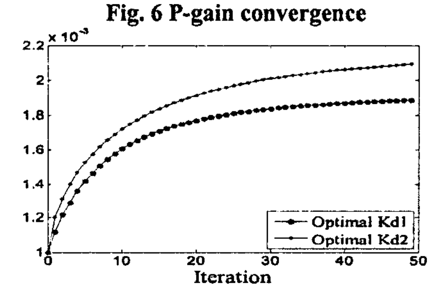

Fig. 7 D-gain convergence

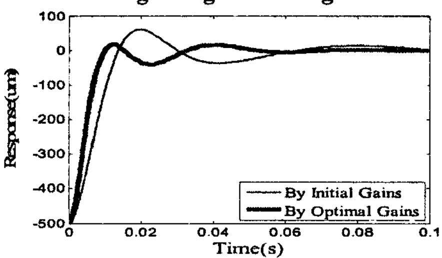

Fig. 8 Control response at bearing 1

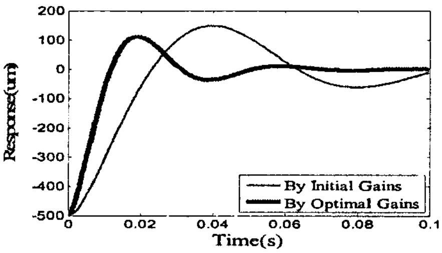

Fig. 9 Control response at bearing 2

$$\mathbf{k}_{d,op} = \begin{bmatrix} k_{d1,op} & k_{d2,op} \end{bmatrix} = \begin{bmatrix} 0.0018 & 0.0019 \end{bmatrix} \qquad (4\text{-}3)$$

It is noted that the gains of bearing 2 are larger than one of bearing 1. Finally, we can find the control response become faster and less fluctuating on Fig. 8 and 9.

5. Conclusions

In this paper, the simultaneous optimization method of PD controller is proposed considering the structural dynamics of flexible rotor-AMB system. Under the assumption of linearized force equation relative to neutral rotor's position, the equivalent bearing parameters w.r.t PD gains are determined whereby the reduced modal model is obtained from finite element model. Since the modal properties of the flexible rotor-AMB system are varied with gains, the eigenpair sensitivity approach is adopted to find local minimum of the performance index defined in quadratic form.

Given starting PD gains, suggested search algorithm for optimization, steepest descent method, find the optimal PD gains successfully. The control performance for optimal gains is shown to become better than that of starting. Reminding the bearing 2 is closer to the mass center than the bearing 1, it is physically acceptable to have larger optimal gains for bearing 2. Therefore, this suggest algorithm can be used effectively in searching optimal gains of the flexible rotor-AMB system.

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