Passive Millimeter-Wave Image Deblurring Using Adaptively Accelerated Maximum Entropy Method

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Abstract— **In this paper we present an adaptive method for accelerating conventional Maximum Entropy Method (MEM) for restoration of Passive Millimeter-Wave (PMMW) image from its blurred and noisy version. MEM is nonlinear and its convergence is very slow. We present a new method to accelerate the MEM by using an exponent on the correction ratio. In this method the exponent is computed adaptively in each iteration, using first-order derivatives of deblurred image in previous two iterations. Using this exponent the accelerated MEM emphasizes speed at the beginning stages and stability at later stages. In accelerated MEM the non-negativity is automatically ensured and also conservation of flux without additional computation. Simulation study shows that the accelerated MEM gives better results in terms of RMSE, SNR, moreover, it takes only about 46% lesser iterations than conventional MEM. This is also confirmed by applying this algorithm on actual PMMW image captured by 94 GHz mechanically scanned radiometer.**

Index Terms— **Image deblurring, Maximum Entropy, ill-conditioned, nonlinear method.**

I. Introduction

Passive millimeter-wave (PMMW) image have an inherently poor resolution and highly blurred due to limited aperture dimension and the consequent diffraction limits. Thus, an efficient PMMW image deblurring algorithm is required to get high quality image in order to use for practical purpose. Image deblurring is a longstanding linear inverse problem and is encountered in many application areas such as remote sensing, medical imaging, seismology, and astronomy [1]-[3]. Generally many linear inverse problems are ill-conditioned, since either the inverse of linear operators does not exist or is nearly singular yielding highly noise sensitive solutions. Most methods given to solve ill-conditioned problems are classified into following two categories: a) Methods based on regularization [2], [3] and b) Methods based on Bayesian theory [1], [2], [4] - [8].

The main idea of regularization and Bayesian approach is the use of *a priori* information expressed by Regularization/ prior term. Prior term gives a higher score to most likely images. However, modeling a prior for real-word images is not trivial and subjective matter. Many directions for prior modeling have been proposed such as derivative energy in the Wiener filter [2],[3], compound Gauss Markov random field [2], [13], Markov random fields with non quadratic potentials [2], [11], [13], Entropy [1], [8], [10], and heavy tailed densities of images in wavelet domain [12]. But in the absence of any prior information about the original image, entropy is considered as the best choice to define prior term [4].

MEM developed under Bayesian framework is nonlinear and solved iteratively [8], [10]. However, it has the drawbacks of slow convergence and being computationally expensive. Many techniques for accelerating the iterative method have been proposed, these can also be used for accelerating the MEM [1], [9]. All these methods use correction terms - may be negative at times – which are computed in every iteration, multiplied with acceleration parameter, and added to the results obtained in previous iteration. Because the correction term may be negative at times, the non-negativity of pixel intensity in restored image is not guaranteed. In these acceleration methods postivity is enforced manually at the end of iterations. The main drawback of these acceleration methods is the selection of optimal acceleration parameter. Large acceleration parameter speeds up the algorithm, but it may introduce error. If error is amplified during iteration, it can lead to instability. Thus these methods require a correction procedure in order to ensure the stability. This correction procedure reduces the gain obtained by acceleration step and also needs extra computation.

In this paper we propose a new adaptive acceleration method for MEM in order to cope with the problem of earlier acceleration method. The proposed acceleration method requires minimum information about the iterative process. We use an exponent on multiplicative correction as an acceleration parameter which is computed adaptively in each iteration using first order derivative of deblurred image from previous two iterations. The positivity of pixel intensity in the proposed acceleration method is automatically ensured since multiplicative correction term is always positive. Maintaining the total flux is important for applications where the blurring does not change the total number of photons or electrons detected. In this method we also achieve flux conservation without extra computational overhead. Section II discusses the accelerated MEM and in Section III adaptive acceleration of MEM is presented. In Section IV simulation and experiment results are presented. Section V gives the conclusion, which is followed by references.

II. Accelerated Maximum Entropy Method With Flux Conservation For Image Deblurring

Let an original image, size M x N, blurred by shift-invariant point spread function (PSF) and corrupted by Poisson noise. This can be written in matrix form as [14]:

$$y = Hx + n, \tag{1}$$

where H is MN x MN block Toeplitz matrix representing a linear shift-invariant PSF; x, y, and n are vectors of size MN x 1 containing the original image, observed image, and sample of noise, respectively, arranged in column lexicographic ordering. The aim in image deblurring is to find an estimate of an original image x for a given blurred image y blurring operator H and distribution of noise n.

We derive the MEM, in Bayesian framework, with Poisson type noise n. The basic idea of Bayesian framework is to incorporate the prior information, about the solution. A prior information is included using *a priori* distribution. In MEM, *a priori* distribution, $p(x)$, is defined using entropy as

$$p(x) = \exp(-E(x)), \tag{2}$$

where $E(x)$ is the entropy of the original image x. We use the following entropy function

$$E(x) = -\sum_i x_i \log x_i. \tag{3}$$

When n is zero in Eq. (1), we consider only blurring, the expected value at the i^{th} pixel in the blurred image is $\sum_j h_{ij} x_j$. Where h_{ij} is $(i, j)^{th}$ element of H and x_j is the j^{th} element of x. Because of Poison noise, the actual i^{th} pixel value y_i in y is one realization of Poisson distribution with mean $\sum_j h_{ij} x_j$. Thus we have following relation:

$$p(y_i/x) = \left(\sum_j h_{ij} x_j\right)^{y_i} \exp\left(-\sum_j h_{ij} x_j\right) \Big/ y_i!. \tag{4}$$

Each pixel in blurred and noisy image, y, is realized by an independent Poisson process. Thus the likelihood of getting noisy and blurred image y is given by

$$p(y/x) = \prod_i \left[\left(\sum_j h_{ij} x_j\right)^{y_i} \exp\left(-\sum_j h_{ij} x_j\right) \Big/ y_i!\right]. \tag{5}$$

MEM method with flux conservation for image deblurring, seeks an approximate solution of (1) that maximizes the a posteriori probability $p(x/y)$ or $\log p(x/y)$, subject to the constraint of flux conservation, $\sum_j x_j = N$, where N is the sum of pixel values in observed image. We consider the maximization of following function

$$L(x,\mu) = \log p(x/y) - \mu\left(\sum_j x_j - N\right). \tag{6}$$

where μ is the Lagrange multiplier for flux conservation. Now from Bye's theorem substitution of $p(x/y)$ in terms of $p(y/x)$ in (6), and then using $p(x)$, $p(y/x)$ from (2), (5) we get

$$L(x,\mu) = \sum_i\left[-\sum_j h_{ij} x_j + y_i \log\left(\sum_j h_{ij} x_j\right)\right] - \sum_j x_j \log x_j - \mu\left(\sum_j x_j - N\right). \tag{7}$$

For maximization of L, $\partial L(x,\mu)/\partial x_k = 0$, we get the following relation

$$1+\mu = \sum_i\left[h_{ik}\left\{\left(y_i \Big/ \sum_j h_{ij} x_j\right) - 1\right\}\right] - \log(x_k). \tag{8}$$

Eq. (8) is nonlinear in x_k, and is solved iteratively. By adding a positive constant C and raising exponent q both sides of (8), and then multiply both sides by x_k, we arrive at the following iterative procedure:

$$x_k^{l+1} = A x_k^l\left[\sum_i\left(h_{ik} y_i \Big/ \sum_j h_{ij} x_j^l\right) - 1 + \log x_k^l + C\right]^q, \tag{9}$$

where $A = [\ 1 + \mu + C\ ]^{-q}$. For ensuring the non-negativity of x_k^l, which allow the computation of $\log x_k^l$ in the next iteration, a suitable constant C is selected. The constant A is recalculated at the end of each iteration using constraint $\sum_j x_j^l = N$. Accordingly, we get following:

$$A(l) = N\left[\sum_k x_k^l\left\{\sum_i\left(h_{ik} y_i \Big/ \sum_j h_{ij} x_j^l\right) - 1 - \log x_k^l + C\right\}\right]^{-q}. \tag{10}$$

It is found that the iteration given in (9) converges for $1 \le q \le 3$. Large values of q give faster convergence but the risk of instability increases. Smaller values of q lead to slow convergence and reduce the risk of instability. Thus, relation (9) with adaptive selection of an exponent q leads to the adaptively accelerated MEM. For q = 1, relation (9) gives conventional MEM. It is found that the convergence speed does not depend on the choice of C.

III. Adaptive Seletion Of Exponent

The choice of q in Eq. (9) mainly depends on the noise, n, and its amplification during iterations. If noise is high, smaller value of q is selected and vice-versa. Thus the convergence speed of the proposed method depends on the choice of the parameter q. Drawback of this accelerated form of MEM is that the selection of exponent q has to be done manually by trial and error. We overcome such serious limitation by proposing a method in which q is computed adaptively as iterations proceed. Proposed expression for q is as follows:

$$q(l+1) = \exp\left(\frac{\left\|\nabla x^{l}\right\|}{\left\|\nabla x^{l-1}\right\|} \right) - \frac{\left\|\nabla x^{2}\right\|}{\left\|\nabla x^{1}\right\|}, \qquad (11)$$

where ∇x^l stands for the first-order derivative of x^l and $\| \; \|$ denotes the L_2 norm. Main idea in using first-order derivative is to utilize the sharpness of image. Because of the blurring, the image becomes smooth, sharpness decreases, and edges are lost or become weak. Deblurring makes image non-smooth, and increases the sharpness. Hence the sharpness of deblurred image, ∇x^l, increases as iterations proceed. For different level of blur and different classes of images, it has been found by experiments that L_2 norm of gradient ratio $\left\|\nabla x^l\right\| / \left\|\nabla x^{l-1}\right\|$ converges to 1 as the number of iterations increase. Accelerated MEM emphasizes speed at the beginning stages of iterations by forcing q around three. When the exponential term in (11) is greater than three, the second term, $\left\|\nabla x^2\right\| / \left\|\nabla x^1\right\|$, limits the value of q within three to prevent divergence. As iterations increase the second term forces q towards the value of one which leads to stability of iteration. By using the proposed exponent, q, the method emphasizes speed at the beginning stages and stability at later stages of iteration. Thus selecting q given by (11) for iterative solution (9) gives accelerated MEM with adaptive selection of acceleration parameter. The non-negativity of pixel intensity is automatically ensured, since correction ratio (9) is always positive. In order to initialize the proposed method, first two iterations are computed using some fixed value of q ($1 \le q \le 3$).

IV. Simulation And Experiment Results

For simulation, we choose the gray scale test image "Cameraman" (8-bit, 256 x 256), uniform 5x5 Box-car PSF, and Poisson noise. The blurred signal to noise ratio (*BSNR*) as defined in [14] is set to 40 dB. The RMSE and SNR criteria are used for performance comparison of conventional and adaptive accelerated MEM. Figure 1 (a), (b) show the original and noisy blurred images of this simulation. Figure 1 (c), (d) show the results of the MEM and accelerated MEM corresponding to maximum SNR. Figures 3 (a), (b), show the variation of SNR, RMSE versus iterations for MEM and accelerated MEM. It is observed that the accelerated MEM has faster increase in SNR and faster decrease in RMSE in comparison of MEM method. Figure 4 shows the variation of exponent q versus iteration number.

Result of real PMMW image, captured by 94 GHz mechanically scanned radiometer, using estimated PSF of [15] is shown in figure 2.

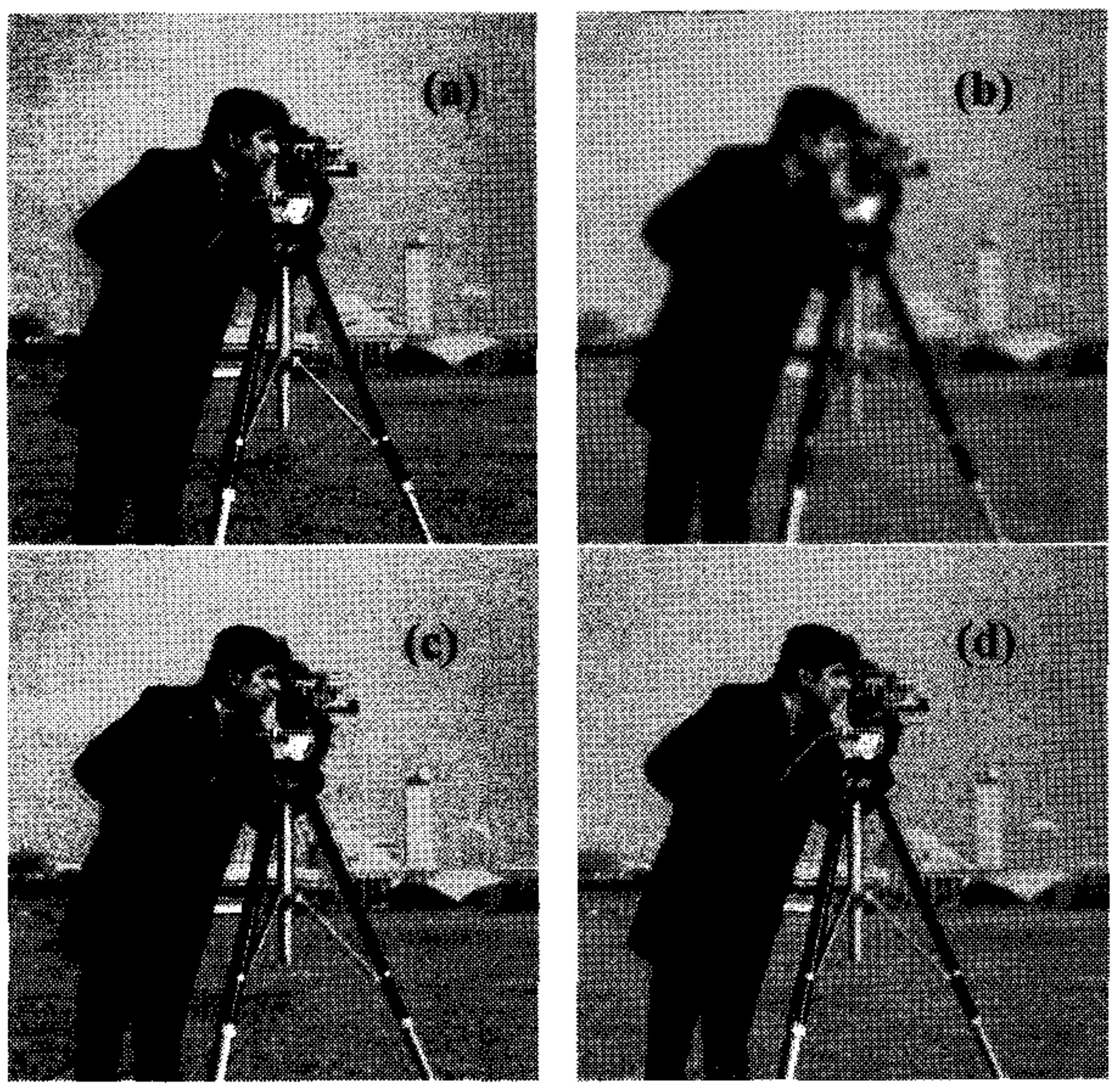

Figure 1. "Cameraman" a) Original b) Noisy and Blurred c) Restored by MEM corresponding maximum SNR in 367 iteartion d) Retored Image by Adaptively Accelerated MEM corresponding maximum SNR in 200 iterations.

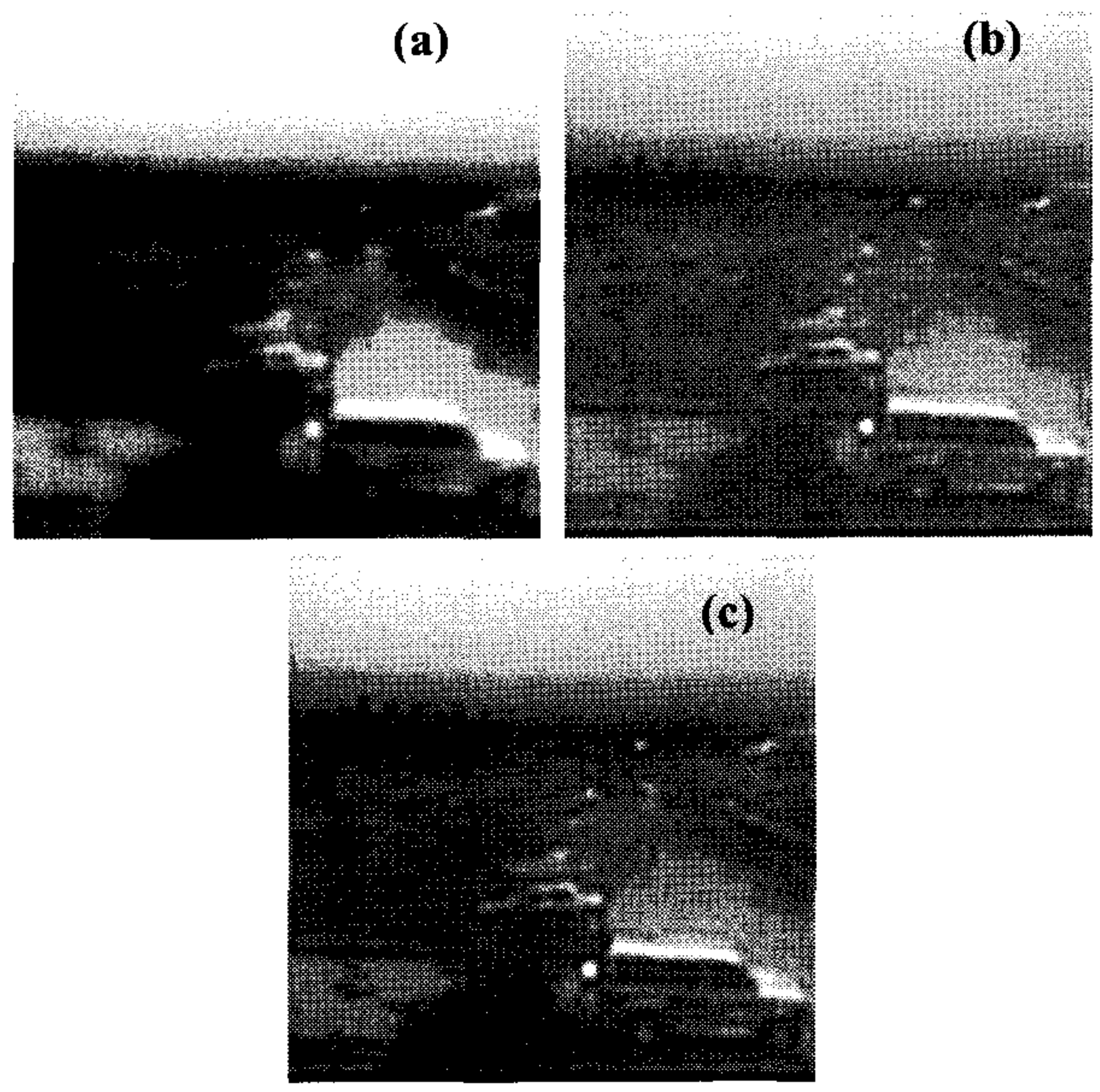

Figure 2. a) Observed 94 GHz PMMW image b) Adaptively Accelarated MEM in 10 iteration c) Conventional MEM in 35 iteration.

V. Conclusion

We have given a new method to accelerate the conventional MEM. This method adaptively computes exponent of correction term in each iteration using the first-order derivative of the restored image in previous two iteration. The aim of adaptive selection of the exponent, q, is to emphasize speed and stability at early and late stages of iteration respectively. From simulation and experiment, it is found that accelerated MEM gives better results in terms of RMSE, high SNR, approximately in 46% lesser iterations than the conventional MEM method. While computations required per iteration in MEM as well as accelerated MEM are almost same. This adaptive acceleration method has simple form and can be very easily implemented. Accelerated MEM automatically preserves the non-negativity and flux, without additional computations.

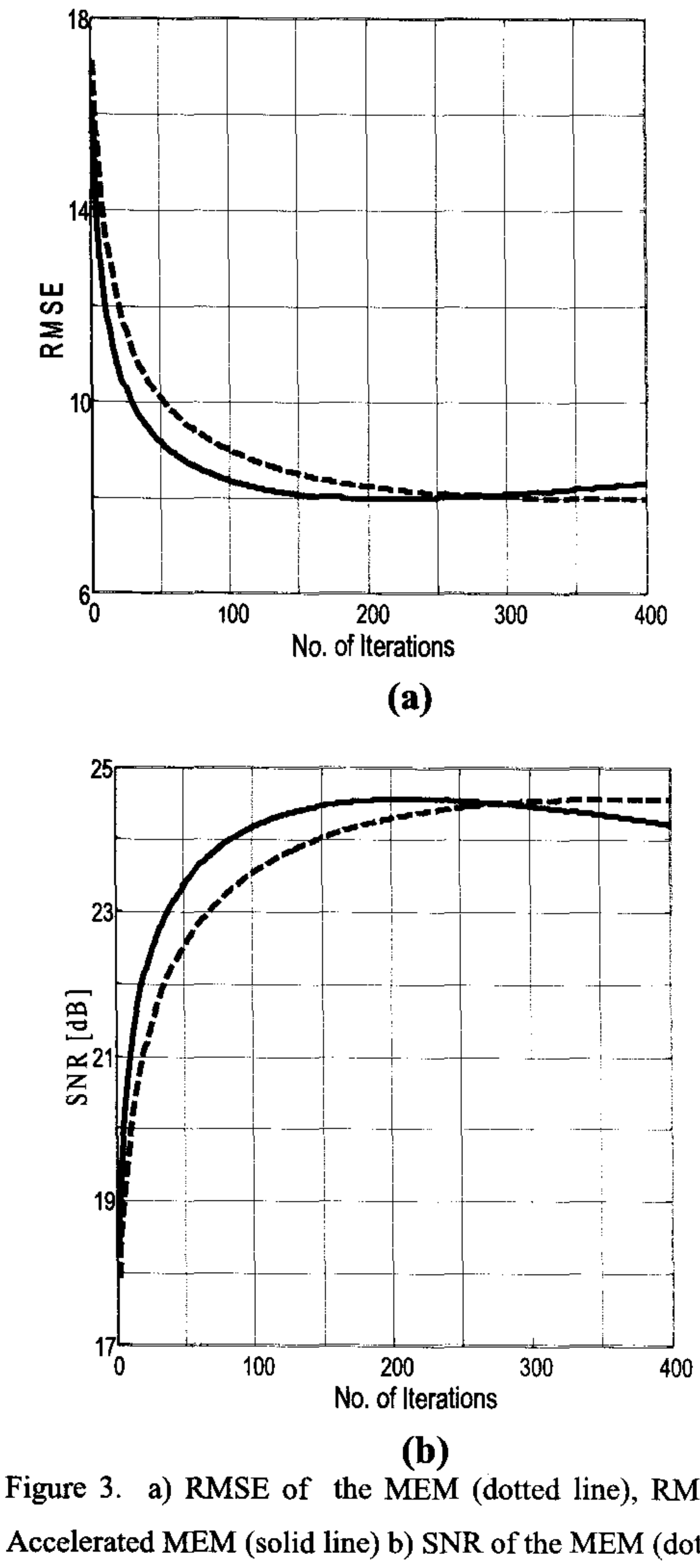

Figure 3. a) RMSE of the MEM (dotted line), RMSE of the Accelerated MEM (solid line) b) SNR of the MEM (dotted line), SNR of the Accelerated (solid line).

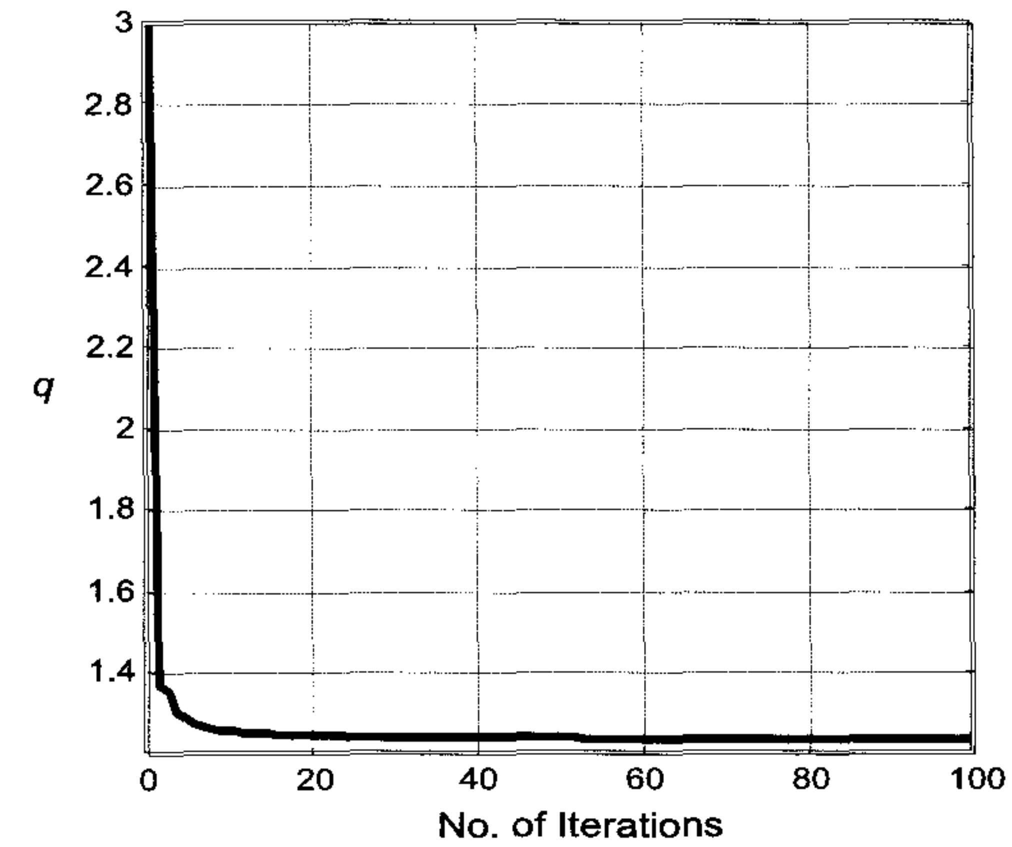

Figure 4. Iteration Vs. q

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