SAR Despeckling with Boundary Correction

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Abstracts: In this paper, a SAR-despeckling approach of adaptive iteration based a Bayesian model using the lognormal distribution for image intensity and a Gibbs random field (GRF) for image texture is proposed for noise removal of the images that are corrupted by multiplicative speckle noise. When the image intensity is logarithmically transformed, the speckle noise is approximately Gaussian additive noise, and it tends to a normal probability much faster than the intensity distribution. The MRF is incorporated into digital image analysis by viewing pixel types as states of molecules in a lattice-like physical system. The iterative approach based on MRF is very effective for the inner areas of regions in the observed scene, but may result in yielding false reconstruction around the boundaries due to using wrong information of adjacent regions with different characteristics. The proposed method suggests an adaptive approach using variable parameters depending on the location of reconstructed area, that is, how near to the boundary. The proximity of boundary is estimated by the statistics based on edge value, standard deviation, entropy, and the 4th moment of intensity distribution.

Keywords: SAR, Despeckling, Point-Jacobian Iteration, Adaptive MAP Estimation, Boundary Correction, Homogeniety.

1. Introduction

In the last couple of decades, the use of Synthetic Aperture Radar (SAR) has become increasingly popular because there are several well-known advantages of SAR data over other imaging systems (Leberl, 1990) including its capacity of imaging regardless weather conditions. However, the radar wave coherence produces "speckle" in SAR imagery. This phenomenon gives to the images a granular appearance that complicates image analysis and interpretation in remote sensing tasks. Although it is a deterministic phenomenon due to the coherent processing of terrain backscattering signals, the speckle contribution is often considered as noise that degrades the quality of SAR imagery. Speckle filtering is a common requirement in many SAR image applications. Up to now, speckle reduction remains a major issue in SAR imagery processing.

The iterative *maximum a posteriori* (MAP) approaches using a Bayesian model based on the lognormal distribution for image intensity and a Gibbs random field (GRF) for image texture have been proposed for despeckling the SAR images that are corrupted by multiplicative speckle noise (Lee, 2007a; Lee, 2007b). When the image intensity is logarithmically transformed, the speckle noise is approximately Gaussian additive noise, and it tends to a normal probability much faster than the intensity distribution (Arsenault and April, 1976). The MRF is incorporated into digital image analysis by viewing pixel type s as states of molecules in a lattice-like physical system defined on a GRF (Georgii, 1979). Because of the MRF-GRF equivalence resulted from the Hammersley–Clifford theorem (Kindermann and Snell, 1982), the assignment of an energy function to the physical system determines its Gibbs measure, which is used to model molecular interactions. The MAP estimation method using the Point-Jacobian iteration was first proposed (Lee, 2007a), and Lee (2007b) has then modified the previous method by using the smoothing parameters adaptively estimated at each iterations. Compared to the results of the conventional schemes, the results of both approaches have shown considerable improvement. In this study, an iterative MAP approach, which employs variable parameters depending on the location, is proposed. In the new scheme, different values are given for the parameters according to how near to the boundary. The proximity to the boundary is measured by the statistics based on edge value, standard deviation, entropy, and the 4th moment of intensity distribution, which was suggested by Cheng *et al.* (2003).

2. Bayesian Function for MAP Estimation

The image model of SAR imagery is usually given by

$$z_k \cong v_k \eta_k \,. \qquad (1)$$

where η_k follows a log-normal distribution. If $Y = \{y_k = \ln z_k, k \in I_n\}$, $X = \{x_k = \ln v_k, k \in I_n\}$, and σ_k^2 is a variance of $\ln \eta_k$, then $Y \sim N(X, \Sigma)$ where $\Sigma = diagonal\{\sigma_k^2, k \in I_n\}$.

Image processes are assumed to combine the random fields associated with intensity and texture respectively. The objective measure for determining the optimal restoration of this "double compound stochastic" image process is based on Bayes' theorem. Given an observed image *Y*, the Bayesian method is to find the MAP estimate from the mode of the posterior probability distribution of the noise-free vector *X*, or equivalently, to maximize the log-likelihood function

$$\ell PN = \ln P(Y \mid X) + \ln P(X) \,. \qquad (2)$$

In the proposed algorithm, the MRF is used to quantify the spatial interaction probabilistically, that is, to provide a type of prior information on the image texture.

If R_i is the index set of neighbors of the *i*th pixel, $R = \{R_i \mid i \in I_n\}$ is a "neighborhood system" for I_n. A "clique" of $\{I_n, R\}$, *c*, is a subset of I_n such that every pair of distinct indices in *c* represents pixels which are mutual neighbors, and *C* denotes the set of all cliques. A GRF relative to the graph $\{I_n, R\}$ on *X* is defined as

$$\begin{aligned} P(X) &= Z^{-1} \exp\{-E(X)\} \\ E(X) &= \sum_{c \in \boldsymbol{C}} V_c(X) \quad \text{(energy function)} \end{aligned} \qquad (3)$$

where *Z* is a normalizing constant and V_c is a potential function which has the property that it depends only on *X* and *c*. Specification of *C* and V_c is sufficient to formulate a Gibbs measure for the region-class model. A particular class of GRF, in which the energy function is expressed in terms of non-symmetric "pair-potentials," is used in this study (Kindermann and Snell, 1982). Here, the energy function of the GRF is specified as a quadratic function of ***X***, which defines the probability structure of the texture process:

$$E_p(X) = \sum_{i \in I_n} \sum_{(i,j) \in \boldsymbol{C}_p} \alpha_{ij} \left(x_i - x_j\right)^2 \qquad (4)$$

where α_{ij} is a nonnegative coefficient vector which represents the "bonding strength" of the *i*th and the *j*th pixels.

The log-likelihood function of Eq. (2) using the log-normal intensity model and the GRF texture model is:

$$\ell PN \propto -(Y - X)' \Sigma^{-1} (Y - X) - X' \mathbf{B} X \qquad (5)$$

where $\mathbf{B} = \{\beta_{ij}\}$ is the bonding strength matrix.

3. Point-Jacobian Iteration MAP Estimation

Since the log-likelihood function of Eq. (5) is convex, the MAP estimate of *X* is obtained by taking the first derivative:

$$\Sigma^{-1}(Y - X) - \mathbf{B}X = 0 \,. \qquad (6)$$

By solving Eq. (6) with the Point-Jacobian iteration (Varga, 1962), the noise-free intensity can be recovered iteratively (Lee, 2007a): given an initial estimate, $\hat{x}_i^0$, at the *h*th iteration for $\forall i \in I_n$

$$\hat{x}_i^h = \frac{1}{\sigma_i^{-2} + \beta_{ii}} \left(\sigma_i^{-2} y_i - \sum_{(i,j) \in C_p} \beta_{ij} \hat{x}_j^{h-1} \right). \qquad (7)$$

Various regions constituting an image can be characterized by textural components. The bonding strength coefficients of Eq. (4) are associated with local interaction between neighboring pixels and can provide some contextual information on the local region. It is important to choose the coefficients suitable for the analyzed image. Given a constant r, the Bayesian MAP estimation of Eq. (5) can be considered as an optimization problem:

$$\underset{X}{\arg\min}\left\{\sum_{i\in I_n}\sum_{(i,j)\in C_p}\alpha_{ij}(x_i-x_j)^2\right\} \quad (8)$$

$$\text{subject to } \sigma_k^{-2}(y_k-x_k)^2<r,\ \forall k\in I_n.$$

From the optimization of Eq. (8), the bonding strength coefficient $\beta_{ij}=\phi_i\alpha_{ij}$ can be estimated as (Lee, 2007a)

$$\hat{\phi}_i=\sqrt{\frac{r}{\sigma_i^2\sum_{(i,j)\in C_p}\alpha_{ij}(y_i-y_j)^2}},\ \forall i\in I_n$$

$$\hat{\alpha}_{ik}=\begin{cases}\dfrac{(y_i-y_k)^2}{\sum_{(i,j)\in C_p}(y_i-y_j)^2} & \text{for } (i,k)\in C_p\\ 0 & \text{otherwise}\end{cases}. \quad (9)$$

The constant r is a parameter related to the distribution of Y and its appropriate choice is unit value.

4. Adaptive Iteration MAP Estimation

The adaptive iteration method differs from the Point-Jacobian iteration only in the estimation of bonding strength coefficients. The coefficients are computed in Eq. (9) using the values estimated at the previous iteration for the adaptive approach (Lee, 2007b): for the hth iteration,

$$\hat{\alpha}_{ik}=\begin{cases}\dfrac{(\hat{x}_i^{h-1}-\hat{x}_k^{h-1})^2}{\sum_{(i,j)\in C_p}(\hat{x}_i^{h-1}-\hat{x}_j^{h-1})^2} & \text{for } (i,k)\in C_p\\ 0 & \text{otherwise}\end{cases}.$$

$$\hat{\phi}_i=\sqrt{\frac{r}{\sigma_i^2\sum_{(i,j)\in C_p}\hat{\alpha}_{ij}(\hat{x}_i^{h-1}-\hat{x}_j^{h-1})^2}},\ \forall i\in I_n \quad (10)$$

In the adaptive approach, the observed process is newly defined at each iteration by considering the observation, which is needed to be despeckled, as the estimate generated at the previous iteration.

In most applications, the variances of pixels are not known, and the true intensity image of observed scene, which is required to estimate them, are also not given. In this study, the variance σ_{ij}^2 was estimated using the average value of observed intensities in the neighbor-window related to the clique system:

$$\hat{\sigma}_{ij}^2=\frac{\sum_{pq\in W_{ij}}(x_{pq}-\hat{\mu}_{ij})^2}{n_w},\quad \hat{\mu}_{ij}=\frac{\sum_{pq\in W_{ij}}x_{pq}}{n_w} \quad (11)$$

where $\{x_{ij}\}$ is the observed process, n_w is the number of pixels in the neighbor-window, and W_{ij} is the set of indices belonging to the window centered at the (i,j)th pixel. It is difficult to find a correct textural component in the noisy observation.

5. Adaptive Estimation with Boundary Correction

The MAP estimation using the neighbor window would use wrong information from adjacent regions for the pixels located in the region close to or on the boundary. To overcome this problem, the new method is designed to use smaller sizes of the neighbor window and lower values of the bonding coefficients as the location of pixels is closer to the boundary. Small windows can reduce the possibility to involve the pixel values of adjacent region with different characteristics, and the low values of the coefficients can make the estimation fit to the own value of the pixel more than the values of neighbor pixels.

Homogeneity is mainly related to the local information of an image and reflects how uniform an image region is. Since a region including the boundary is non-uniform, the homogeneity plays important role to find the region close to or on the boundary. Cheng *et al.* (2003) suggested a homogeneity measurement which is in [0, 1], and the larger the value, the more homogeneous the region is. The measurement is computed using edge value, standard deviation, entropy, and the 4th moment of intensity distribution for a given window centered at the (i,j)th pixel and its set of pixel indices, W_{ij}^U.

Edge value measures the abrupt changes in gray levels. Sobel operator is used to calculate the edge value for the (i,j)th pixel. At each pixel location, there are two components: s_1, corresponding to the result from the row mask, and s_2, the result from the column mask. The magnitude of the gradient at location (i,j) as the measurement (Gonzales and Woods, 2002):

$$e_{ij} = \sqrt{s_1^2 + s_2^2}\ . \qquad (12)$$

Standard deviation describes the dispersion within a local region. It can be calculated as in Eq. (11):

$$s_{ij} = \sqrt{\frac{\sum_{pq \in W_{ij}^U} (x_{pq} - \hat{\mu}_{ij})^2}{n_w}}\ . \qquad (13)$$

Entropy can also be used to describe the distribution variation in a region (Dash and Chatterji, 1991). Entropy of pixel (i,j) can be calculated as

$$h_{ij} = \frac{1}{\ln n_w} \sum_{k=1}^{L} P_k \ln P_k \qquad (14)$$

where P_k is the probability of the kth gray level, and L is the total number of gray levels in the window.

The 4th moment of intensity distribution can be used to describe the impulsiveness of the distribution (Sterling and Pollack, 1968) which can be computed through

$$\gamma_{ij} = \frac{\sum_{pq \in W_{ij}^U \text{ and } pq \neq ij} (x_{ij} - \hat{\mu}_{ij})^4}{n_w - 1}\ . \qquad (15)$$

Homogeneity represents the uniformity. If the region is perfectly uniform, the values of Eqs. (12 – 15) are all 0. A uniformity measure can be defined as

$$U_{ij} = \left[1 - e_n(W_{ij}^U)\right] \times \left[1 - s_n(W_{ij}^U)\right] \times \left[1 - h_n(W_{ij}^U)\right] \times \left[1 - \gamma_n(W_{ij}^U)\right] \qquad (16)$$

where

$$e_n(W_{ij}^U) = \frac{e_{ij}}{\max_{\forall pq}\{e_{pq}\}},\ s_n(W_{ij}^U) = \frac{s_{ij}}{\max_{\forall pq}\{s_{pq}\}},$$
$$h_n(W_{ij}^U) = \frac{h_{ij}}{\max_{\forall pq}\{h_{pq}\}},\ \gamma_n(W_{ij}^U) = \frac{\gamma_{ij}}{\max_{\forall pq}\{\gamma_{pq}\}}.$$

The proposed scheme is designed to use smaller sizes of the neighbor window related to the clique system and lower values of r in Eq. (10) for the adaptive iteration MAP estimation. The window size of the uniformity measure must be larger than one of the neighbor window.

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