A MULTI-OBJECTIVE OPTIMIZATION FOR CAPITAL STRUCTURE IN PRIVATELY-FINANCED INFRASTRUCTURE PROJECTS

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Abstract

Private financing is playing an increasing role in public infrastructure construction projects worldwide. However, private investors/operators are exposed to the financial risk of low profitability due to the inaccurate estimation of facility demand, operation income, maintenance costs, etc. From the operator's perspective, a sound and thorough financial feasibility study is required to establish the appropriate capital structure of a project. Operators tend to reduce the equity amount to minimize the level of risk exposure, while creditors persist to raise it, in an attempt to secure a sufficient level of financial involvement from the operators. Therefore, it is important for creditors and operators to reach an agreement for a balanced capital structure that synthetically considers both profitability and repayment capacity. This paper presents an optimal capital structure model for successful private infrastructure investment. This model finds the optimized point where the profitability is balanced with the repayment capacity, with the use of the concept of utility function and multi-objective GA (Generic Algorithm)-based optimization. A case study is presented to show the validity of the model and its verification. The research conclusions provide a proper capital structure for privately-financed infrastructure projects through a proposed multi-objective model.

Keywords: Infrastructure, Multi-objective optimization, Optimal capital structure, Project financing, Genetic algorithm

1. Introduction

The amount of public funds required to build new infrastructures and to maintain existing infrastructures is insufficient worldwide. In order to maintain the high standard of living quality that any modern society requires, additional funding sources are highly sought. In this context, private financing is playing an increasingly more important role, in order to keep the service of modern infrastructure at a satisfactory level. Without this increase of private funds, it is extremely challenging to meet the ever-increasing societal demand for new infrastructures. However, there exist many risk variables that need to be given careful consideration due to the uncertain nature of a project's future profitability. For the success of privately-funded infrastructure, a high level of financing skill is necessary to produce a reasonable and convincing plan that satisfies the range of stakeholders involved.

Founding a special-purpose company (SPC) as a separate entity has become a quite common practice to fund all or part of mega-scale infrastructure construction projects. An SPC has interesting and unique characteristics, as compared to other general companies, in view of off-balance sheets and non-recourse financing. An off-balance sheet indicates that the business activities of the SPC are not reflected in the balance sheet of the SPC shareholders. In other words, the impact that the business failure of the SPC can have on the shareholders is limited; the maximum loss that the shareholders can incur from the SPC failure depends only on the total investment amount. More to an off-balance sheets, non-recourse financing indicates that the SPC can attract additional investment from other lenders, based on its potential future profits, without security. This nature of an SPC enables it to become a popular business model for privately-financed infrastructure investment.

Shareholders of an SPC desire to minimize their investment in the SPC, in order to reduce the amount of their risk exposure as much as possible. This explains why the shareholders want to establish the SPC in the first place. However, other lenders who participate in the infrastructure projects that the SPC pursues want to demand a significant portion of investment from the SPC shareholders, to ensure that the shareholders assume the proper amount of risk. Therefore, the SPC needs to find an optimal point where the SPC's perspective is balanced with that of the lenders. In other words, it is necessary to determine the capital structure with which the SPC's profitability can be maximized under the constraints imposed by the lenders. This paper presents an optimal capital structure model for successful private infrastructure investment from the standpoint of an SPC. This model uses the concept of utility function and multi-objective optimization. A case study is presented to show the validity of the model and its verification.

2. Previous Studies

Since the mid-1990s, the number of studies of privately-financed infrastructure projects has significantly increased. Firstly, Dias and Ioannou (1995) closely examined the relationship between debt-service coverage and the optimal capital structure of privately-financed projects, using the capital asset price method (CAPM). Wooldridge et al. (2001) described the role of financial information among the components such as condition assessment, planning, financing, and acquisition. Also, they provided the basis for characterizing the effects of accounting and budgeting on capital allocation through case studies of infrastructure development and recent applications of a decision-support system that assists engineers and planners with the analysis and comparison of infrastructure production strategies. Recently, Bakatjan et al. (2003) and Zhang (2005) presented schemes optimizing capital structure in privately-financed investment projects, using a financial feasibility study. Bakatjan optimized the capital structure with various financial feasibility indices through variation of the equity ratio, using linear programming. Zhang, on the other hand, proposed a scheme of optimizing capital structure through financial viability analysis, including a risk concept, using Monte Carlo simulation (MCS). Though both Bakatjan and Zhang presented the optimization of capital structure, their researches were limited in that they did not reflect the uncertainty of the determination procedure of capital structure. For instance, all variables in a model were assumed as fixed values and further the decision criteria of equity ratio were impractically simplified by using a single criterion—its potential profit.

Associated with previous international researches, the studies on privately-financed investment projects in Korea began in the mid of 1990s, but have been much focused on policy-supporting researches, such as rules and regulations, procedures of a project initiation, and criteria for a candidate selection, performed in the KDI (Korea Development Institute) and the KRIHS (Korea Research Institute for Human Settlement). Recently, the range of the research has gradually expanded to include various topics such as profitability, financial viability, and risk analysis in privately-financed infrastructure projects.

3. Decision Model for Optimizing Capital Structure

3.1 Assumptions for Decision Model

For building a decision model for optimal capital structure, it is necessary to decide the relevant financial indices and quantify those variables. For this reason, the boundary conditions for the decision model need to be assumed first. The following are the assumptions for this model:

1. The scope of this research is limited to a Built- Operate-Transfer (BOT) and a Built-Transfer-Operate (BTO) project.
2. Project financing consists of combination with equity and debt, and net cash flow runs negative (-) during the construction period; however, net cash flow is positive (+) during the operations period.
3. It is possible to loan from one or two more funds with annual equal repayment conditions.
4. The grace period for debt service is the same as the construction period.
5. Land expropriation cost is added to the basic cost
6. Cash flow during the construction period is measured in advance.
7. The structure made from the total project cost is depreciated at a fixed rate during its life span.
8. The entire value of the total project cost is completely terminated during the concession period.
9. The development cost is the construction cost.
10. The less the ratio of equity capital, the more the risk of the lenders; therefore, the debt interest rate is described as a function of equity level

3.2 Financial Formulation and Variables for Capital Structure

To build the decision model for optimizing capital structure, the formulations and variables of the previous financial models (Dias and Ioannou, Bakatjan, and Zhang) can be useful. The financial variables consist of total project cost (TPC), net annual cash available (NAC), profit before interest and tax (PBIT), depreciation (DEP), and so on. Using these variables, financial viability indices such as net present value (NPV), internal ratio of return (IRR), and debt-service coverage ratio (DSCR) are calculated. Among these financial viability indices, while both NPV and IRR are the economic decision criteria from the shareholders' point of view, DSCR is the only criterion from the perspective of the lenders.

IRR and NPV are the most common and fundamental economic decision criteria employed in practice (Lohmann 1988). The NPV from the point of view of the shareholders in a special purpose company is;

$$NPV = -\sum_{j=1}^{c} \frac{E_i}{(1+d)^{j-1}} + \sum_{i=1}^{m} \frac{NCA_i}{(1+d)^{i+c}} \quad \text{Eqn. (1)}$$

$$\text{for } i = 1,2,...,m, \text{ for } j = 1,2,...,c$$

where m = concession period; c = construction duration; E_i = equity drawing in i th year of construction; NCA_i = net annual cash available j th year of operation; and d = the discount rate (Bakatjan 2003). The IRR is decided when the NPV is zero (=0).

On the other hand, DSCR_{avg} is the most generic financial decision standard in deciding a loan. Therefore, it is the lender's main criterion for a project's financial viability. It is calculated as;

$$DSCR_i = \frac{PBIT_i + DEP_i - TAX_i}{D_i} \text{ for } i = 1,2,...,m \quad \text{Eqn. (2)}$$

Hence

$$DSCR_{avg} = \frac{\sum_{1}^{m} DSCR_i}{m} \text{ for } i = 1,2,...,m \quad \text{Eqn. (3)}$$

Where $DSCR_i$ = debt service coverage ratio i th year of operation; $PBIT_i$ = profit before interest and tax i th year; DEP_i = depreciation; TAX_i = corporation tax i th year of operation; D_i = annual debt installment i th year. Using these formulations, this research presents the object function for maximizing IRR with the proper proportion of DSCR_{avg}.

3.3. Risk Utility Function for Optimization

This research optimized capital structure using multi-criteria, IRR and DSCR_{avg} among financial viability indices, because IRR represents the shareholder's position and DSCR describes the lender's at a glance. The term "optimization" refers to the study of problems in which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within an allowed set in the same dimension. However, it is impossible to optimize the multi-object function composed of IRR and DSCR_{avg} directly, because the dimension of IRR value is quite different from that of the DSCR value. Moreover, IRR is a discrete value calculated from the formulation of NPV, where NPV reaches zero (=0); however, DSCR_{avg} is an average value of DSCR computed from cash flow in each time. Therefore, it is unreasonable to find the optimal point of capital structure by directly comparing IRR with DSCR_{avg}. For transforming two independent values with different dimensions into one value with the same dimensions, it is necessary to introduce to the model the concept of utility function in microeconomics. According to a decision-maker's risk attitude, utility function is divided into three types: risk-taking, risk-neutral, and risk-averse. According to previous researches, it is generally known that the broad configuration of risk utility function describes one of the exponential functions (Clemen 2001). Based on this research result, the risk utility function applies to the parameter function of IRR and $\mathrm{DSCR}_{\mathrm{avg}}$, respectively.

3.3.1 Risk Utility Function for IRR

The following is the proposed procedure for deriving risk utility function for IRR:

1. IRR, the ratio of required profit for accomplishing a project, must be at least larger than the interest rate for debt($IRR \geq IRR_{\min}$)
2. It is assumed that the utility value is zero (= 0) when IRR is less than the interest rate and the utility value is 1 when IRR is the same as the ratio of required profit ($IRR_{req.}$).
3. From the operator's point of view, the type of utility function is a form of risk-taking because the utility of IRR increases when the same increment of IRR is closer at the point of the ratio of required profit than at the minimum criterion point. Therefore, the broad shape of utility function for IRR is concave as shown in the following figure 1.

According to Clemen (2001), The formulation of utility function which is risk-taking is;

- Fundamental form: $U_{IRR} = \alpha + \beta(1 - e^{IRR})$ Eqn. (4)
- Boundary condition: $IRR < IRR_{\min}$, $U_{IRR} = 0$ Eqn. (5)
  $IRR \geq IRR_{req.}$, $U_{IRR} = 1$ Eqn. (6)
- Accordingly, the risk utility function for IRR can be formulated:

$$U_{IRR} = 0.5 \times \left(\frac{\left| e^{IRR} - e^{IRR_{\min}} \right| - \left| e^{IRR} - e^{IRR_{req.}} \right|}{e^{IRR_{req.}} - e^{IRR_{\min}}} \right) + 0.5 \qquad \text{Eqn. (7)}$$

Figure 1: Risk Utility Function for IRR

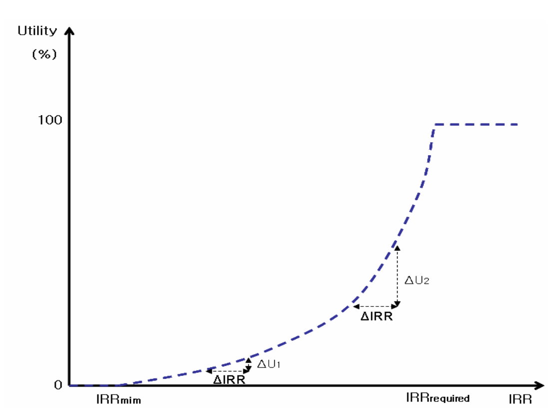
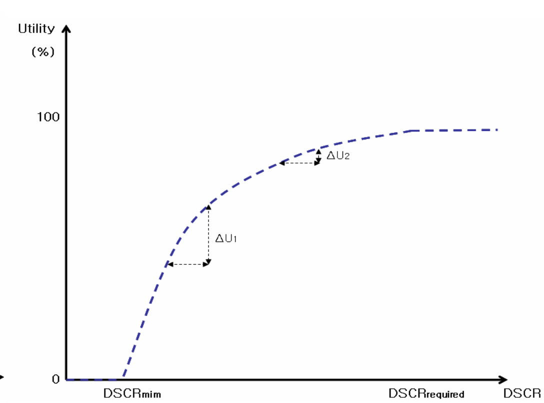


Figure 2: Risk Utility Function for DSCR



3.3.2 Risk Utility Function for DSCR

The following is another procedure for risk utility function of DSCR:

1. DSCR, the ratio of cash flow available to meet annual interest and principal payments on debt, must be at least larger than 1 ($DSCR \geq DSCR_{\min}$)
2. It is assumed that the utility value is zero (= 0) when DSCR is less than 1 and the utility value is 1 when DSCR is the same as the ratio of required debt service coverage ($DSCR_{req.}$).
3. From the operator's point of view, the type of utility function is a form of risk-averse function, because the utility of DSCR decreases when the same increment of DSCR is closer at the point of the ratio of required debt service coverage than at the minimum criterion point. Therefore, the broad shape of utility function for DSCR is convex as shown in figure 2.

The formulation of utility function which is risk-averse is;

- Fundamental Form: $U_{IRR} = \alpha + \beta(1 - e^{-DSCR})$ Eqn. (8)
- Boundary condition: $DSCR < DSCR_{\min}(=1)$, $U_{DSCR} = 0$ Eqn. (9)

  $DSCR \geq DSCR_{req.}$, $U_{DSCR} = 1$ Eqn. (10)
- Accordingly, the risk utility function for DSCR can be formulated:

$$U_{DSCR} = 0.5 \times \left(\frac{\left| e^{-DSCR} - e^{-DSCR_{req.}} \right| - \left| e^{-DSCR} - e^{-DSCR_{\min.}} \right|}{e^{-DSCR_{req.}} - e^{-DSCR_{\min}}} \right) + 0.5 \quad \text{Eqn. (11)}$$

3.4. Object Function for Multi-objective Optimization

For optimizing capital structure, it is important to maximize IRR, but at the same time to maintain a stable DSCR. Accordingly, this model should be satisfied with two objectives for optimization respectively. To address this condition, this paper uses multi-objective optimization, and formulates the object function as a total risk utility function. The total-risk utility function for optimization is described by the summation of the utility function of IRR and DSCR in association with each weight.

The following are the multi-objective function and boundary condition for satisfying this requirement:

$$U_T = k_1 U_{IRR} + k_2 U_{DSCR},\ k_1 + k_2 = 1,\ 0 \leq k_1 \leq 1,\ 0 \leq k_2 \leq 1 \quad \text{Eqn. (12)}$$

Therefore, the multi-objective function for optimizing capital structure is the function by which the total utility function can be maximized. The following is the object function:

$$U_T = k_1 U_{IRR} + k_2 U_{DSCR} \quad \text{Eqn. (13)}$$

$$= k_1 \times \left(0.5 \times \left(\frac{\left| e^{IRR} - e^{IRR_{\min}} \right| - \left| e^{IRR} - e^{IRR_{req.}} \right|}{e^{IRR_{req.}} - e^{IRR_{\min}}} \right) + 0.5 \right)$$

$$+ k_2 \times \left(0.5 \times \left(\frac{\left| e^{-DSCR} - e^{-DSCR_{req.}} \right| - \left| e^{-DSCR} - e^{-DSCR_{\min.}} \right|}{e^{-DSCR_{req.}} - e^{-DSCR_{\min}}} \right) + 0.5 \right)$$

3.5. Multi-objective optimization using multi-objective genetic algorithm

There are many solutions and methods to solve optimization problems: linear programming, dynamic programming, genetic algorithm, etc. These optimization methods are affected by forms of object function and constraint function, the number of determinant variables and so on. Basically, however, the most significant purpose of these optimization methods is deciding the proper values of determinant variables for providing optimal value under the specific constraints. Accordingly, it is necessary to choose the proper optimization method for the object function. Bakatjan (2003) applied the linear programming method to optimize capital structure in a BOT project. However, the result of the model using linear programming was not exactly the optimal point, because the form of the functions of IRR and DSCR is not linear by nature as previously demonstrated. Even if the gap between the actual point and calculated point is small, it is not an exact optimal point in a straightforward sense. Therefore, this paper uses a genetic algorithm for optimizing capital structure with the non-linear object functions.

A genetic algorithm is a search technique used in computing to find true or approximate solutions to optimization and search problems, and is often abbreviated as GA. Genetic algorithms are categorized as global search heuristics (Michalewicz 1994). Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover (also called recombination). For solving an optimization problem with two or more object functions, it is necessary to apply multi-objective genetic algorithm, so-called MOGA. The MOGA was first introduced by Fonseca and Fleming (1993).

The MOGA functions by seeking to optimize the components of a vector-valued objective function. Unlike single-objective optimization, the solution to a multi-objective optimization problem is a family of points known as the Pareto-optimal set. Each point in the set is optimal in the sense that no improvement can be achieved in one component of the objective vector that does not lead to degradation in at least one of the remaining components. Using this algorithm, this paper optimizes the object function with IRR and DSCR. As an application program for simulating MOGA, this paper utilizes GATOOLS in MATLAB version 7.0. GATOOLS is the graphic user interface toolbox in MATLAB and can be easily applied to the programming for MOGA.

3.6. Decision support model for multi-objective optimization

The model for optimizing capital structure consists of the following five steps:

1. STEP I is the data collecting and processing module. This module collects and processes the basic data of a privately-financed investment project, macroeconomic indices, and period for analysis
2. STEP II is the presumed financial statement module. This module composes the presumed financial statement from the financial stochastic variables macro-economic indices. Moreover, it analyzes the cash flow of the project and derives financial viability index equations such as IRR and DSCR from the presumed financial statement.
3. STEP III is the risk utility function module. This module transfers financial viability index equations into a risk-utility function for eliciting the object function for optimization.
4. STEP IV is the multi-objective optimization module. This module optimizes the object function on the boundary condition using a multi-objective genetic algorithm.

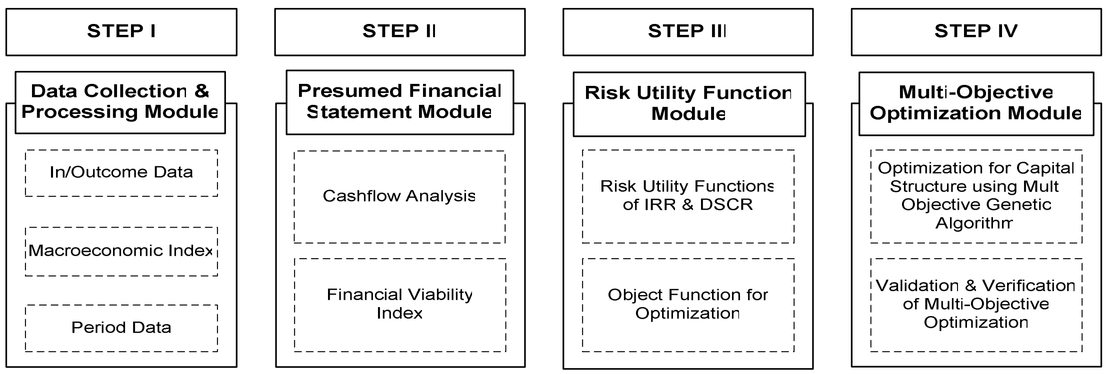


Figure 3: Step-wised approach for capital structure optimization

4. Case Study: Incheon International Airport Railway Project

4.1. Building the Model

For verification and validity of the model for capital structure in this research, this section illustrates the application of the model to an actual project. The project for case study is the Incheon international airport railway project, which is under construction with engaging a contract in 2001. Though a business plan and financial report on the project are needed for a substantial case study, these documents are generally not opened to the public because they are strictly confidential and very sensitive in nature. Therefore, this research uses and reconstitutes the data of the contract document between the special-purpose company for the project and the Government partner—Korea Railroad Network Agency. For application of the model, the following "Eqn. (14)" is set up as the object function for multi-objective optimization with risk utility functions for IRR and DSCR of the project:

$$U_T = 0.5 \times U_{IRR} + 0.5 \times U_{DSCR} \qquad \text{Eqn. (14)}$$

On this function, basic values are assumed that $IRR_{\min}$ is 7.6%, $IRR_{req.}$ is 10.43%, $DSCR_{\min}$ is 1, and $DSCR_{req.}$ is 3, based on the actual contract condition. Also, equity ratio (e) in the capital structure ranges from 20% to 50%. The weight of each term in the function decided in the boundary condition: $k_1 + k_2 = 1$, $0 \le k_1 \le 1$, $0 \le k_2 \le 1$. The object function has the default weight values ($k_1 = k_2 = 0.5$) and their sensitivities depending on the different weights are also tested.

Applying this object function, this research runs the simulation for optimization using programming of a multi-objective genetic algorithm (MOGA). The following is the constitution of the model for optimizing capital structure. Since the basic calculation of MOGA is fitted to the minimum optimization, the objective function is multiplied by -1 for maximum optimization.

- Object Function: Maximize $U_T = -(0.5 \times U_{IRR} + 0.5 \times U_{DSCR})$ Eqn. (15)
- Constraints: Subject to $0.2 \le e \le 0.5$ Eqn. (16)
  $IRR \ge IRR_{\min}$ or $NPV > 0$ Eqn. (17)
  $DSCR_{avg.} \ge 1$, for $i = 1,2,\ldots,m$ Eqn. (18)

Table 1: Conditions of the Model for Optimizing Capital Structure using GATOOL

Items	Values	Remarks
Population Size	100	
Generation Number	100	
Initial Range *(e)*	[0.2 0.5]	
Crossover Function	Scattered	
Mutation Function(initial)	Gaussian Function	Scale=1.0, Shrink=1.0

4.2 Result from the simulation for optimization

Firstly, the optimization model is simulated, using default values without uncertainty of input variables where k_1, the weight of U_{IRR}, is 0.5 and k_2, the weight of U_{DSCR}, is 0.5. As a result, when the maximum value of object function, U_T, is 0.852, the equity ration (e) is optimized at 22.3%.

Table 2: The Simulation Results for Optimization

Simulation Result	Base Line(mean)
U_T	0.852
Equity Ratio(e)	22.3%
IRR	10.43%
DSCR	1.938

Hence, the decision-maker, the operator of the project, can be suggested to decide the equity ratio with less than 22.3% for securing the financial viability of the project reliably in negotiation with the lender.

5. Conclusion

As a result of optimization simulation, the optimal equity ratio is actually calculated by 22.3%, which is quietly lower than 30%, the equity ratio presented in the actual contract. This result indicates that the profit of the project would be established, from the operator's point of view, if the operation income is sustained, even in the case that the operator decides lower equity ratio than base rate. When equity ratio goes down, the financial viability is generally on the decline because the more increase in debt, the more increase in financial cost.

This research can contribute to project management for privately-financed investment project in three aspects. First, this paper proposes a method to decide the proper equity ratio at the negotiation stage in privately-financed investment projects through a financial viability assessment considering possible project uncertainty. In addition, this research contributes toward the reduction of the operator's financial cost by lowering the equity ratio as the operator's ability to analyze the project viability is improved.

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