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유사측도에 기반한 퍼지 엔트로피구성

Fuzzy Entropy Construction based on Similarity Measure

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Abstract

In this paper we derived fuzzy entropy that is based on similarity measure. Similarity measure represents the degree of similarity between two informations, those informations characteristics are not important. First we construct similarity measure between two informations, and derived entropy functions with obtained similarity measure. Obtained entropy is verified with proof. With the help of one-to-one similarity is also obtained through distance measure, this similarity measure is also proved in our paper.

Key Words : Similarity measure, distance measure, fuzzy entropy, one-to-one correspondence

1. Introduction

Similarity represents the degree of similarity between two or more data. Hence computation of similarity is essential for the fields of decision making, pattern classification, or etc.. Quantity of difference can be useful to discriminate or cluster for various informations. Until now the research of designing similarity measure has been made by numerous researchers[1-6]. For fuzzy set, there is an uncertainty knowledge in fuzzy set itself. Hence information of the data can be obtained from analyzing the fuzzy membership function. Thus most studies about fuzzy set are focussed on designing similarity measure based on membership function. In the previous results, similarity measures are obtained through fuzzy number[1-4]. Fuzzy number provide similarity measure easily. However considering similarity measures are restricted within triangular or trapezoidal membership functions[1-4]. In this paper we try to analyze relations between fuzzy entropy and similarity. Furthermore with the help of distance measure, we design the similarity measure. Proposed similarity measure produce fuzzy entropy based on relation between fuzzy entropy and similarity measure.

In this paper, first we introduce the properties of fuzzy entropy, distance measure and similarity measure. Similarity measure is also proposed using distance measure. Proposed similarity measure construct fuzzy entropy with the relation of fuzzy entropy and similarity measure. Notations of this paper are used Liu's [7].

2. Similarity measure and fuzzy entropy analysis

In this chapter, introduction of fuzzy entropy and similarity measure are carried out. Furthermore similarity measures are also proposed with distance measure.

2.1 Introduction of fuzzy entropy

Axiomatic definitions of fuzzy entropy and similarity measure represents the difference or closeness for different fuzzy membership functions [7]. By this definition, we design a similarity measure.

Definition 2.1 [7] A real function e: $F(X) \to R^+$ is called an entropy on $F(X)$, if e has the following properties:

(E1) $e(D)=0,\ \forall D \in P(X)$

(E2) $e([1/2]) = max_{A \in F(X)} e(A)$

(E3) $e(A^*) \le e(A)$, for any sharpening A^* of A

(E4) $e(A) = e(A^c),\ \forall A \in F(X)$

where $[1/2]$ is the fuzzy set in which the value of the membership function is $1/2$, $R^+ = [0, \infty)$, X is the universal set, $F(X)$ is the class of all fuzzy sets of X, $P(X)$ is the class of all crisp sets of X, and D^c is the complement of D. There are a lot of fuzzy entropy satisfying Definition 2.1. Hamming distance is commonly used as distance measure between fuzzy sets A and B,

$$d(A,B) = \frac{1}{n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$$

where $X = \{x_1, x_2, \cdots x_n\}$, $|k|$ is the absolute value of k. $\mu_A(x)$ is the membership function of $A \in F(X)$. Basically fuzzy entropy means the difference between two fuzzy membership functions. Next we will introduce the similarity measure, and it describes the degree of closeness between two fuzzy membership functions. It is also found in literature of Liu.

Definition 2.2 [7] A real function s: $F^2 \to R^+$ is called a similarity measure, if s has the following properties:

(S1) $s(A,B) = s(B,A), \forall A,B \in F(X)$

(S2) $s(D,D^c) = 0\ \ \forall D \in P(X)$

(S3) $s(C,C) = max_{A,B \in F} s(A,B),\ \forall C \in F(X)$

(S4) $\forall A,B,C \in F(X)$, if $A \subset B \subset C$, then $s(A,B) \ge s(A,C)$ and $s(B,C) \ge s(A,C)$.

Liu also pointed out that there is an one-to-one relation between all distance measures and all similarity measures, $d+s=1$. Fuzzy normal similarity measure on F is also obtained by the division of $\max_{C,D \in F} s(C,D)$. With Definition 2.2, we propose the following theorem as the similarity measure.

And the similarity measure construction with the distance measure is obtained in Theorem 2.1.

Theorem 2.1 For any set $A,B \in F(X)$, if d satisfies Hamming distance measure and $d(A,B) = d(A^C,B^C)$, then

$$s(A,B) = 1 - d((A \cap B^C),[0]) - d((A \cup B^C),[1]) \quad (1)$$

is the similarity measure between set A and set B.

proof. Commutativity of (S1) is proved through

$$\begin{aligned} s(A,B) &= 1 - d((A \cap B^C),[0]) - d((A \cup B^C),[1]) \\ &= 1 - d((A \cap B^C)^C,[0]^C) - d((A \cup B^C)^C,[1]^C) \\ &= 1 - d((B \cup A^C),[1]) - d((B \cap A^C),[0]) \\ &= s(B,A). \end{aligned}$$

To show the property of (S2),

$$\begin{aligned} s(A,A^c) &= 1 - d((A \cap (A^c)^c),[0]) - d((A \cup (A^c)^c),[1]) \\ &= 1 - (d(A,[0]) + d(A,[1])) \\ &= 1 - 1 \cdot 1 = 0 \end{aligned}$$

is clear. (S3) is clear from the relation

$$\begin{aligned} s(A,B) &= 1 - d((A \cap B^c),[0]) - d((A \cup B^c),[1]) \\ &\leqq 1 - d((D \cap D^c),[0]) - d((D \cup D^c),[1]) \\ &= s(D,D), \end{aligned}$$

where the inequality is proved by

$$d((A \cap B^c),[0]) \geqq d((D \cap D^c),[0])$$

and $d((A \cup B^c),[1]) \geqq d((D \cup D^c),[1])$.

Finally, $\forall A,B,C \in F(X)$ and $A \subset B \subset C$ imply

$$\begin{aligned} s(A,B) &= 1 - d((A \cap B^c),[0]) - d((A \cup B^c),[1]) \\ &= 1 - d([0],[0]) - d((A \cup B^c),[1]) \\ &\geqq 1 - d(A \cap C^c,[0]) - d(A \cup C^c,[1]) \\ &= 1 - d([0],[0]) - d(A \cup C^c,[1]) \\ &= s(A,C). \end{aligned}$$

$$\begin{aligned} s(B,C) &= 1 - d((B \cap C^c),[0]) - d((B \cup C^c),[1]) \\ &= 1 - d([0],[0]) - d((B \cup C^c),[1]) \\ &\geqq 1 - d(A \cap C^c,[0]) - d(A \cup C^c,[1]) \\ &= 1 - d([0],[0]) - d(A \cup C^c,[1]) \\ &= s(A,C) \end{aligned}$$

is also satisfied with

$d((A\cup B^c),[1]) \geq d((A\cup C^c),[1])$ and $d((B\cup C^c),[1]) \geq d((A\cup C^c),[1])$.

We have proposed the similarity measure that are induced from distance measure. This similarity is useful for the non interacting fuzzy membership function pair. Another similarity is also obtained, and it can be found in our previous literature [6].

Theorem 2.2 For any set $A,B\in F(X)$ if d satisfies Hamming distance measure, then

$$s(A,B)=2-d((A\cap B),[1])-d((A\cup B),[0]) \quad (2)$$

To be a similarity measure, Theorem 2.2 does not need condition $d(A,B)=d(A^C,B^C)$. Because commutativity is clear from the theorem itself. Also this similarity (2) is useful for the interacting membership function pair. In the next section, we derive similarity measure that is generated by distance measure. Furthermore entropy is derived through similarity measure by the properties of Liu.

2.2 One-to-one correspondence

It is obvious that next Hamming distance is represented as

$$d(A,B)=d((A\cap B),[1])-(1-d((A\cup B),[0])).$$

Where $A\cap B=min(\mu_A(x_i),\mu_B(x_i))$, $A\cup B=max(\mu_A(x_i),\mu_B(x_i))$ are satisfied. With the Proposition 3.4 of Liu[7], we can generate the similarity measure or distance measure through distance measure or similarity measure [7].

Proposition 2.1[7] There exists an one-to-one correlation between all distance measures and all similarity measures, and a distance measure d and its corresponding similarity measure s satisfy $s+d=1$. With the property of $s=1-d$, we can construct the similarity measure generated by distance measure d, that is $s<d>$.

$$d(A,B)=d((A\cap B),[1])+d((A\cup B),[0])-1$$
$$=1-s(A,B)$$

Therefore we propose the similarity measure with above expression.

$$s<d>=2-d((A\cap B),[1])-d((A\cup B),[0]) \quad (3)$$

This similarity measure is exactly same with (2). At this point, we verified the one-to-one relation of distance measure and similarity measure. In the next chapter, we verify that the entropy of fuzz set is derived through similarity (2).

3 Entropy derivation with similarity measure

Liu also suggested propositions about entropy and similarity measure. He also insisted that the entropy can be generated by similarity measure and distance measure, those are denoted by $e<s>$ and $e<d>$. Proposition 3.5 and 3.6 of reference [7] are summarized as follows.

Proposition 3.1 [7] If s is a similarity measure on F, define

$$e(A)=s(A,A^c),\ \forall A\in F.$$

Then e is an entropy on F. Similarly, it is obvious that the following proposition is satisfied. Now we check whether our similarities (1) and (2) satisfy Proposition 2.2. Proof can be obtained by checking whether

$s(A,A^c)=2-d((A\cap A^c),[1])-d((A\cup A^c),[0])$ satisfy from (E1) to (E4) of Definition 2.1.

For (E1), $\forall D\in P(X)$,

$$s(D,D^c)=2-d((D\cap D^c),[1])-d((D\cup D^c),[0])$$
$$=2-d([0],[1])-d([1],[0])=0$$

(E2) represents that crisp set $1/2$ has the maximum entropy value. Therefore, the entropy $e([1/2])$ satisfies

$$s([1/2],[1/2]^c)=2-d(([1/2]\cap[1/2]^c),[1])-d(([1/2]\cup[1/2]^c),[0])$$
$$=2-d([1/2],[1])-d([1/2],[0])$$
$$=2-1/2-1/2=1$$

In the above equation, $[1/2]^c=[1/2]$ is satisfied.

(E3) shows that the entropy of the sharpened version of fuzzy set A, $e(A^*)$, is less than or equal to $e(A)$.

$$\begin{aligned} s(A^*,A^{*c}) &= 2-d((A^*\cap A^{*c}),[1])-d((A^*\cup A^{*c}),[0]) \\ &\leqq 2-d((A\cap A^c),[1])-d((A\cup A^c),[0]) \\ &= s(A,A^c) \end{aligned}$$

Finally, (E4) is proved directly

$$\begin{aligned} s(A,A^c) &= 2-d((A\cap A^c),[1])-d((A\cup A^c),[0]) \\ &= 2-d((A^c\cap A),[1])-d((A^c\cup A),[0]) \\ &= s(A^c,A) \end{aligned}$$

From the above proof, our similarity measure
$s(A,A^c)=2-d((A\cap A^c),[1])-d((A\cup A^c),[0])$
generate fuzzy entropy.

Next another similarity (1) between A and A^c is

$$\begin{aligned} s(A,A^c) &= 1-d((A\cap A),[0])-d((A\cup A),[1]) \\ &= 1-d(A,[0])-d(A,[1]) \end{aligned}$$

This similarity always satisfies zero.
For (E1), $\forall D\in P(X)$,

$$s(D,D^c)=1-d(D,[0])-d(D,[1])=0$$

(E2),

$$s([1/2],[1/2]^c)=1-d([1/2],[0])-d([1/2],[1])=0$$

For (E3), $s(A^*,A^{*c})=s(A,A^c)=0$

Finally, (E4) is proved similarly

$$s(A,A^c)=s(A^c,A)=0.$$

From the above proof, our similarity measure
$s(A,A^c)=1-d((A\cap A),[0])-d((A\cup A),[1])$
generate fuzzy entropy trivially.

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4. Conclusions

We have derived the similarity measure that is derived from distance measure. The proposed similarity usefulness is proved. Furthermore with the relation between fuzzy entropy and similarity measure we also verified that the fuzzy entropy is induced through similarity measure. In this paper our proposed similarity measure provide the fuzzy entropy. Finally we can see that proposed similarity measure can be applied to the general types of fuzzy membership functions.

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