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Robust Adaptive Fuzzy Observer Based Synchronization of Chaotic Systems

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요 약

This paper proposes an alternative robust adaptive high-gain fuzzy observer design scheme and its application to synchronization of chaotic systems. The structure of the proposed observer is represented by Takagi-Sugeno fuzzy model and has the integrator of the estimation error. This improves the performance of high-gain observer and makes the proposed observer robust against noisy measurements, uncertainties and parameter perturbations as well. Using Lyapunov stability theory, an adaptive law is derived and the stability of the proposed observer is analyzed. Some simulation result is given to present the validity of theoretical derivations and the performance of the proposed observer.

Key Words : *T-S fuzzy model, Robust, Adaptive, Observer, Lyapunov theory, Synchronization*

I. INTRODUCTION

Chaotic systems have been researched and known to exhibit complex dynamical behavior in the past two decades. The interest in chaotic systems lies on nonstandard control problems. Chaotic model following control has unpredictable behavior, and extreme sensitivity to initial conditions as well as parameter variation. Many researches have been proposed to achieve synchronization with exactly known or uncertain conditions [1][12].

T-S fuzzy model has the mathematical simplicity advange of analysis. T-S fuzzy model has been investigated widely in the control and synchronization of chaotic systems [1][8]. Fuzzy systems are supposed to work in situation where there is a large uncertainty or unknown variation in plant parameters and structures. Adaptive schemes for nonlinear systems that incorporate fuzzy systems have been enormously popular [21].

The technique, known as a high-gain observer, is to design the observer gain that makes the observer robust against model uncertainties in nonlinear functions. However, high gains may excite hidden dynamics and amplify measurement noise.

In this paper, an alternative robust adaptive high-gain fuzzy observer design scheme is proposed and applied to synchronization. The proposed observer improves the robustness of the existing high-gain observer against uncertainties and parameter perturbation.

II. PROBLEM FORMULATION

The following chaotic system is considered for the synchronization problem.

$$\dot{x}_{n-1} = x_n$$
$$\dot{x}_n = f(\boldsymbol{X}) + g(\boldsymbol{X})u \qquad (1)$$
$$y = x_1$$

where, $f(\boldsymbol{X})$, $g(\boldsymbol{X})$ are unknown but bounded continuous nonlinear functions. $u \in R$ is a control input and $y \in R$ is an output of the system respectively and it is assume that only y is measurable and the system (1) is observable.

$\boldsymbol{X} = [x_1 \ \ x_2 \ \ \dots \ \ x_n]^T \in R^n$ is the state vector of the system, which is assumed to be unmeasurable.

The state space representation of the system can be described as follows.

$$\dot{X} = AX + B[f(X) + g(X)u] \quad (2)$$
$$y = CX$$

where, $A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$,

$C = [1\,0 \dots 0\,0]$.

The output y is used as a premise variable. Chaotic function can be expressed by T-S fuzzy model as follows.

$$f(X) = \sum_{i=1}^{r} h_i(y) a_i^T X \quad (5)$$

$$g(X) = \sum_{i=1}^{r} h_i(y) b_i \quad (6)$$

The object of this paper is to design a robust adaptive high-gain fuzzy observer for synchronization.

$$f(\hat{X} \mid \hat{\theta}_f) = \sum_{i=1}^{r} h_i(y) \hat{a}_i^T \hat{X} = \hat{\theta}_f^T \xi(\hat{X}) \quad (7)$$

$$g(\hat{X} \mid \hat{\theta}_g) = \sum_{i=1}^{r} h_i(y) \hat{b}_i = \hat{\theta}_g^T \eta(\hat{X}) \quad (8)$$

where, $\hat{\theta}_f = \hat{a}_i \in R^n$ and $\hat{\theta}_g = \hat{b}_i \in R$. $\hat{X} = [\hat{x}_1\, \hat{x}_2 \dots \hat{x}_n]^T \in R^n$ is an estimated state vector. $\xi(\hat{X}) = [h_1(y)\hat{X} h_2(y)\hat{X} \cdots h_r(y)\hat{X}]$ and $\eta(\hat{X}) = [h_1(y) \;\; h_2(y) \;\; \cdots \;\; h_r(y)]$.

Hence, the proposed observer structure can be expressed as follows.

<The proposed observer structure>

IF y *is* M_i *THEN*

$$\dot{\hat{X}} = A\hat{X} + B\left[\hat{\theta}_f^T \xi(\hat{X}) + \hat{\theta}_g^T \eta(\hat{X}) u\right] + L(y - \hat{y}) + M\sigma \quad (9)$$

$$\dot{\sigma} = y - \hat{y} \quad (10)$$

$$\hat{y} = C\hat{X} \quad (11)$$

where, $\hat{a}_i$ and $\hat{b}_i$ are adaptive parameters and $L = E[L_1 \;\; L_2 \;\; \cdots \; L_n]^T$ as an observer gain vector. $M \in R^n$ is an integral gain

$$e := X - \hat{X} \quad (13)$$

$$\dot{e} = (A - LC)e + B\left[f(X) + g(X)u - \hat{\theta}_f^T \xi(\hat{X}) - \hat{\theta}_g^T \eta(\hat{X}) u + \omega - \omega\right] - M\sigma \quad (14)$$

$$\dot{\sigma} = y - \hat{y} \quad (15)$$

$$\dot{e} = (A - LC)e + B\left[(\theta_f^* - \hat{\theta}_f)^T \xi(\hat{X}) + (\theta_g^* - \hat{\theta}_g)\eta(\hat{X})^T u - \omega\right] - M\sigma \quad (16)$$

$$\dot{\sigma} = y - \hat{y} \quad (17)$$

$$\dot{Z} = \begin{bmatrix} \dot{\sigma} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & C \\ -M & A - LC \end{bmatrix} \begin{bmatrix} \sigma \\ e \end{bmatrix} + \quad \left[(\theta_f^* - \hat{\theta}_f)^T \xi(\hat{X}) + (\theta_g^* - \hat{\theta}_g)\eta(\hat{X})^T u - \omega\right]$$

$$\dot{Z} = \quad Z + \quad \zeta(\theta, z_2, \omega, u) \quad (18)$$

where, $= \begin{bmatrix} 0 & C \\ -M & A - LC \end{bmatrix}$ and $= \begin{bmatrix} 0 \\ B \end{bmatrix}$

vector. σ is a new state describing the integral regulation error between the system output and the observer output, and

$$E = \begin{bmatrix} \frac{1}{\epsilon} & 0 & \cdots & 0 \\ 0 & \frac{1}{\epsilon^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\epsilon^n} \end{bmatrix} \in R^{n \times n}$$

III. OBSERVER DESIGN

An alternative robust adaptive high-gain fuzzy observer is developed in this section.

A. Fuzzy Observer System Structure

Adaptive schemes incorporate with fuzzy systems to improve the accuracy of the proposed method. We denote $\hat{f}(X) = f(\hat{X} \mid \hat{\theta}_f)$ and $\hat{g}(X) = g(\hat{X} \mid \hat{\theta}_g)$. Finally, $f(\hat{X} \mid \hat{\theta}_f)$ and $g(\hat{X} \mid \hat{\theta}_g)$ replace $f(X)$ and $g(X)$. We can rewrite $f(\hat{X} \mid \hat{\theta}_f)$ and $g(\hat{X} \mid \hat{\theta}_g)$ as

B. Observation Error Dynamics

Define the minimum fuzzy approximation error as follows.

$$\boldsymbol{\omega} = \left[f(\hat{X}|\boldsymbol{\theta}_f^*) - f(\boldsymbol{X})\right] + \left[g(\hat{\boldsymbol{X}}|\boldsymbol{\theta}_g^*) - g(\boldsymbol{X})\right]u \quad (12)$$

where, the optimal parameters are defined as

$$\boldsymbol{\theta}_f^* = \arg\min_{\hat{\boldsymbol{\theta}}_f \in R^{n\times r}}\left[\sup_{\boldsymbol{X},\hat{\boldsymbol{X}}\in R^n}\left|f(\hat{\boldsymbol{X}}|\hat{\boldsymbol{\theta}}_f) - f(\boldsymbol{X})\right|\right]$$

$$\boldsymbol{\theta}_g^* = \arg\min_{\hat{\boldsymbol{\theta}}_g \in R^{r}}\left[\sup_{\boldsymbol{X},\hat{\boldsymbol{X}}\in R^r}\left|g(\hat{\boldsymbol{X}}|\hat{\boldsymbol{\theta}}_g) - g(\boldsymbol{X})\right|\right]$$

Thus, $f(\hat{\boldsymbol{X}}|\boldsymbol{\theta}_f^*)$ and $g(\hat{\boldsymbol{X}}|\boldsymbol{\theta}_g^*)$ are the best approximators of $f(\boldsymbol{X})$ and $g(\boldsymbol{X})$ respectively. The observation error, $\boldsymbol{e}$ is defined as the difference between the original state vector, $\boldsymbol{X}$ and the estimated state vector, $\hat{\boldsymbol{X}}$;

After differentiating $\boldsymbol{e}$, the error dynamic equation can be obtained as (14) by using substituting (2) and (9) into (13).

Using (12), (14) can be rewritten as (16) and (17).

Let $\boldsymbol{Z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \sigma \\ \boldsymbol{e} \end{bmatrix}$ as a new state vector, then the dynamic equation (16) and (17) can take the form (18), which is the proposed observation error dynamics.

Theorem 1.

The system expressed by (18) is an asymptotical observer system for the system (2) if all eigenvalues of have negative real parts, and $\hat{\boldsymbol{\theta}}_f$ and $\hat{\boldsymbol{\theta}}_g$ estimate the optimal parameters $\boldsymbol{\theta}_f^*$ and $\boldsymbol{\theta}_g^*$.

Proof.

It is readily proved by the stability analysis of linear systems. If $\hat{\boldsymbol{\theta}}_f \rightarrow \boldsymbol{\theta}_f^*$ and $\hat{\boldsymbol{\theta}}_g \rightarrow \boldsymbol{\theta}_g^*$ then, zero-input response of (18) is considered only. For the minimum fuzzy approximation error, $\boldsymbol{\omega}$, the fuzzy systems can be chosen such that $\boldsymbol{\omega}$ is sufficiently small enough to ignore. Hence, $\dot{\boldsymbol{Z}} = \quad \boldsymbol{Z}$ is asymptotically stable if and only if all eigenvalues of have negative real parts.

IV. ADAPTIVE LAW AND STABILITY ANALYSIS

In this section, indirect adaptive law is derived and stability analysis based on Lyapunov theory is performed.

Remark 1. According to Lyapunov equation [18], if there exist $\boldsymbol{P}$ and $\boldsymbol{Q}$, which satisfy that

$$^T\boldsymbol{P} + \boldsymbol{P} \quad = \; -\boldsymbol{Q}$$

then, a matrix is *Hurwitz*; that is, all eigenvalues of have negative real parts ($Re\lambda_i < 0$). Where, P, Q are symmetric positive definite matrices.

This adaptive law and V guarantee the asymptotic stability of $\boldsymbol{Z}$ for the equilibrium point $\boldsymbol{Z} = 0$

V. SIMULATION

<Duffing Forced-Oscillation Model [8]>

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1^3 - 0.1x_2 + 12\cos(t) + u$$

Assume that $y \in [-d \quad d]$ and $d > 0$. Then we can have the following fuzzy model as well:

Rule 1

IF y *is* M_1, *THEN* $\dot{\boldsymbol{X}} = \boldsymbol{A}_1\boldsymbol{X} + \boldsymbol{B}u^*$

Rule 2

IF y *is* M_2, *THEN* $\dot{\boldsymbol{X}} = \boldsymbol{A}_2\boldsymbol{X} + \boldsymbol{B}u^*$

Theorem 2.

For (18), consider a Lyapunov candidate as

$$V = \frac{1}{2}\boldsymbol{Z}^T\boldsymbol{P}\boldsymbol{Z} + \frac{1}{2\gamma_1}(\boldsymbol{\theta}_f^* - \hat{\boldsymbol{\theta}}_f)^T(\boldsymbol{\theta}_f^* - \hat{\boldsymbol{\theta}}_f) + \frac{1}{2\gamma_2}(\boldsymbol{\theta}_g^* - \hat{\boldsymbol{\theta}}_g)^T(\boldsymbol{\theta}_g^* - \hat{\boldsymbol{\theta}}_g) \quad (19)$$

and adaptive law as

$$\dot{\hat{\boldsymbol{\theta}}}_f = \gamma_1\boldsymbol{Z}^T\boldsymbol{P} \quad \boldsymbol{\xi}(\hat{\boldsymbol{X}}) \quad (20)$$

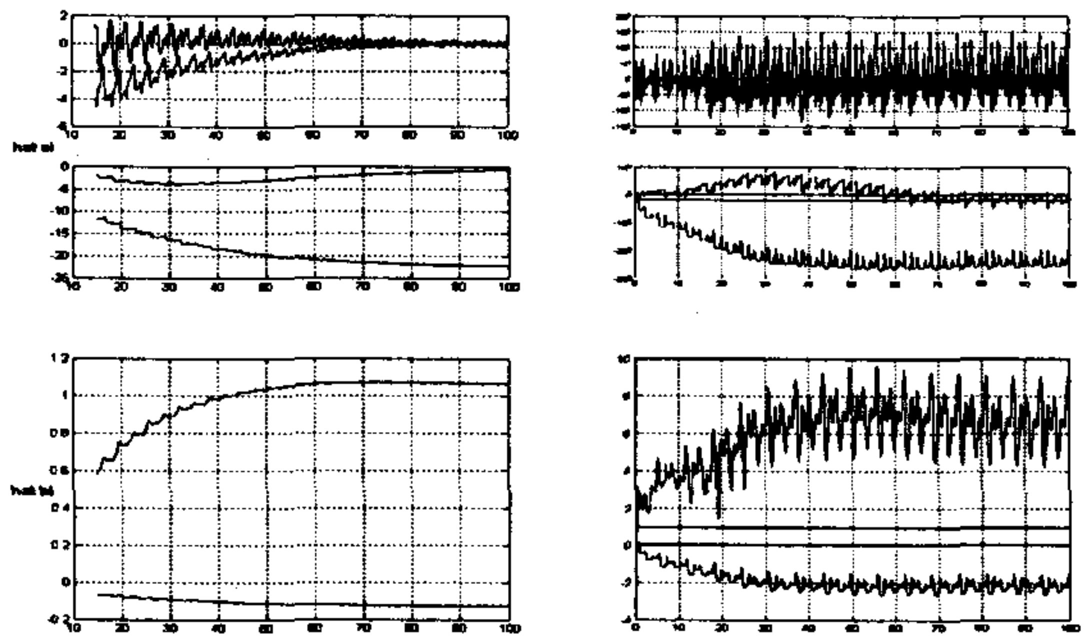
$$\dot{\hat{\boldsymbol{\theta}}}_g = \gamma_2\boldsymbol{Z}^T\boldsymbol{P} \quad \boldsymbol{\eta}(\hat{\boldsymbol{X}})u \quad (21)$$

where, V : a positive definite and radially unbounded function

$\boldsymbol{P}$: a symmetric positive definite matrix

γ_1 and γ_2 : positive adaptation constant gains.

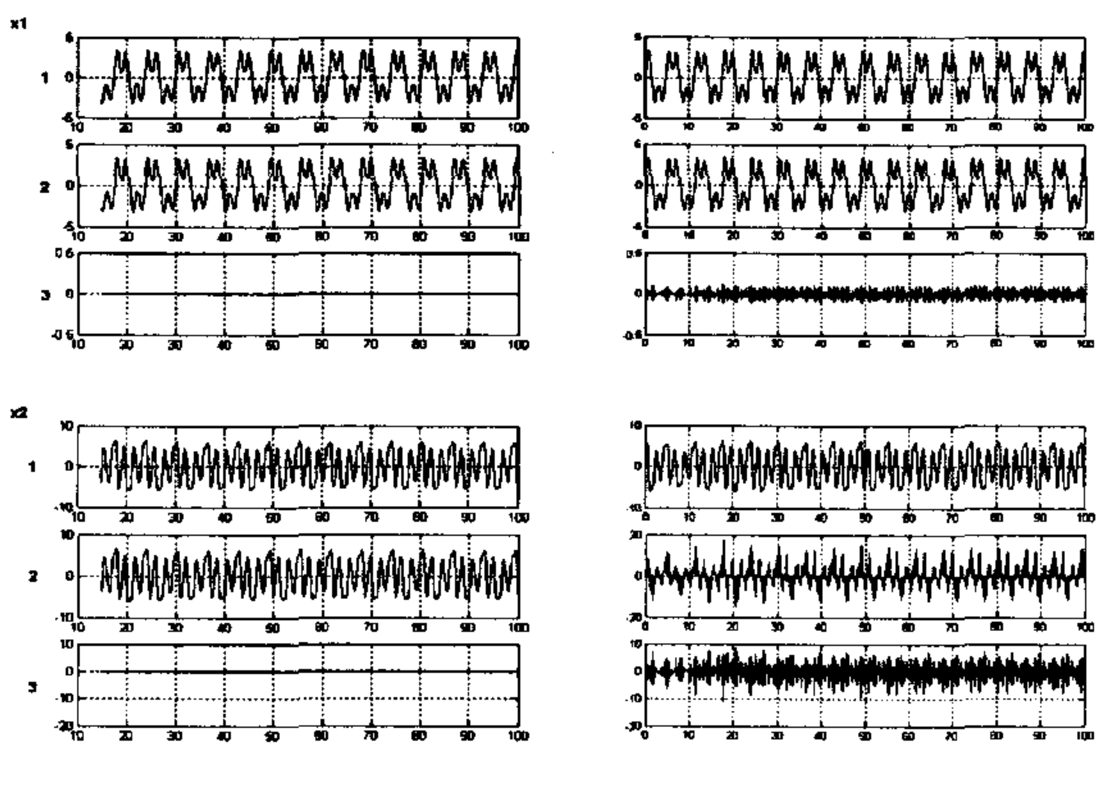
Here, $\boldsymbol{X} = [x_1 \quad x_2]^T$ and $u^* = u + 12\cos(t)$, $\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}$, $\boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 \\ -d^2 & -0.1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $M_1(y) = 1 - \frac{y^2}{d^2}$, $M_2(y) = \frac{y^2}{d^2}$, $d = 5$ in this paper.



(a) The proposed scheme (b) The existing scheme

<Fig. 1> Simulation results of the perturbed parameter estimation, $\hat{a}_i$ and $\hat{b}_i$

The unknown parameters are set as $\boldsymbol{A}_1$: $a_1 = 0$, $a_2 = -0.1$, $\boldsymbol{A}_2$: $a_1 = -25$, $a_2 = -0.1$ and $\boldsymbol{B}$: $b_1 = 0$, $b_2 = 1$. As fig.1 shows, the proposed scheme is more robust against parameter perturbation than the existing scheme, and estimates the original values of parameters well whereas the existing scheme does not. Both the proposed scheme and the existing scheme seem to estimate the original state well. However, fig.2(**3**) presents that the proposed observer has a smaller observation error than the existing one. It means that the proposed scheme is more accurate than the existing scheme.



(a) The proposed scheme (b) The existing scheme

<Fig. 2> Estimation of the state $\boldsymbol{X}$: **1** original states **2** observer states **3** observation error

VI. CONCLUSION

We proposed an alternative obust adaptive high-gain fuzzy observer design scheme and its application to synchronization of chaotic systems. It improved the parameter adaptation accuracy and the robustness of the proposed observer. With the proposed observer, the asymptotic synchronization of chaotic systems was achieved.

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REFERENCES

[1] Hyun C.-H., Kim J.-H., Kim E., and Park M. Adaptive fuzzy observer based synchronization design and secure communications of chaotic systems, Chaos, Solitons & Fractals, 2006;27(4):930-940

[2] Hyun C.-H, Park C.-W. and Park M. A robust indirect adaptive fuzzy state feedback regulator based on Takagi-Sugeno fuzzy model, J. Korea Fuzzy Logic and Intelligent Systems Society, 2002;12(6):554-558

[3] Ryu T.-Y., Hyun C.-H., Kim E. and Park M., Robust adaptive fuzzy observer based control for unknown nonlinear systems, Proc. of KFIS Spring Conf., 2006;16(1):420-424

[7] Khalil H. K., Nonlinear systems 3rd, Prentice-Hall, 2002

[8] Tanaka K, Wang H. O. Fuzzy control systems design and analysis, Wiley-Interscience, 2001

[12] Lian K.-Y, Liu P, Chiang T.-S. and Chiu C.-S. Adaptive synchronization design for chaotic systems via a scalar driving signal, IEEE Trans. Circuits and systems, 2002;49(1):17-27

[21] Wang L.-X. A Course in Fuzzy systems and Control, Prentice-Hall International, Inc., 1997

[22] Slotine J. -J. E. and Li W. Applied nonlinear control, Englewood Cliffs, NJ: Prentice-Hall, 1991