비 컨벡스 퍼지 소속함수에 대한 유사측도구성

Similarity Measure Construction for Non-Convex Fuzzy Membership Function

Park, Hyun Jeong[1], Sungshin Kim[2] and Sang H. Lee[3]

[1] 서울시 서대문구 이화여자대학교 수학교육과
E-mail: hyunpj88@naver.com
[2] 부산시 금정구 부산대학교 전기공학과
E-mail: sskim@pusan.ac.kr
[3] 경남 창원시 창원대학교 메카트로닉스 공학부
E-mail: leehyuk@changwon.ac.kr

Abstract

The similarity measure is constructed for non-convex fuzzy membership function using well known Hamming distance measure. Comparison with convex fuzzy membership function is carried out, furthermore characteristic analysis for non-convex function are also illustrated. Proposed similarity measure is proved and the usefulness is verified through example. In example, usefulness of proposed similarity is pointed out.

Key Words : Similarity measure, non-convex fuzzy membership function, distance measure

1. Introduction

Research on the similarity measure between fuzzy sets has been can be applied to the pattern classification or many other fields. Similarity measure represents the degree of similarity between two or more informations. It also has been noticed as the complementary meaning of the distance measure, i.e, $s+d=1$, where d and s are distance and similarity measure respectively. Until now the research of designing similarity measure has been made by numerous researchers [ ]. For fuzzy set, there is an uncertainty knowledge in fuzzy set itself. Hence information of the data can be obtained from analyzing the fuzzy membership function. Thus most studies about fuzzy set are focussed on designing similarity measure based on membership function. At this point we have an interest about the similarity measure for non-convex fuzzy membership. We already design similarity for general fuzzy membership [ ]. In this paper we verify the usefulness of similarity between fuzzy sets with our proposed similarity measure. Applying similarity measure to non-convex fuzzy membership function, we analyze the similarity characteristics for fuzzy sets.

In the next section, the axiomatic definitions of entropy, distance measure and similarity measure of fuzzy sets are introduced and fuzzy entropy is constructed through distance measure. In Chapter 3, similarity measures are constructed and proved through the distance measure. Similarity application to non-convex fuzzy set is proposed by considering support average. Conclusions are followed in Chapter 4. Notations of Liu's are used in this paper [4].

2. Preliminaries about similarity

We introduce some preliminary results about axiomatic definitions of distance measure and similarity measure as follows [4].

Definition 2.1 [4] A real function d: $F^2 \to R^+$ is called a distance measure on $F(X)$, if d satisfies the following properties:

(D1) $d(A,B) = d(B,A), \forall A, B \in F(X)$

(D2) $d(A,A) = 0$, $\forall A \in F(X)$

(D3) $d(D, D^c) = max_{A,B \in F}\, d(A,B)$, $\forall D \in P(X)$

(D4) $\forall A, B, C \in F(X)$, if $A \subset B \subset C$, then $d(A,B) \le d(A,C)$ and $d(B,C) \le d(A,C)$.

Fuzzy normal entropy on F is obtained by the division of $\max_{C,D \in F} s(C,D)$. Liu also pointed out that there is an one-to-one relation between all distance measures and all similarity measures, $d+s=1$. In this paper, among distance measures, Hamming distance is commonly used as distance measure between fuzzy sets A and B,

$$d(A,B) = \frac{1}{n}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$$

where $X = \{x_1, x_2, \cdots x_n\}$, $|k|$ is the absolute value of k. We had proposed another fuzzy entropy induced by distance measure which is different from Theorem 3.1 of Fan, Ma and Xie [7]. Proposed entropy needs only A_{near} crisp set, and it has the advantage in computation of entropy. Furthermore we considered another entropy, which considers only A_{far}, and it has more compact form than another one. Proofs can be found in [9]. Proposed entropies have some advantages to the Liu's, they don't need extra assumptions of Liu. Furthermore they use only one crisp sets A_{near} and A_{far}, respectively.

Definition 2.2 [4] A real function s: $F^2 \to R^+$ or $P^2 \to R^+$ is called a similarity measure, if s has the following properties:

(S1) $s(A,B) = s(B,A), \forall A, B \in F(X)$

(S2) $s(A, A^c) = 0$ $\forall A \in F(X)$

(S3) $s(D,D) = max_{A,B \in F}\, s(A,B)$, $\forall A, B \in F(X)$

(S4) $\forall A, B, C \in F(X)$, if $A \subset B \subset C$, then $s(A,B) \ge s(A,C)$ and $s(B,C) \ge s(A,C)$.

Fuzzy normal similarity measure on F is also obtained by the division of $\max_{C,D \in F} s(C,D)$. With the definition of similarity measure we derive the similarity between fuzzy sets. Consider the two gaussian type membership functions as in Fig. 2.

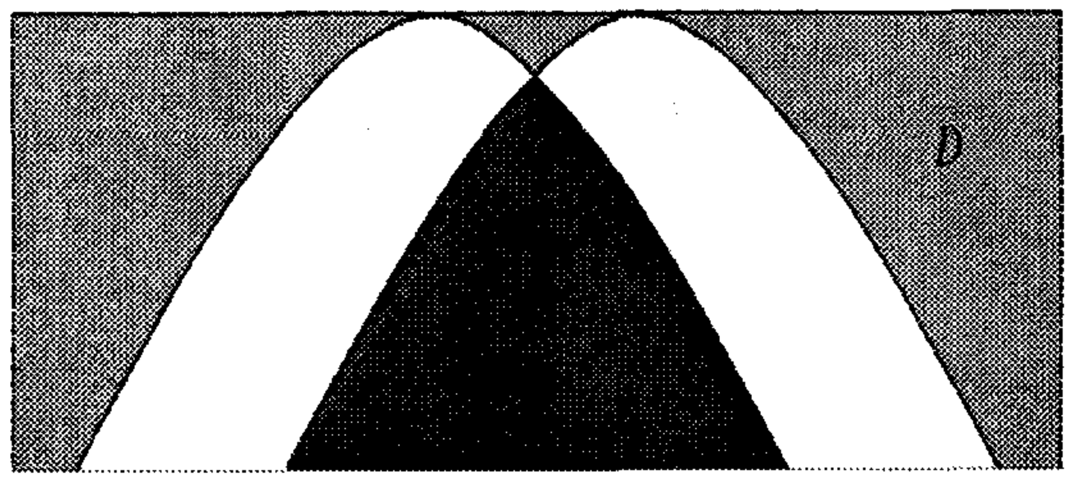

Fig. 1 Gaussian type two membership functions

From Fig. 1, we conjecture that the shaded area can be considered as component of similarity measure. Hence we obtain those areas as equations, then next two equations

$$1 - d((A \cap B), [1]) \text{ and } 1 - d((A \cup B), [0])$$

will be denoted as **C** and **D**. Part **C** is common area of membership function, whereas **D** also represents the common information of the two membership functions. As the area **C** goes to 1, area **D** satisfies 0. If **D** satisfies 1, then two membership functions are the exact same membership function. Hence proper similarity measure is obtained through combining two values.

3. Derivation of Similarity Measure

We obtain the fuzzy entropy with the distance measure in previous section. Generally, fuzzy entropy is expressed through distance measure, i.e., $e(A) = e(d(A))$. In our result, entropy is represented distance measure itself, $e(A) = e(d(A))$. Hence, by the result of Liu's,

$$d(A) + s(A) = 1 \tag{1}$$

we modify the similarity measure as $s(A) = 1 - e(A)$, that means fuzzy set A matches to the crisp set A_{near} nearly as $s(A)$

approaches to 0. We illustrate the similarity measure with distance measure in the following theorems.

Theorem 3.1 For fuzzy set $A \in F(X)$, if d satisfies distance measure, then

$$s(A,B) = 2 - d((A \cap B),[1]) - d((A \cup B),[0]) \quad (2)$$

is the similarity measure between fuzzy set A and B.

proof. We prove that the eq. (2) by satisfying the similarity definition. (S1) means the commutativity of set A and B, hence it is clear from (7) itself. From (S2), $s(A, A^c) = 0$ is shown as

$$\begin{aligned} s(A,A^c) &= 2 - d((A \cap A^c),[1]) - d((A \cup A^c),[0]) \\ &= 2 - d([0],[1]) - d([1],[0]) \\ &= 2 - 1 \cdot 1 - 1 \cdot 1 = 0. \end{aligned}$$

For all $A, B \in F(X)$, inequality of (S3) is proved by

$$\begin{aligned} s(A,B) &= 2 - d((A \cap B),[1]) - d((A \cup B),[0]) \\ &\leq 2 - d((D \cap D),[1]) - d((D \cup D),[0]) \\ &= s(D,D). \end{aligned}$$

Inequality is satisfied from $d((A \cap B),[1]) \geq d((D \cap D),[1])$ and $d((A \cup B),[0]) \geq d((D \cup D),[0])$.

Finally, (S4) is $\forall A, B, C \in F(X)$, $A \subset B \subset C$,

$$\begin{aligned} s(A,B) &= 2 - d((A \cap B),[1]) - d((A \cup B),[0]) \\ &= 2 - d(A,[1]) - d(B,[0]) \\ &\geq 2 - d(A,[1]) - d(C,[0]) \\ &= s(A,C) \end{aligned}$$

also

$$\begin{aligned} s(B,C) &= 2 - d((B \cap C),[1]) - d((B \cup C),[0]) \\ &= 2 - d(B,[1]) - d(C,[0]) \\ &\geq 2 - d(A,[1]) - d(C,[0]) \\ &= s(A,C) \end{aligned}$$

is satisfied. Inequality is also satisfied with $d(B,[0]) \leq d(C,[0])$ and $d(B,[1]) \leq d(A,[1])$.

Therefore proposed similarity measure (2) satisfy Definition 2.2. Similarly, we propose another similarity measure in the following theorem.

Theorem 3.2 For fuzzy set $A \in F(X)$ and distance measure d,

$$s(A,B) = 1 - d((A \cap B),[0]) - d((A \cup B),[1]) \quad (3)$$

is the similarity measure of fuzzy set A and crisp set B.

proof. Proofs are shown similarly as Theorem 3.1.

We have proposed the similarity measure that are induced from distance measure. By analyzing the similarity (2) and (3), similarity is proportional to the common area of two membership functions. Summation of areas C and D represent similarity. At this point we have a question how about non-convex membership functions are ? For the same area of between convex to convex and convex to non-convex which pair has better similarity ? Even though two pairs have same similarity measure, their geometrical description may not identical. Hence another measure is required to discriminate tow pairs. Commonly, different fuzzy membership function pair has different mean values of universe of discourse. Therefore we use "support" as measure for similarity to obtain implicit result.

$$support_A(x_i) = \frac{1}{n}\sum_{i=1}^{n}|x_i|,\ x_i \in A$$

Support between set A and B is represented as follows.

$$support\ (A,B) = \frac{1}{n}\sum_{i=1}^{n}|x_{Ai}| - \frac{1}{m}\sum_{j=1}^{m} x_{Bj}|, \quad x_{Ai} \in A, x_{Bj} \in B$$

Now we consider another similarity between set A and B is

$$s_2(A,B) = \frac{1}{support\,(A,B)+1} \quad (4)$$

Next we combine similarity measure as follows

$$s(A,B) = \omega_1 s_1(A,B) + \omega_2 s_2(A,B). \quad (5)$$

Where similarity measure (2) and (3) are replaced into $s_1(A,B)$, ω_1, ω_2 are weighting factors. Similarity proof of (4) is can be obtained easily. Now we consider the membership functions type 1 and 2 in Fig. 2 and 3. In the following figures, area between μ_A and μ_B are the same. Furthermore similarity measures between μ_A and μ_B $s_1(A,B)$ are same. Then which case is more similar ?

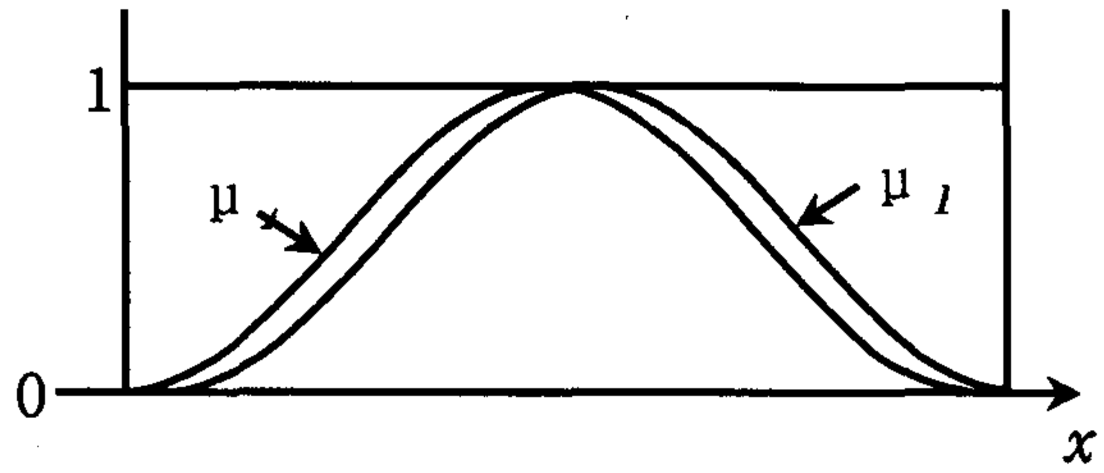

Fig.2 Membership functions type 1

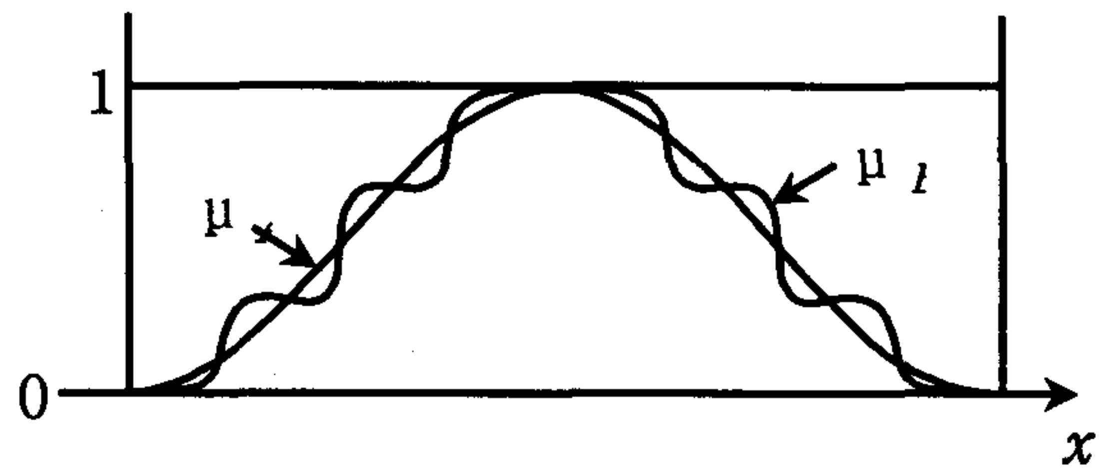

Fig. 3 Membership functions type 2

In this case, the first part of (5) is the similarity measure, hence the value of Fig. 2 and Fig. 3 are the same. However the last part of (5) represents the difference of average support. Now we check whether the eq. (5) satisfy the similarity measure definition or not. For (S1), commutativity is clear from (4) itself. $s_2(A,A^c)$ is minimum value. (S3) is clear, same membership function of D has the maximum value. (S4) is also clear because farther average support value invokes less s_2 value. Finally proof of (5) is clear because $s_1(A,B)$ and $s_2(A,B)$ are similarity measure themselves.

ACKNOWLEDGEMENTS

This work has been supported by 2nd BK21 Program, which is funded by KRF(Korea Research Foundation).

4. Conclusions

We introduce the distance measure and similarity measure, similarity measure can be represented by the function of distance measure. By the one to one correspondence of distance measure and similarity measure, we construct the similarity measure using distance measure. With the proposed similarity measure we analyze the similarity between fuzzy membership function pair, especially non-convex fuzzy membership function. Furthermore modified similarity measure is constructed through "support" characteristic. We verify that the proposed measure is the similarity measure.

References

[1] D. Bhandari, N. R. Pal, "Some new information measure of fuzzy sets", Inform. Sci. 67, 209-228, 1993.

[2] A. Ghosh, "Use of fuzziness measure in layered networks for object extraction: a generalization", Fuzzy Sets and Systems, 72, 331-348, 1995.

[3] B. Kosko, Neural Networks and Fuzzy Systems, Prentice-Hall, Englewood Cliffs, NJ, 1992.

[4] Liu Xuecheng, "Entropy, distance measure and similarity measure of fuzzy sets and their relations", Fuzzy Sets and Systems, 52, 305-318, 1992.

[5] N.R. Pal, S.K. Pal, "Object-background segmentation using new definitions of entropy", IEEE Proc. 36, 284-295, 1989.

[6] J. L. Fan, W. X. Xie, "Distance measure and induced fuzzy entropy", Fuzzy Set and Systems, 104, 305-314, 1999.

[7] J. L. Fan, Y. L. Ma, and W. X. Xie, "On some properties of distance measures", Fuzzy Set and Systems, 117, 355-361, 2001.

[8] S. Aksoy and R.M. Haralick, "Feature normalization and likelihood-based similarity measures for image retrieval", Pattern Recognition Letters, 22, 563-582, 2001.

[9] S.H. Lee, K.B. Kang and S.S. Kim, "Measure of fuzziness with fuzzy entropy function", Journal of Fuzzy Logic and Intelligent Systems, vol. 14, no. 5, 642-647, 2004.

[10] J.S.R. Jang, C.T.Sun and E. Mizutani, Neuro-Fuzzy and Soft Computing, Prentice Hall, Upper Saddle RIver, 1997.