Real Time Error Correction of Hydrologic Model Using Kalman Filter

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Abstract

Accuracy of flood forecasting is an important non-structural measure on the flood control and mitigation. Hence, combination of horologic model with real time error correction became an important issue. It is one of the efficient ways to improve the forecasting precision. In this work, an approach based on Kalman Filter (KF) is proposed to continuously revise state estimates to promote the accuracy of flood forecasting results. The case study refers to the Wi River in Korea, with the flood forecasting results of Xinanjiang model. Compared to the results, the corrected results based on the Kalman filter are more accurate. It proved that this method can take good effect on hydrologic forecasting of Wi River, Korea, although there are also flood peak discharge and flood reach time biases. The average determined coefficient and the peak discharge are quite improved, with the determined coefficient exceeding 0.95 for every year.

Key words: Flood forecasting, Real time error correction, Kalman Filter, Xinanjiang model

1. Introduction

It is not a good method that only to rely on hydrological model to promote the prediction precise because of the difficulty to know the characteristics of watersheds well. The appropriate hydrological model is an important part of flood forecasting, but real time error correction can be very helpful to improve in the accuracy of flood forecasting. Recently, the real time correction methods, such as the method of minimum squares, error autoregressive method and adaptive algorithm and so on, are used widely. Kalman filter can optimally estimate the system state variables and get the least-squares with the flood periods. The first method for forming an optimal estimate from noisy data is the method of linear least squares[3]. So kalman filter is a good method for real time error correction of the Wi River, Korea, and it was proved that the overall result is successful.

2. Klaman Filter

Kalman Filter (KF) is an optimal state estimation process applied to a dynamic system that involves random perturbations. More precisely, the kalman filter gives a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from noisy data taken at discrete real-time intervals[1]. KF is essentially a set of mathematical equations that

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implement a predictor-corrector type estimator. It provides an efficient recursive solution of the least-squares method when some presumed conditions are met[2]. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown.

2.1. The State-space description of Kalman Filter

With the assumption that the model noise and measurement noise to be independent of each other, zero-mean Gaussian white noise, and the control input set to be zero, the description of Kalman Filter for a linear system is as following:

1) The state equation

$$X_k = \Phi_{k|k-1} X_{k-1} + G_{k|k-1} U_{k-1} + \omega_{k-1} \tag{2-1}$$

2) The measured equation

$$Z_k = H_k X_k + v_k \tag{2-2}$$

Where X_k, Z_k are the actual state and measurement vectors; $\Phi_{k|k-1}$, G_{k-1} are transition matrices. The matrix $\Phi_{k|k-1}$ relates the state at time step k-1 to the state at step k; The matrix G_{k-1} relates the control input to the state X, it is set to zero to simplify the notation as the influence of the forcing term is purely deterministic and does not affect the estimation process. U_{k-1} is the forcing term and set to zero; ω_{k-1} is the model noise with zero-mean and variance Q_k, v_k is the vector of measurement noise with zero-mean and variance R_k; The matrix H_k changed with each time step in the measurement equation relates the state to the measurement Z_k.

2.2 The Computational Origins of the Filter

1) The priori estimate error covariance:

$$P_{k|k-1} = E[\widetilde{X}_{k|k-1}, \widetilde{X}_{k|k-1}{}^T] \tag{2-3}$$

where $\widetilde{X}_{k|k-1} = X_k - \hat{X}_{k|k-1}$, $\hat{X}_{k|k-1}$ is given to be forecasted value at step k estimated by the state vector at step k-1

2) The posteriori estimate error covariance:

$$P_{k|k} = E[\widetilde{X}_{k|k}, \widetilde{X}_{k|k}{}^T] \tag{2-4}$$

where $\widetilde{X}_{k|k} = X_k - \hat{X}_k$, $\hat{X}_k$ is given to be a filter value at step k

2.3 The Kalman Filter Algorithm

From the derived equations for the Kalman filter we can get the estimation algorithm:

1) Filtering :

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k (Z_k - H_k \hat{X}_{k|k-1}) \tag{2-5}$$

2) Forecasting :

$$\hat{X}_{k|k-1} = \Phi_{k|k-1} \hat{X}_{k-1} + G_{k-1} U_{k-1} \tag{2-6}$$

3) The Kalman gain

$$K_k = P_{k|k-1} H_k^{\ T} (H_k P_{k|k-1} H_k^{\ T} + R_k)^{-1} \tag{2-7}$$

4) The associated matrix of prediction errors for state vector:

$$P_{k|k-1} = \Phi_{k|k-1} P_{k|k-1} \Phi_{k|k-1}^{\ T} + Q_{k-1} \tag{2-8}$$

5) The associated matrix of filtering errors for state vector:

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{2-9}$$

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements[2]. The first task during the measurement update is to compute the Kalman gain K_k in equation (2-7). The next step is to actually measure the process to obtain Z_k, and then to generate an a posteriori state estimate by incorporating the measurement as in equation (2-5). Again equation (2-5) repeated here for completeness. The final step is to obtain an a posteriori error covariance estimate via equation (2-9). After each time and measurement update complete, the process is repeated with the previous. Posteriori estimates used to predict the new priori estimates.

3. The application of Kalman filter

Wi River is a tributary of the main Nakdong River. There are forecasting errors by Xinanjiang model which has applied in Wi Stream, in Korea. In order to make the forecasting result more accurate, the procedure to reduce these errors is based on Kalman Filter so that the observed and simulated discharges are equal.

3.1 The initial values of Kalman Filtering

Before starting the model calculation, we set the initial associated matrix of Q^0, R^0, Φ^0, H^0 through the calibration of flood event based on the difference of measured flow and forecasting flow. Table 3.1 showed the Initial values of Kalman filter.

Table 3.1 Initial values of Kalman filter

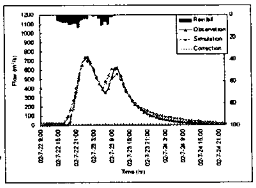
Q^0			R^0	Φ^0			H^0		
Q^0_{11}	Q^0_{22}	Q^0_{33}	R^0	Φ^0_{11}	Φ^0_{12}	Φ^0_{13}	H^0_{11}	H^0_{12}	H^0_{13}
500	500	500	500	0.724	-0.028	1.281	1	0	0

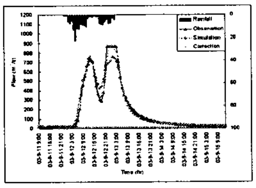
3.2 Analysis and results

Table 3.2 shows the real time corrected results obtained by the method using Kalman filter. Fig 3.3 shows the hydrolographs of corrected results for the selected eight flood events. The figure shows that the corrected results are quite more similar with the measured series. In particular, the timing and peak of the flood are in better agreement with those of the measured data than simulated. The average Determined Coefficient (DC) can reach 0.97 and the peak discharges are quite improved, such as 020722, 960616, 970705 and so on. Although the results are satisfactory, there are also lag time of flood events such as 030911, 040822. The reason is that usually there is one lag time to correct the results with the limits of Kalman filter method, it uses the information of prior to forecast the current state variant. Also, it is assumed that the noise is white noise with constant. In fact, it is not constant which is difficult to be estimate and measured.

Table 3.2 The real time correction results for selected flood events

	Determined Coefficient		Relative Error(%)		Peak Discharge					Time Interval(hr)	
Event	Sim	Corr	Sim	Corr	Obs	Sim	Corr	Rel(Sim)	Rel(Corr)	Sim	Corr
020722	0.97	0.99	9.97	2.47	743.67	713.81	721.02	4.02	3.05	-1	0
030911	0.96	0.99	1.07	0.25	867.00	763.28	881.52	11.96	1.67	3	3
040619	0.88	0.96	0.34	0.97	867.00	839.85	909.35	3.13	4.88	2	1
040822	0.97	0.99	3.20	0.62	867.00	859.52	821.51	0.86	5.25	0	2
900718	0.90	0.96	5.31	1.35	471.71	523.00	552.71	10.87	17.17	1	0
920715	0.91	0.97	4.73	1.52	426.45	438.44	472.87	2.81	10.89	2	0
920824	0.86	0.98	7.88	1.96	485.67	573.67	547.85	18.12	12.80	1	0
960616	0.98	0.99	5.78	1.04	1127.00	1082.53	1149.54	3.95	2.00	1	0
970624	0.96	0.97	9.05	2.28	628.50	526.41	597.19	16.24	4.98	0	1
970705	0.91	0.97	9.57	2.28	1331.90	1026.12	1311.62	22.96	1.52	0	0
970803	0.93	0.98	6.87	1.73	956.75	782.79	964.99	18.18	0.86	1	1


020722 flood event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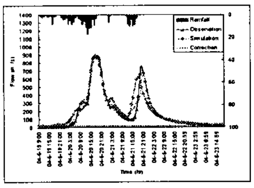
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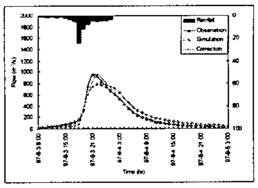
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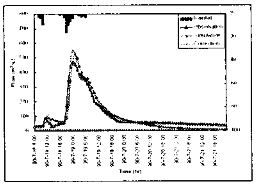
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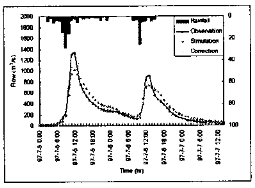
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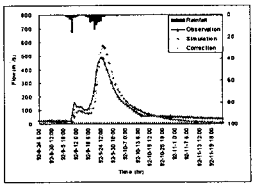
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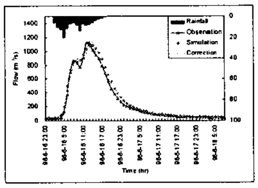
960616 flood event

Fig. 3.3 Observed, Simulated and corrected hydrographs of selected flood events

4. Conclusion

The real time correction for the flood forecasting results of Xinanjiang model based on Kalman Filter was carried out in the Wi River, Korea. The results indicate that this correction method based on Kalman Filter can improve the lead time and accuracy of flood prediction. It can be a useful flood forecasting tool for Wi River, Korea, although there are limits:

1) The calculation method of Kalman filter is based on white noise. In fact, the noise is not white noise.
2) The real time correction result is not well when the result error of Xinanjiang model changed unregularly.

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