

Spring Flow Prediction affected by Hydro-power Station Discharge using the Dynamic Neuro-Fuzzy Local Modeling System

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ABSTRACT: This paper introduces the new generic dynamic neuro-fuzzy local modeling system (DNFLMS) that is based on a dynamic Takagi-Sugeno (TS) type fuzzy inference system for complex dynamic hydrological modeling tasks. The proposed DNFLMS applies a local generalization principle and an one-pass training procedure by using the evolving clustering method to create and update fuzzy local models dynamically and the extended Kalman filtering learning algorithm to optimize the parameters of the consequence part of fuzzy local models. The proposed DNFLMS is applied to develop the inference model to forecast the flow of Waikoropupu Springs, located in the Takaka Valley, South Island, New Zealand, and the influence of the operation of the 32 Megawatts Cobb hydropower station on springs flow. It is demonstrated that the proposed DNFLMS is superior in terms of model accuracy, model complexity, and computational efficiency when compared with a multi-layer perceptron trained with the back propagation learning algorithm and well-known adaptive neural-fuzzy inference system, both of which adopt global generalization.

1 INTRODUCTION

Recently some researchers have focused on the so-called neuro-fuzzy modeling that combines fuzzy system and learning algorithms of artificial neural networks in sciences and engineering. Neuro-fuzzy modeling techniques are designed to overcome inherent drawbacks with fuzzy system, namely, the choice of appropriate fuzzy IF-THEN-rules and membership function [Hong *et al.*, 2002]. A number of different neuro-fuzzy modeling schemes have been developed [Jang, 1993; Lin and Lee, 1996] over the past decades. Neuro-fuzzy modeling techniques have been used for modeling hydrological processes and have been found to show satisfactory results in some hydrological applications [Hong *et al.*, 2002; Hong and Song, 2004]. The most existing neuro-fuzzy modeling techniques apply the global generalization principle to simultaneously optimize all the parameters of fuzzy systems. This global generalization principle has significant disadvantages for the dynamic hydrological modeling tasks: (1) slow training speed due to the significantly higher computational complexity, (2) higher sensitivity to noise data and sparsely distributed data, and (3) difficulty in online training simulation. In contrast, the local generalization principle is to consider the optimization of the model parameters of fuzzy systems as individual problem. The parameters of each fuzzy model are estimated separately region by region using subsets of training data. Therefore, it has a fast training speed and a capability of online simulation, a higher interpretation for the specific operating regime of the system, and a reduction of over-fitting.

To overcome inherent drawbacks with existing neuro-fuzzy systems using the global generalization principle, we propose a new dynamic neuro-fuzzy local modeling system (DNFLMS), based on a dynamic Takagi-Sugeno (TS) [Takagi and Sugeno, 1985] type fuzzy inference system, by applying the online one-pass clustering method [Song and Kasabov, 2001] to dynamically create and update each fuzzy local model, and by implementing the local generalization principle with the extended Kalman filtering (EKF) [Maybeck, 1982] learning algorithm to optimize each fuzzy model parameters depending on the region. Our developed DNFLMS technique is applied to develop a prediction model of Waikoropupu Springs (Pupu Springs) flow in Takaka valley catchment, New Zealand. Flow in Pupu Springs is influenced by Cobb hydropower station discharge, by flow in Takaka River and Waingarō River, and by rainfall. Comparative simulation results of developed DNFLMS with artificial neural networks [Haykin, 1994] and another well-known adaptive neural-fuzzy inference system (ANFIS) [Jang, 1993], together with numerical and graphical results, are presented.

2 DYNAMIC NEURO-FUZZY LOCAL MODELING SYSTEM (DNFLMS)

2.1 Dynamic Takagi-Sugeno (TS) Fuzzy Model

A dynamic model assumes that the new system state of a dynamic system can be predicted by the past inputs and outputs of the system. A number of dynamic model structures exist in the system theory literature [Ljung, 1999]. For the i -th fuzzy local model (rule), the Takagi-Sugeno (TS) type fuzzy NARX (Nonlinear Autoregressive with eXogenous) model [Hong et al., 2002] for a dynamic system can be represented by:

$$\begin{aligned}
 & y(k) \text{ is } \alpha_1^i \text{ and } y(k-1) \text{ is } \alpha_2^i \text{ K K and } y(k-n_y) \text{ is } \alpha_{n_y}^i \\
 & \text{and } u_1(k-n_{k_1}) \text{ is } \beta_{11}^i \text{ and } u_1(k-n_{k_1}-1) \text{ is } \beta_{12}^i \text{ K and } u_1(k-n_{k_1}-n_{u_1}+1) \text{ is } \beta_{1n_{u_1}}^i \\
 & \text{and } u_m(k-n_{k_m}) \text{ is } \beta_{m1}^i \text{ and } u_m(k-n_{k_m}-1) \text{ is } \beta_{m2}^i \text{ K and } u_m(k-n_{k_m}-n_{u_m}+1) \text{ is } \beta_{mn_{u_m}}^i \\
 & \hat{y}^i(k+1) = A_1^i y(k) + A_2^i y(k-1) + \Lambda + A_{n_y}^i y(k-n_y) \\
 & \quad + B_{11}^i u_1(k-n_{k_1}) + B_{12}^i u_1(k-n_{k_1}-1) + \Lambda + B_{1n_{u_1}}^i u_1(k-n_{k_1}-n_{u_1}+1) \\
 & \quad + B_{m1}^i u_m(k-n_{k_m}) + B_{m2}^i u_m(k-n_{k_m}-1) + \Lambda + B_{mn_{u_m}}^i u_m(k-n_{k_m}-n_{u_m}+1) + c^i \\
 & \quad i = 1, 2, K, r
 \end{aligned} \tag{1}$$

Here $y(k) \in Y \in R$ is the output vector, $u(k) \in U \in R^m$ is the input vector with m inputs. n and k denote the number of data samples and the discrete time samples, respectively. n_y and n_{u_m} are the maximum lags considered for the output, and input terms, respectively. n_{k_m} represents the time delay between a change in the inputs and the observed change in the output. The function $f(\cdot)$ is a TS fuzzy model that maps the past inputs and outputs to future outputs.

Eq. (1) can be rewritten in the following vector form:

$$R^i : \text{ IF } x(k) \text{ is } \Theta^i \text{ THEN } \hat{y}^i(k+1) = \Psi^{iT} X + c^i \tag{2}$$

$$\Theta^i = [\alpha_1^i \dots \alpha_{n_y}^i \quad \beta_{11}^i \dots \beta_{1n_{u_1}}^i \dots \beta_{m1}^i \dots \beta_{mn_{u_m}}^i]^T \tag{3}$$

$$\Psi^i = [A_1^i \dots A_{n_y}^i \quad B_{11}^i \dots B_{1n_{u_1}}^i \dots B_{m1}^i \dots B_{mn_{u_m}}^i]^T \tag{4}$$

Here $\hat{y}^i(k+1)$ is the local output of the i -th fuzz rule. Θ^i is the antecedent fuzzy set of the i -th fuzzy rule. ψ^i and c^i are the consequence parameters, which are linear combinations of the input variables. Therefore, the TS fuzzy system in Eq. (2) is called a first-order TS fuzzy inference system. R^i denotes the number of fuzzy rules. The antecedent fuzzy set Θ^i can be defined by the multivariate fuzzy membership function $\mu(x(k)): R^p \rightarrow [0, 1]$, $p = n_y + \sum_{j=1}^m n_{u_m}^j + 1$. In this work, $\mu(x(k))$ are defined by the Gaussian fuzzy membership function:

$$\mu^i(x(k)) = \prod_{j=1}^p \Phi_j^i = \prod_{j=1}^p \exp \left(-\frac{1}{2} \left(\frac{x_j(k) - v_j^i}{\sigma_j^i} \right)^2 \right) \tag{5}$$

For an input vector $x(k)$, the overall fuzzy model output is calculated as a weighted average of each rule's local output value:

$$\hat{y}(k+1) = \frac{\sum_{i=1}^r \mu^i(x(k)) \cdot \hat{y}^i(k+1)}{\sum_{i=1}^r \mu^i(x(k))} \quad (6)$$

2.2 Dynamic Neuro-Fuzzy Local Modeling System (DNFLMS)

For neuro-fuzzy system, the learning procedure to create the fuzzy membership and fuzzy rule is implemented in following two ways: (1) input space partitioning using a clustering algorithm; (2) parameters optimization of the consequence part [Hong and Song, 2004].

In the DNFLMS, we create the antecedent for each dynamic TS fuzzy rule depending on the region centre and the region size using Eq. (5) after the evolving clustering method (ECM) [Song and Kasabov, 2001] algorithm clusters the input space into m regions and finds their centers (m rule nodes, m fuzzy rules). The ECM algorithm is a distance-based clustering method based on a normalized Euclidean distance and applies an one-pass algorithm that the data enter in series, or as a data stream, and each datum is only used once. In the clustering process of the ECM, the data examples come from a data stream and this process starts with an empty set of clusters. When a new cluster is created, its cluster centre is located and its cluster radius is initially set to zero. With more examples presented one after another, some already created clusters will be updated through changing their centre positions and increasing their cluster radiuses. Which cluster will be updated and how it will be changed depend on the position of the current data example. A cluster will not be updated any more when its cluster radius has reached the value that is greater than or equal to a distance threshold value [Song and Kasabov, 2001].

After creating the antecedent for each dynamic TS fuzzy model (rule) using the ECM algorithm, the parameters (Ψ and c) of each TS fuzzy rule consequence part in Eq. (2) should be optimized. There are two different approaches for the parameters optimization of the consequence part in Eq. (2): global and local generalization principle. In the global generalization principle, all parameters of consequent part in Eq. (2) are estimated simultaneously in an optimization algorithm with all training data. Therefore this global generalization principle has significant disadvantages: (1) slow training due to the significantly higher computational complexity (e.g., the high input dimensional problem) and (2) higher sensitivity to noise data and sparsely distributed data. This is the most common approach which is used in adaptive-network-based fuzzy inference systems, ANFIS [Jang, 1993]. In contrast, the local generalization principle is to consider the optimization of the fuzzy rule consequent part parameters as individual problem. The parameters of each TS fuzzy model are estimated separately region by region using subsets of training data. Therefore, it has a fast training speed and a capability of online simulation, a higher interpretation for the specific operating regime of the system, and a reduction of over-fitting. The local generalization principle for a TS fuzzy system is applied in [Song and Kasabov, 2000; Nelles, 2001].

To overcome drawbacks with existing neuro-fuzzy system using the global generalization principle, the DNFLMS uses the local generalization principle with the extended Kalman filtering (EKF) [Maybeck, 1982] learning algorithm to optimize the parameters (Ψ and c in Eq. (2)) in the consequence for each TS fuzzy model using m training data subsets. In the EKF learning algorithm, the unknown parameter vector, $W = [\Psi \text{ and } c]$ for each TS fuzzy model is represented by a state-space representation:

$$W_{k+1} = W_k + d_k : \text{System model} \quad (7)$$

$$Y_k = \psi(W_k, X_k) + v_k : \text{Measurement model} \quad (8)$$

where W is a vector consisting of the parameters of the consequence part in Eq. (2), Y the measured output, and X the input vector. The measurement mapping $\psi(\bullet)$ is approximated by the TS fuzzy model defined by Eq. (6). The measurements are assumed to be corrupted by noise v_k , which in our case we model as zero mean, uncorrelated Gaussian noise with covariance R . The process noise d_k may represent the uncertainty in how the parameters evolve, modeling errors or unknown inputs. We assume the process noise to be a zero mean,

uncorrelated Gaussian process with covariance Q . The EKF equations for updating the estimate of the parameter vector of the consequence part (system state) can be derived as follows:

$$K_{k+1} = P_{k+1|k} H_k^T [H_k P_{k+1|k} H_k^T + R_k]^{-1} \quad (9)$$

$$\hat{W}_{k+1} = \hat{W}_{k+1|k} + K_{k+1} [Y_{k+1} - \psi(X_k, \hat{W}_{k+1|k})] \quad (10)$$

$$P_{k+1} = P_{k+1|k} - K_{k+1} H(k) P_{k+1|k} \quad (11)$$

where \hat{W}_{k+1} is the Kaman filter estimate of the parameter vector W at step k , P_{k+1} is an error covariance matrix, K is called the Kalman gain matrix which is computed at each step and is used to update the parameter vector W and error covariance matrix P , and H is the Jacobian matrix resulting from linearizing the TS fuzzy model. The time update equations of the system state from:

$$H_k = \psi^T(\psi(X_k, \hat{W}_{k+1|k})) = \frac{\partial \psi(X_k, \hat{W}_{k+1|k})}{\partial \hat{W}_{k+1|k}} \quad (12)$$

where \hat{W}_{k+1} is the Kaman filter estimate of the parameter vector W at step k , P_{k+1} is an error covariance matrix, K is called the Kalman gain matrix which is computed at each step and is used to update the parameter vector W and error covariance matrix P , and H is the Jacobian matrix resulting from linearizing the TS fuzzy model. The time update equations of the system state from:

$$\hat{W}_{k+1|k} = \hat{W}_k \quad (13)$$

$$P_{k+1|k} = P_k + Q_k \quad (14)$$

In the DNFLMS, the parameters of each fuzzy local model are estimated separately region by region using subsets of training data using the EKF algorithm. Therefore, the DNFLMS has a fast training speed and a capability of online simulation, a higher interpretation for the specific operating regime of the system, and a reduction of over-fitting. In this work, the software of the DNFLMS is developed using a graphical user interface (GUI) under MATLABTM. The DNFLMS program has a capability of on-line visualization of inputs partitioning, fuzzy rules evolution, fuzzy membership function, inputs display, and prediction result.

3 STUDY AREA

Pupu Springs, also known as Waikoropupu Springs, are one of the largest freshwater springs of the world. They are located 3 km west of Takaka town in Golden Bay, north-west of Nelson on the South Island of New Zealand (Figure 1). The average flow from the Pupu Springs is approximately 13.9 m³/sec from the eight vents at a constant temperature of 11.7° Celsius. Potential recharge sources of the Pupu Springs flow have been identified as: rainfall, Waingaro River flow, and Takaka River flow [Mueller, 1991; White et al., 2000]. Recharge to the aquifer comes from the Takaka River, and its tributaries, which flow over karstified marble downstream of Lindsay's Bridge. In summer sometimes the total flow of the Takaka River discharges to groundwater downstream of Lindsay's Bridge, approximately 15m south-east of Pupu Springs (Figure 1). Williams (1977) showed, by pulse-train analysis, the connection of Takaka River flow at Lindsay's Bridge with the Pupu Springs flow. Recent work [White et al., 2000] also identifies the connection of the Takaka River flow at Lindsay's Bridge and the discharge due to power generation from the Cobb hydropower station with the Pupu Springs (Figure 1). The Cobb hydropower station releases, during periods of electricity generation cycle, up to 7153 L/sec into the Takaka River. Flow from the Cobb hydropower station travels down the Takaka River and discharge to the aquifer downstream of Lindsay's Bridge and then cause increased flow in Pupu Springs. Figure 2 shows observed Pupu Springs flow and Takaka River flow at the Hardwoods recorder, rainfall, and discharge flow at the Cobb hydropower station in the 1995 year.

4 DNFLMS MODEL BUILDING FOR THE PUPU SPRINGS FLOW PREDICTION

The data set contains daily values for each of the four variables (rainfall, Waingaro River flow, Takaka River flow, and discharge flow from Cobb hydropower station) starting on the 21st December 1992 and ending on the

31st December 1998. The data is split into two sets: (1) a training set including 1435 data points (21/12/1992-31/05/1996), and (2) a test set including the remaining 915 data points (01/06/1996-31/12/1998). This training set is used to construct a TS dynamic neuro-fuzzy model by DNFLMS. The testing set is used to assess the trained TS dynamic neuro-fuzzy model with data not used during the training phase.

In this work, we aim to develop the one-day ahead predictive inference model of Pupu Springs flow rate using the DNFLMS. The heuristic searching algorithm [Hong *et al.*, 2002] is implemented to find the best combination of past inputs (rainfall, river flows, Cobb hydropower station discharge) and output (Pupu Springs flow). The following set of input variables resulted in the best performance by heuristic searching algorithm: (1) past rainfall values, $Ra(t-1)$, $Ra(t-3)$, $Ra(t-4)$; (2) past Takaka River flow values, $Ta(t-10)$, $Ta(t-6)$, $Ta(t-5)$, $Ta(t-3)$, $Ta(t-2)$; (3) past Waingaro River flow values, $Wa(t-9)$, $Wa(t-7)$, $Wa(t-5)$, $Wa(t-4)$, $Wa(t-2)$; (4) past Cobb power station discharge rate values, $Cobb(t-11)$, $Cobb(t-7)$, $Cobb(t-5)$, $Cobb(t-3)$; where $(t-k)$ represents the value k days before the forecasting date, t . These 17 input variables are presented in the DNFLMS as continuous variables.

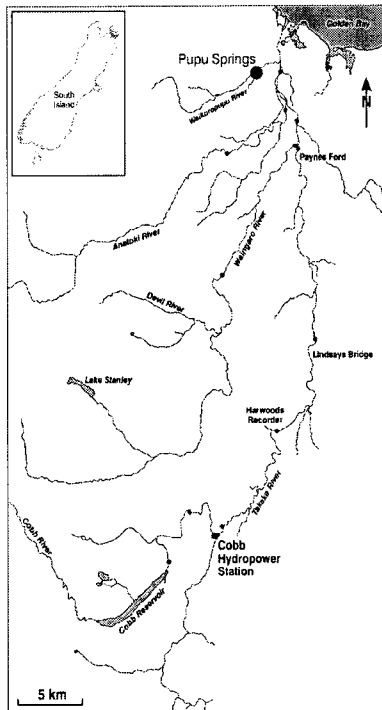


Fig. 1. Study area: Takaka Valley, South Island, of New Zealand.

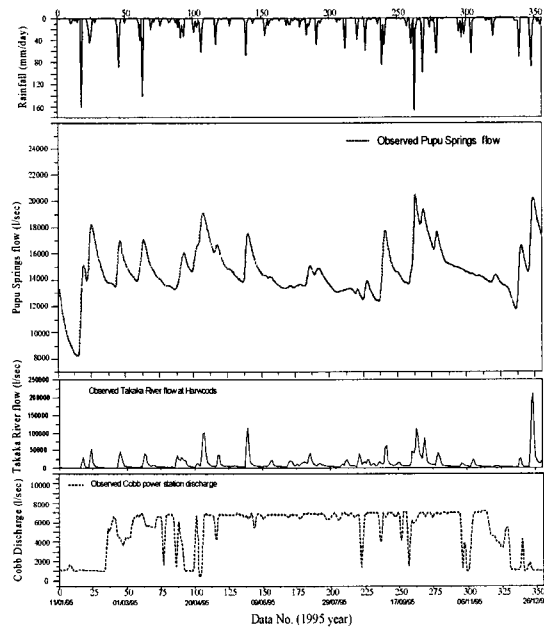


Fig. 2. Rainfall, Flow in the Pupu Springs, Flow in the Takaka River, and discharge flow from the Cobb hydropower station in the 1995 year.

For the l -th fuzzy local model (rule), the first-order TS dynamic fuzzy NARX model for a MISO (multiple-input, single-output) Pupu Springs flow forecasting system in the DNFLMS can be represented by:

To provide comparisons, a multi-layer perceptron (MLP) trained with the back propagation learning algorithm (MLP-BP) [Haykin, 1994] and an adaptive neural-fuzzy inference system (ANFIS) [Jang, 1993] are used for the same inputs and output data. Table 1 lists the prediction results of DNFLMS, MLP-BP and ANFIS. The number of fuzzy rules or neurons in hidden layer of the MLP-BP, training epochs, training time based on CPU-time, and RMSE for training and testing data are listed in Table 1.

Table 1. Results for the Pupu Springs flows prediction.

| Method | No. of neurons or fuzzy rules | Training time (second) | Epochs | RMSE | |
|--------|-------------------------------|------------------------|--------|----------|---------|
| | | | | Training | Testing |
| MLP-BP | 14 | - | 5000 | 1044 | 1378 |
| ANFIS | 86 | 21960 | 200 | 758 | 893 |
| DNFLMS | 58 | 93 | 1 | 783 | 906 |

The structure of the MLP-BP model is optimized by a genetic algorithm [Goldberg, 1989]. The optimized MLP-BP model for the prediction of Pupu Springs flow employs 17 inputs, one hidden layer with 14 neurons, 11 of which use a sigmoid transfer function and 3 of which use a hyperbolic transfer function. The training is carried out with 5000 epochs. The performance of the MLP-BP model is also shown in Figures 4 and 5, giving the RMSE on the training set of 1044 l/sec and on the testing set of 1378 L/sec (Table 1). The DNFLMS performs better than the MLP-BP model in terms of statistical and visual comparison, and generalization capability (Figures 4 and 5). For example, predictions of the DNFLMS are similar to observed Pupu Springs flow, especially when the Pupu Springs flow is relatively low. By contrast, the MLP-BP model prediction does not follow the observed pattern of low Pupu Springs flow during summer season. This discrepancy of the MLP-BP model is due to the weakness of the MLP-BP model as a global model because it tends to use a large number of parameters in order to cover the full range of data. In contrast, the DNFLMS is adapted through local models which represent the transition of the state of Pupu Springs as a response to changes in the state of input variables (Cobb discharge rate, rainfall, Takaka River flow, and Waingaro River flow. Therefore, different fuzzy local models in the DNFLMS provide some insight on the response of Pupu Springs flow due to Cobb hydropower station operation, for instance, from the high discharge mode to low discharge mode during periods of very low, or no rainfall. This advantage in the proposed DNFLMS model brings significantly better prediction of Pupu Springs flow due to changes in the Cobb hydropower station operation.

To provide a benchmark comparison, the well-known adaptive neural-fuzzy inference system (ANFIS) [Jang, 1993] is applied to Pupu Springs flow prediction. The ANFIS uses a direct nonlinear optimization algorithm to optimize the fuzzy rule premise parameters and global least squares in a nested or staggered approach to optimize the fuzzy rule consequent parameters. The Fuzzy Logic Toolbox (MATLAB version 6.5) is used for the simulation. For the application of ANFIS, the number of fuzzy memberships and fuzzy rules is fixed by trial and error. The best result of ANFIS for the Pupu Springs flow prediction is found with 86 fuzzy rules. Over-fitting often becomes a serious problem when the number of fuzzy rules exceeds 90.

The ANFIS approach shows a slightly better result compared to DNFLMS (Table 1). A large number of fuzzy rules and very long learning time due to the optimization of a large number of parameters, however, are required to achieve the similar range of training and testing results obtained by DNFLMS (Table 1). The main criticism of this ANFIS approach is that it produces TS fuzzy models which no longer represent the model locally due to global parameter estimation and have a fixed structure during the training procedure. In contrast to ANFIS, the proposed DNFLMS has a flexible structure where all its fuzzy rules can be created and updated into the fuzzy rule set before or during the one-pass training procedure. They can also be extracted from the fuzzy rule set during or after the one-pass training procedure by using the ECM and the EKF learning algorithm. The one-pass training procedure and local generalization cause the DNFLMS to have faster computational speed than the ANFIS and MLP-BP, both of which adopt global generalization.

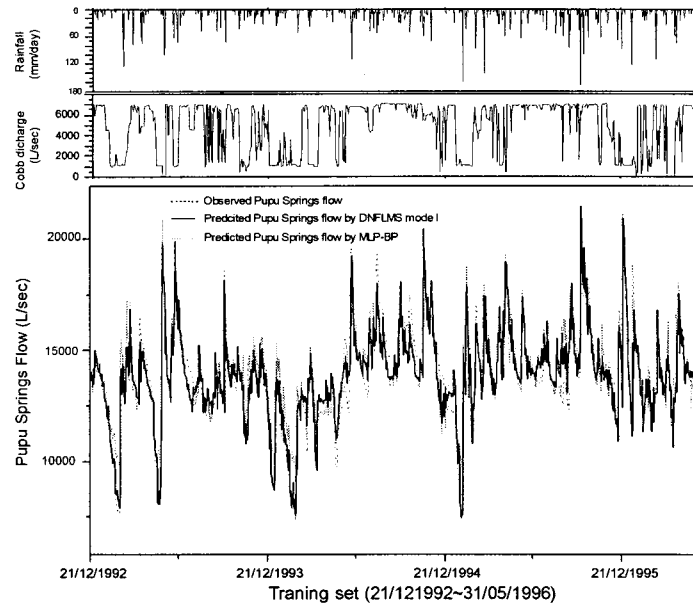


Fig. 4. Pupu Springs flow prediction of the DNFLMS for a training set (21/12/92-31/05/96).

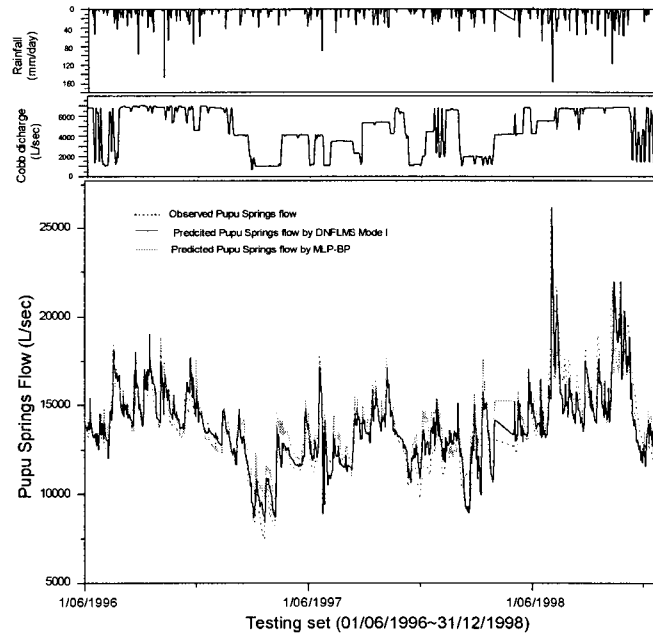


Fig. 5. Pupu Springs flow prediction of the DNFLMS for a testing set (01/06/1996-31/12/1998).

6 CONCLUSION

In this paper, we develop a new dynamic neuro-fuzzy local modeling system (DNFLMS), based on a dynamic Takagi-Sugeno (TS) type fuzzy inference system, by applying the one-pass evolving clustering method (ECM) and the local generalization principle with the extended Kalman filtering (EKF) learning algorithm.

The real hydrological application of the developed DNFLMS demonstrated that the DNFLMS shows superiority in terms of model accuracy when compared with MLP-BP and ANFIS, and also better performance in terms of model complexity (e.g., number of fuzzy models created) and training time with ANFIS. Due to

computational efficiency, dynamic simulation capability, and high levels of accuracy implemented in the DNFLMS, there is a great deal of potential for using it as a dynamic hydrological modeling tool, suitable for a variety of other complex dynamic or real-time tasks.

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