Variation of Global Coherence on Propagation in Coherent Mode Representation

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Every electromagnetic field found in nature has some fluctuations associated with it and one can deduce their existence from suitable experiments for correlations between the fluctuations at two or more space-time points. In the optical coherence theory for the scalar field, the simplest space-time correlation function of a fluctuating field is called the mutual coherence function defined by

$$\Gamma(x_1, x_2; t_1, t_2) = \langle V^*(x_1, t_1) V(x_2, t_2) \rangle,$$

where $V(x,t)$ represents the complex analytic signal associated with the field and the angular brackets denote the ensemble average. Sometimes, the mutual coherence function depends on t_1 and t_2 only through the difference $\tau = t_2 - t_1$ and then one can introduce the cross spectral density defined by

$$W(x_1, x_2; \omega) = \int \Gamma(x_1, x_2; \tau) e^{i\omega\tau} d\tau.$$

This cross spectral density is the space-frequency correlation function and allows the frequency by frequency analysis which is often more desirable.

The complex degree of spectral coherence, which is the normalized version of the cross spectral density, is defined by

$$\mu(x_1, x_2; \omega) = \frac{W(x_1, x_2; \omega)}{\sqrt{W(x_1, x_1; \omega) W(x_2, x_2; \omega)}}.$$

The complex degree of spectral coherence reflects the local coherence properties of field in the sense that it shows the correlation between two spatial points. According to Schwarz's inequality, the modulus of μ takes on values between zero and unity. When $|\mu| = 0$, thefield is completely incoherent, when $|\mu| = 1$, the field is completely coherent, and when $0 < |\mu| < 1$, the field is partially coherent between two points x_1 and x_2.

While μ gives a local description of coherence, there is another scheme to reveal the coherence properties of the field from a different point of view. This new scheme is based on the so-called mode functions defined globally on the entire domain of the field. This new scheme is often called the coherent mode representation of the field. When a cross spectral density becomes a Hilbert-Schmidt kernel, then the cross spectral density can be written as a factorized form

$$W(x_1, x_2; \omega) = \sum_n \lambda_n(\omega) \phi_n^*(x_1; \omega) \phi_n(x_2; \omega),$$

where λ_n and ϕ_n are the eigenvalues and eigenfunctions of the integral equation

$$\int_D W(x_1, x_2; \omega) \phi_n(x_1; \omega) dx_1 = \lambda_n(\omega) \phi_n(x_2; \omega).$$

Some concrete examples of the coherent mode representation have been carried out mainly for Gaussian Schell model sources and for Bessel-correlated fields. According to the above analyses, the coherent mode representation seems to reflect the global coherence properties of fields. It turns out that the relevant parameter β for global coherence is the ratio of the coherence length l_c to the linear dimension L of the source or field, i.e., $\beta = l_c / L$. The coherence length l_c is determined by μ. When β is small, there are many modes contributing to the cross spectral density and the field is rather incoherent. However, when β is large, relatively few modes contribute to the cross spectral density. The entropic measure of global coherence has been defined and applied to Gaussian Schell model fields recently.

As the field developes according to the wave equation, so the state of coherence of light may be appreciably changed in the process of propagation. More specifically, even if the light originates from a completely incoherent source, the field at points sufficiently far from the source may be highly correlated. This is one of the central theorems in the optical coherence theory, formulated by van Cittert and later, in a more general form, by Zernike. In this paper, we investigate how the global coherence of the field may change on propagation. We consider the wave propagation plane by plane for a given field correlation in the input plane, and examine the possible variation in the framework of the coherent mode representation. The entropic measure will be used to quantify the global coherence of the field in the plane. We will also explore the essence of van Cittert-Zernike theorem in the coherent mode representation. In the coherent mode representation, the global properties of the field may be determined by the structure of eigenmodes, in particular, by the effective number of different eigenmodes.