Analytic Study on Wave Propagation in Anisotropic Media with the Index Ellipsoid

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The study of light propagation in anisotropic medium is one of the most important fields of liquid crystal displays (LCDs). As well known, due to its anisotorpic properties, LCDs have a critical problem that images on a screen can be seen to be different in viewing directions. To compensate the viewing-angle dependence, analytic modeling of anisotropic medium should be come first. Berreman and extended Jones matrixes are usually used to analyse the light propagation in oblique incidences[1,2]. Though they give accurate results in the analysis, they are too complicated to understand and follow.

Instead, it is easier to use the index ellipsoid in modeling an anisotropic medium, which visualize the eigen refractive indices and the electric field eigen vectors[3]. With Maxwell's equations and some mathematical manipulation, the surfaces of constant energy density U_e in $\overrightarrow{D}$ space can be written as

$$\frac{x^2}{n_x^2}+\frac{y^2}{n_y^2}+\frac{z^2}{n_z^2}=1, \tag{1}$$

where n_x, n_y and n_z are the principal indices of refraction.

This is the equation of a general ellipsoid with major axes parallel to the x, y and z directions whose respective lengths are $2n_x, 2n_y$ and $2n_z$. The ellipsoid is known as the index ellipsoid, as shown in Fig. 1. In the index ellipsoid, the intersection of the plane through the origin that is normal to $\vec{k}$, which corresponds to the direction of the incident light, shows the indices of refraction and the polarization of normal modes for a given direction of propagation. The length and the direction of major axes in the intersection plane of ellipse correspond to the indices of refraction and the polarization of normal modes, respectively.

When light propagates in anisotropic medium, there are always two independent modes of propagation. In uniaxially birefringent media, they are called the ordinary and the extraordinary waves. But, in biaxially birefringent media, they are acting like two extraordinary waves. In both cases, the two waves feel different refractive indices and propagation distances. In other words, they feel different phase retardations with the direction of propagation.

When light incidents in an anisotropic media, as a result of the birefringence, double refraction occurs. The phase retardation can be written

$$\Gamma = k[n_1\cos\theta_1 - n_2\cos\theta_2], \tag{2}$$

where k is the wavenumber in vacuum, n_1 and n_2 are the eigen indices of refraction of the lights. And, by the continuity conditions

$$\sin\theta = n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where θ_1 and θ_2 are the refracted angles of the lights. In Fig. 2, phase difference between two eigen waves are calculated for uniaxially birefringenct and biaxially birefringent media by the index ellipsoid method.

By now, there have not been easy and analytic results for light propagation in biaxial medium. With the index ellipsoid method, however, we can easily calculate the phase retardation of the light propagating in anisotropic medium, both uniaxial and biaxial medium. As a result, it will be helpful to analyse and compensate th viewing angle dependence of LCDs.

References

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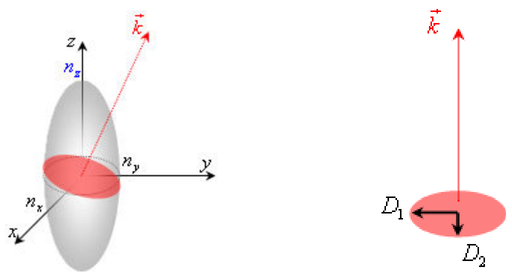
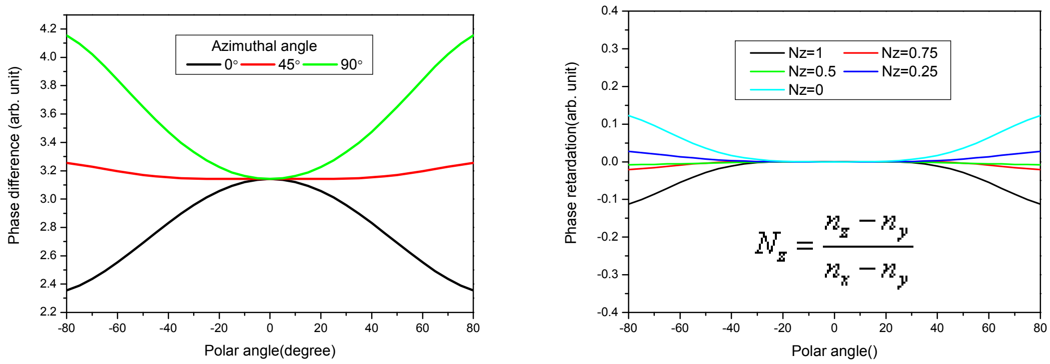


Fig. 1. The index ellipsoid and finding the indices of refraction and the polarization of normal modes for a given direction of propagation.



(a) uniaxial medium (b) biaxial medium

Fig. 2. Phase difference between two eigen waves