Generating entangled states of two ququarts using linear optical elements

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In this paper, we propose experimentally feasible schemes for generating entangled states of two photonic ququarts (four-dimensional quantum systems, i.e., D=4) using linear optical elements and coincidence post-selection. Since the scheme is based on the use of a beamsplitter, entanglement of multiple ququarts may be possible if the two-ququart entangling schemes are properly cascaded.

The individual ququart in our scheme is based on collinear frequency non-degenerate (λ_1 or λ_2) biphoton polarization ($|H\rangle$ or $|V\rangle$) states of spontaneous parametric down-conversion (SPDC). It is then possible to define four orthonormal biphoton states which form the bases states for a single-ququart.

$$\{|H_{\lambda_1},H_{\lambda_2}\rangle,|H_{\lambda_1},V_{\lambda_2}\rangle,|V_{\lambda_1},H_{\lambda_2}\rangle,|V_{\lambda_1},V_{\lambda_2}\rangle \equiv \{|0\rangle,|1\rangle,|2\rangle,|3\rangle\}. \quad (1)$$

The two ququarts, each in spatial modes a and b, can be represented as,

$$|\psi\rangle_a = c_0|0\rangle_a + c_1|1\rangle_a + c_2|2\rangle_a + c_3|3\rangle_a, \qquad |\psi\rangle_b = c_0^{'}|0\rangle_b + c_1^{'}|1\rangle_b + c_2^{'}|2\rangle_b + c_3^{'}|3\rangle_b \quad (2)$$

These ququarts can be generated, for example, by using the scheme shown in Fig. 1(a). We make use of quantum interference of photons at a beamsplitter to generate entangled states of two ququarts (Fig 1(b)). Three choices of the beamsplitters are considered : an ordinary 50/50 beamsplitter (BS), a polarizing beamsplitter (PBS), and a dichroic beamsplitter (DBS).

First, assuming that there is one SPDC pair in each input mode of the beamsplitter ($|\psi\rangle_{in} = |\psi\rangle_a \otimes |\psi\rangle_b$), we investigate the output state $|\psi\rangle_{out}$ (at modes c and d) by evaluating the beamsplitter unitary transformation $U_{\lambda_1}^{BS} \otimes \mathrm{U}_{\lambda_2}^{\mathrm{BS}} \otimes \mathrm{U}_{\lambda_1}^{\mathrm{BS}} \otimes \mathrm{U}_{\lambda_2}^{\mathrm{BS}}$ on two ququarts input mode $|\psi\rangle_{in}$. When the output state $|\psi\rangle_{out}$ is post-selected by the two-ququart detection scheme (four-fold coincidence), we can see, for all three beamsplitters, an entangled sates of two ququarts (each occupying mode c and d) can be found.

For the 50/50 ordinary and polarizing beamsplitters, $|\psi\rangle_{out}$ includes many non-zero amplitudes which cannot be expressed in the ququart bases states defined in eq.(1). But, for a dichroic

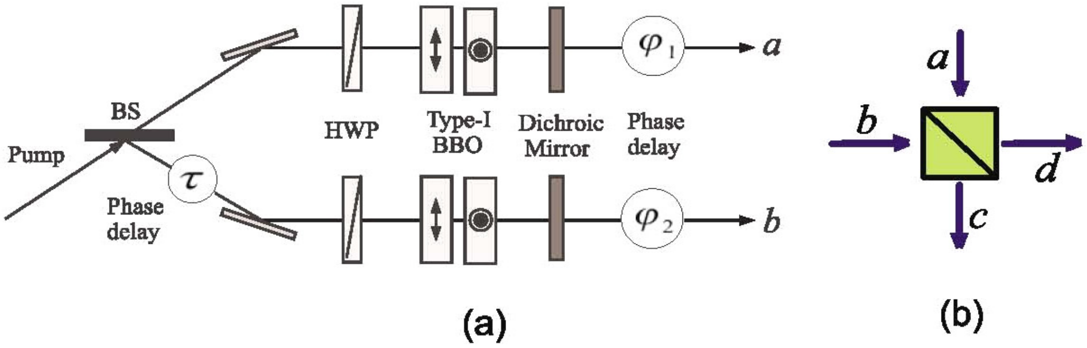

Figure 1. (a) Experimental scheme to generate two biphoton ququarts (b) the spatial mode configuration of a beamsplitter

beamsplitter (DBS), which reflects λ_1 and transmits λ_2, there are 16 non-zero output amplitudes all of them written in the ququart basis. For an experimental demonstration, therefore, a DBS should be used if at all possible. If the input to the DBS is, for example, $|\psi\rangle_a = c_0|0\rangle_a + c_3|3\rangle_a$ and $|\psi\rangle_b = c_0'|0\rangle_b + c_3'|3\rangle_b$, the output state is found to be,

$$|\psi\rangle_{out} = c_0 c_0'|0\rangle_c|0\rangle_d + c_3 c_0'|1\rangle_c|2\rangle_d + c_0 c_3'|2\rangle_c|1\rangle_d + c_3 c_3'|3\rangle_c|3\rangle_d.$$

In addition, we have investigated the effects of the double-pair events ($|\psi\rangle_a|\psi\rangle_a|0\rangle_b$ and $|0\rangle_a|\psi\rangle_b|\psi\rangle_b$) to the final output state. These double pair events cannot be ignored in a real experimental setup involving SPDC. For the cases of 50/50 beamsplitter and polarizing beamsplitter, we find that the double-pair events add total of 128 and 8 additional two-ququart amplitudes to final states. It is, however, interesting and important to note that these double pair events add to the final amplitude coherently. This is quite different from, for example, some of the teleportation and four-photon experiments where the double-pair events add constant background noise to the final state.

Finally, for the case of a dichroic beamsplitter, we find quite interestingly that the double-pair events do not add any amplitudes that can be detected by the two-ququart detection scheme (four-fold coincidence) to the final state. The double-pair events, therefore, can be completely ignored in this case.

We will also discuss a realistic dichroic beamsplitter based experimental scheme, which can generate interesting two-ququart entangled states for the test of Bell's inequality. An experiment to realize this proposal is under way at our laboratory.