CHAOTIC MIXING IN SQUARE CAVITY FLOW

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정사각형 캐비티 유동의 혼돈적 혼합 특성

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The quality of chaotic mixing in square cavity flow was studied numerically by CFD simulation and particle tracking technique. The chaotic mixing was generated by using time-periodic electro-osmotic flow. Finite Volume Method (FVM) was employed to get the stretching and folding field in cavity domain. With adjusting the initial condition of concentration distribution, the best values of modulation period and Peclet number which gave us good mixing performance was determined precisely. From Poicarésection and Lyapunov exponents for characteristic trajectories we find that mixing performance also depends on modulation period. The higher value of modulation period, the better mixing performance was achieved in this case. Furthermore, the results for tracking particle trajectories were also compared between using of Bilinear Interpolation and Higher-order scheme. The values of modulation period for obtaining best mixing effect were matched between using FVM and particle tracking techniques.

Key Words : Chaotic Mixing,Poincaré section, Lyapunov exponent, Bilinear Interpolation.

1. INTRODUCTION

There has been a recently developing surge of fundamental properties of the mixing due to its application in manufacturing, food, pharmacology and other industries.

Some researchers are successfully studied on electro-osmotic flows in the non-uniform zeta-potentials such as Qian and Bau[13], Ajdari[14], H. Aref[15] and etc. Understanding of this article can help to crease a good mixing in cavity flow by switching the various flow fields which are caused by imposing the non-uniform zeta-potential surfaces to the walls.

2. PROBLEM FORMULATION AND ANALYTICAL SOLUTION

We have considered the unsteady two-dimensional motion of an incompressible fluid within a closed square cavity where four electrodes are attached to the bottom and top walls of its as shown in Figure 1. This generates the relevant electro-osmotic slip velocity at the walls.

The governing equations for this problem are written in a dimensionless form as follows

$$\nabla \cdot u = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\nabla p + \frac{1}{Re}\nabla^2 u \tag{2}$$

$$\frac{\partial c}{\partial t} + u \cdot \nabla C = \frac{1}{Pe}\nabla^2 C \tag{3}$$

where C is the concentration, Re(=UL/ν) is the Reynolds number and Pe(=UL/D) is the Peclet number. L is the characteristic length, U is the characteristic velocity, D is the concentration diffusibility, ν is the kinematic viscosity and t is the dimensionless time.

Pattern A and B corresponds with the flow fields applied to bottom wall during the first half of the period and to top wall during the second half of the period, respectively.

The boundary conditions for velocity field and

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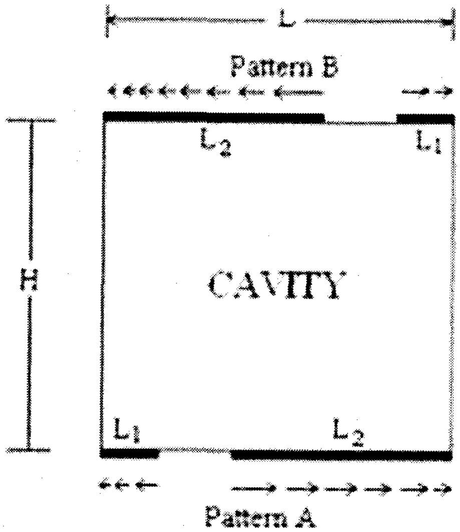


Fig.1 2-D Geometry of the cavity and the periodic non-uniform boundary conditions at the top and the bottom walls. $L_1 = 0.2L$, $L_2 = 0.6L$

concentration are:

At the side walls:

- Velocities: No-slip condition u = v = 0
- Concentration: zero-gradient condition $\frac{\partial C}{\partial x} = 0$

At the top and bottom walls:
Velocities: No-slip condition v = 0

- Concentration: zero-gradient condition $\frac{\partial C}{\partial y} = 0$

At the bottom wall:

- During first half period $0 \le t \le T/2$

$$ub_1 = -\frac{3.33}{2L}x \quad \text{for} \quad 0 \le x \le 0.2 \tag{4}$$

$$ub_2 = \frac{1}{(0.4L-1)}(x-1) \quad \text{for} \quad 0.4 \le x \le 1 \tag{5}$$

where T is modulation period.

- During second half period $T/2 \le t \le T$

$$ut_1 = -\frac{5}{3L}x \quad \text{for} \quad 0 \le x \le 0.6 \tag{6}$$

$$ut_2 = -\frac{0.333}{(0.8L-1)}(x-1) \quad \text{for} \quad 0.8 \le x \le 1 \tag{7}$$

In this case, we used the non-uniform staggered grid system. The governing equations are discretized in time using finite volume method. The Explicit Euler method is used for integration of momentum and concentration equations in time.

Mixing index is defined by the following equation

$$D = \sqrt{\frac{1}{N}\sum_{i,j}^{N}(1 - C_{i,j}/\overline{C}} \tag{13}$$

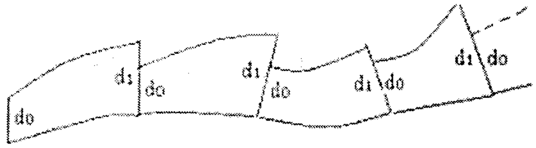


Fig. 2 Schematic for calculation for calculating the Lyapunov exponent.

※ Mixing effect using Poincaré section and Lyapunov exponent

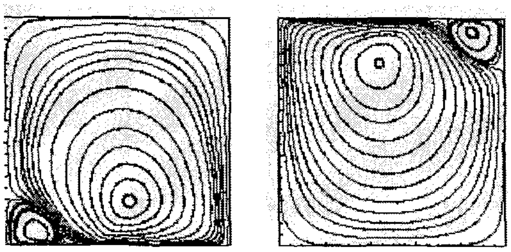
- Poincarésection is a graphical analysis tool to capture interesting features such as mixing zones in cavity flow. Otherwise, it is also a surface in the phase space that cuts across the flow of a given system. With a 2D cavity flow, the positions of a point in calculation domain are advanced by 4th-order Runge-Kutta method.
- Lyapunov exponent describes chaotic mixing by determining the position of two initially nearby particles will be extremely different after a certain time.

The best mixing effect will be obtained when Lyapunov exponent approaches to a maximum value. The largest Lyapunov coefficient should be positive in the chaotic state.

3. Results and Discussion

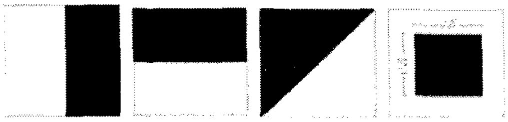
The FORTRAN code has been developed for the 2D case which gave us quite good results. The numerical solutions were obtained for the grid 101x101, which was selected by grid convergence test and for the fixed Reynolds number Re = 10. The mixing process is attained steady state in whole domain of cavity after total time steps 500,000 for Pe = 2000 and 600,000 for Pe = 10,000. In each case, the results are obtained for the five values of modulation periods 1, 5, 10, 15 and 20.

Mixing performance is obtained from solving concentration equation correspond with various initial



(a) (b)

Fig. 3 Streamlines pattern for the cavity flow at steady state corresponding to T=1, (a) first half period and (b) second half period.



(a) (b) (c) (d)

Fig.4 Concentration distribution conditions used in the simulation C = 1 for black and C = 0 for white. (a) VS – horizontal separation (b) HS – vertical separation (c) DS – diagonal *separation and (d) SS – square separation*

conditions of concentration distribution. The Poincaré section and Lyapunov exponent are employed to get mixing index for various boundary conditions of streamline velocity.

With above boundary conditions, we got the velocity field which is symmetric for every half period. The streamlines appear four eddies, two at the bottom wall during first half period and remaining two at the top wall during second half period.

3.1 Mixing Effect with Respect to Various Initial Conditions of Concentration Distribution

The results show that, for the small Peclet number (Pe = 2000), the change in mixing pattern is negligible with varying modulation periods (1 to 20) at a specified time step. However, when we increase the Peclet number (Pe = 10000) the mixing pattern is rather different with respect to different modulation periods. Especially, with the higher value of modulation, the more early mixing process is obtained at a specified Peclet number. Furthermore, we know that the small Peclet number causes a high diffusion, so the mixing process quicker for low Peclet numbers than the higher Peclet numbers.

The best mixing index are collected for every initial condition at Pe = 2000 and Pe = 10000 (Figures. 5). We

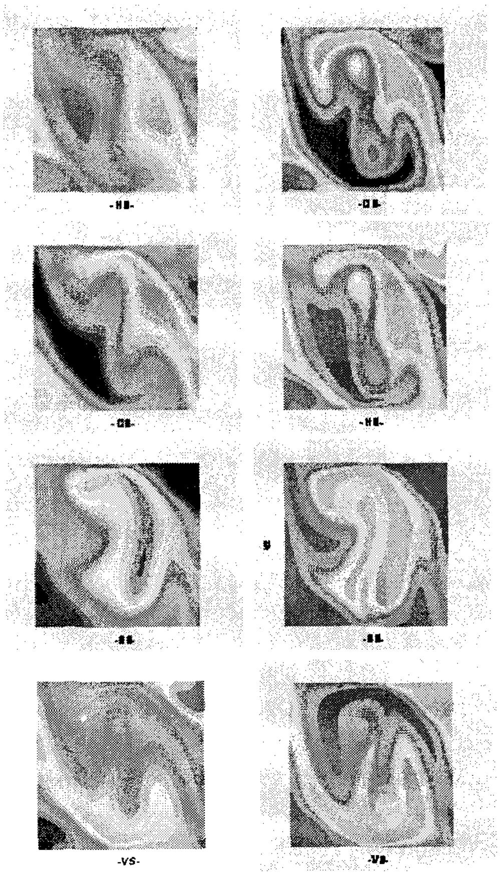


Fig. 5 Mixing effect with respect to various initial conditions of concentration distribution at t= 20 (RHS- results for Pe = 2000; LHS- results for Pe = 10000)

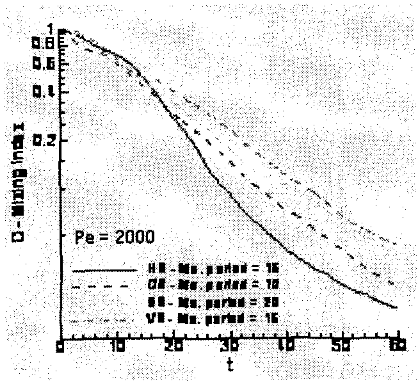


Fig. 5 Variation of Mixing index with respect to dimensionless time for Pe = 2000.

found that mixing effect is specifically affected by the initial condition of C-distribution. Among these results, the best mixing effect is achieved in case of HS (Pe= 2000)

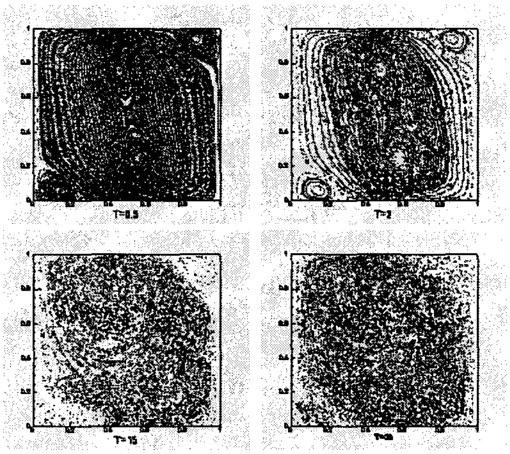


Fig. 6 Poincaré section with respect to various modulation periods.

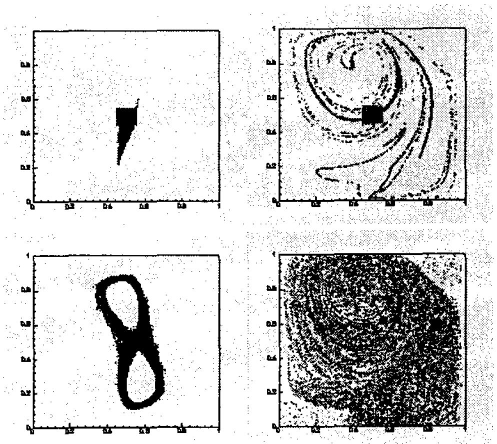


Fig. 7 The deformation of a material of edge size 0.1 initially (t = 0) centered at (0.45, 0.45).

and DS in case of Pe = 10000.

For Pe=2000, the mixing is better compared with Pe = 10000. That means at low Pe number, diffusion is more, so many thin layers will be observed in flow pattern.

3.2 Results for Poincaré Section and Lyapunov-Exponent

We will capture the particle's trajectory by calculating the equation of positions. The 1025 started points are distributed uniformly in cavity domain.

We can see the mixing effect is better gradually when we increase the modulation period. At the smallest modulation period, the trajectories of the particle is clear as streamlines in chaotic and regular domains. But when modulation period increases the particles are distributed uniformly in whole domain of cavity. Therefore, we can get best mixing effect when we input the enough high value of modulation period. The deformation of blob is also considered with respect to various values of modulation periods. After two periods, the square blob has already turned into thin line. When the modulation period is relatively small, the blob stretches, deforms and elongates slowly. At T = 2, all particles just wandered around the small fixed zone after 20 periods.The blob is deformed fully and the fluid particles spread to cover almost the entire cavity domain with respect to T= 20.

Furthermore, when we impose zeta-potential surfaces to another wall which means we change boundary condition at the walls of cavity; we also get the rather different mixing effects.

Appropriate to different flow patterns (shown in Fig. 7), we computed the Lyapunov-exponent for various time periods.

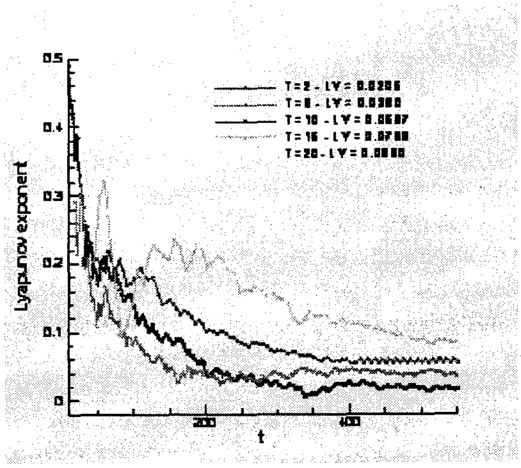


Fig. 14 Lyapunov exponent at various modulation periods.

4. Conclusion

The primary aim of the present work paper was to develop the numerical code of finite element method for solving the chaotic mixing properties of flows generated by solid walls undergoing alternating boundary conditions periodically in every period at the top and bottom walls in a two dimensional cavity. The chaotic mixing is enough good depend on the modulation of the chosen period for imposing of the boundary conditions at the solid wall of cavity and the Pelec number.

The fairly good results of chaotic mixing in this case are to demonstrate FVM is also an advantageous method for simulation of mixing problems. The Poincaré section and Lyapunov exponent are also the good methods to obtain the mixing performance in this case.

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