허용하중집합 개념을 이용한 기계/구조 시스템의 강건 설계

Robust Design of Structural and Mechanical Systems using Concept of Allowable Load Set

곽 병 만*

Kwak, Byung Man

ABSTRACT

The concept of "Allowable Load Set (ALS)" introduced by the author allows an easy understanding of load and strength characteristics of a structure in relation to its integrity under uncertainties. Two criteria of safety are introduced: A relative safety index denotes the distance to the boundary of the ALS and a normalized safety index is a distance in terms of functional value. They have been utilized in several examples, including multi-body mechanical systems such as a biomechanical system. Both formulations amount to robust designs in the sense that designs most insensitive to uncertainties are obtained in the context of newly defined safety indices, without using any input probability information.

Keywords: *robust design, allowable load set, structural integrity, relative safety index, normalized safety index*

1. Introduction

In designing a structural or mechanical system under uncertainty, two types of approaches may be identified: One is reliability based design optimization, in short, RBDO, and the other is robust design. RBDO requires probability information while the basic philosophy of a robust design in narrow sense is making a structure most insensitive to uncertainties without utilizing any probability data of input random variables. Methodologies of RBDO are relatively well developed such as reliability index approach and moment based methods (Seo and Kwak, 2002; Lee and Kwak, 2005; Huh *et al.*, 2006) of more recent development. When no detailed statistical data is available, however, no probabilistic approaches are possible. In addition to the usual gradient based formulation (Han and Kwak, 2001), a non-probabilistic theory based on information-gap models of uncertainty by Ben-Haim(1997) is an example. In 2002, Kwak and Kim(2002) have introduced a new concept called "Allowable load set (ALS)." ALS brings in a new view of structural analysis and design. Here one is interested in finding what kind of loads the structure can support, and finding the dimension or shape such that the set of allowable loads takes an optimal form. To determine the optimum form of ALS, a criterion is necessary and two criteria are developed: a

* 한국과학기술원 기계공학과 삼성석좌교수 Email: bmkwak@khp.kaist.ac.kr

relative safety index (RSI) and a normalized safety index (NSI). The concept of ALS facilitates a new way of understanding characteristics of a structure or a multi-body system and a new design formulation.

2. Allowable Load Set (ALS) (Kwak and Kim, 2002)

ALS is a set of loads that are safe to a given structure or mechanical system. The designer must define the meaning of "allowable" or "safe." When an inequality constraint gi is in terms of stress and displacement, ALS, K is defined as a set in the random variable space, satisfying the inequality conditions as follows:

$$K = \{\mathbf{X} \mid G(\mathbf{X}) \le \mathbf{0}\} \equiv \bigcap_i \{\mathbf{X} \mid g_i(\boldsymbol{\sigma}(\mathbf{X}), \boldsymbol{\delta}(\mathbf{X}), \mathbf{X}) \le 0\} \quad (1)$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ and $\mathbf{X}$ denote the stress, displacement and external load, respectively. If the allowable effective stress and displacement are $\boldsymbol{\sigma}_a$ and $\boldsymbol{\delta}_a$, the constraint equations for point, p, may be written,

$$g_1(\boldsymbol{\sigma}^p) = \frac{V(\boldsymbol{\sigma}^p(\mathbf{X}))}{\sigma_a} - 1 \le 0, \quad g_2(\boldsymbol{\delta}^p) = \frac{D(\boldsymbol{\delta}^p(\mathbf{X}))}{\delta_a} - 1 \le 0 \quad (2)$$

where V(•) represents a general stress criterion such as maximum-shearing stress, distortional energy, etc. The function D(•) is similar to V(•), corresponding to a displacement criterion.

3. Criteria of the Degree of Safety

As can be conjectured from an ALD, the distance from the mean load to the boundary of the ALS can be taken a measure of the safety of a structure. This is a measure called a relative safety index (RSI) studied earlier and can be obtained without any knowledge of probability data of the random load. Consider a structure with an ALS as shown in Fig. 1. When the structure is subject to a random external force X whose mean value is $\overline{X}$, a relative safety index, β_i denotes the smallest distance from the mean value, $\overline{X}$, to the boundary, g_i(X)=0. In notation,

$$\beta_i = \min\{|\mathbf{X} - \overline{\mathbf{X}}| \mid g_i(\mathbf{X}) = 0\} \quad (3)$$

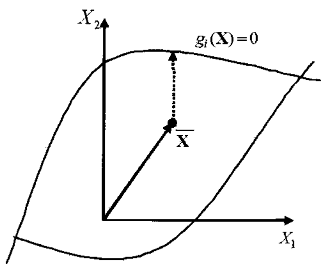


Fig. 1. Definition of a relative safety index in ALS

For a system with m constraints, the system relative safety index is,

$$\beta_i = \min\{\beta_i, i = 1,\ldots,m\} = \min_i \min_X \{|X - \bar{X}| \mid g_i(X) = 0\} \quad (4)$$

It is noted that when probability information of random variables is given, the RSI denotes reliability index, used in first order reliability method.

A very efficient safety index called a normalized safety index (NSI), ν, is next introduced, applicable for normalizable constraint such as the stress constraint given by the first equation in Eq. 2, as follows,

$$\begin{aligned} g(0) &= -1 \\ g(X) &= 0, \text{ when X is on the boundary} \end{aligned} \quad (5)$$

Define a transformed function as follows

$$\bar{g}(\mathbf{X}) = -g(\mathbf{X}) \quad (6)$$

then

$$0 \le \bar{g}(\mathbf{X}) \le 1 \quad \text{for} \quad \mathbf{X} \in \text{ALS} \quad (7)$$

Thus a normalized safety index is defined as

$$\nu_i = \bar{g}_i(\bar{\mathbf{X}}) = -g_i(\bar{\mathbf{X}}) \quad (8)$$

It is reminded that NSI is obtained by only evaluating a constraint function at the mean load, $\bar{X}$. For a system of m constraints,

$$\nu = \min\{\nu_i, i = 1,\ldots,m\} = \min_i\{-g_i(\bar{\mathbf{X}})\} \quad (9)$$

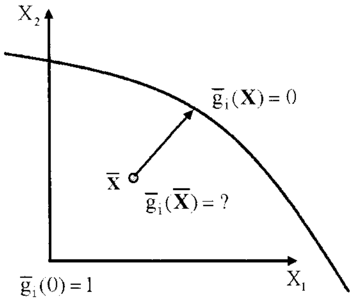


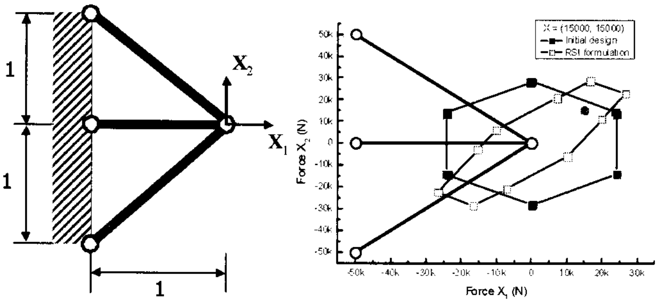
Fig. 2. Normalized safety index $\bar{g}_i(\bar{X}) = -g_i(\bar{X})$

As seen in Fig. 2, the NSI may be interpreted as a distance in terms of functional value instead of the conventional distance or length. NSI can be obtained just by a function evaluation instead of a distance calculation, which requires a minimization procedure.

The idea of ALS can be well adapted to multi-body mechanical systems. A history of loading and sometimes a change of boundary conditions of a component should be considered. For details and examples, refer to Kwak and Kim(2002).

4. Robust design formulation by ALS

The philosophy of an optimum structural design using ALS is to find the optimal ALS by selecting design variables in such a way that the boundary of the set is as far as possible from the mean design load. In Fig. 3(b), it is seen that one of the boundary segments of the ALS of the initial design is very near to the mean external load. The ALS at the optimum solution shows that the boundary is made as far as possible from the mean load.



(a) Description (b) ALS generated at initial design and RSI-based robust design

Fig. 3. A three bar truss problem

The first formulation is to use the relative safety index:

$$\text{Maximize } \beta = \max_{b} \min_{i} \beta_i(b) \tag{10}$$

This can be reformulated to a standard form by introducing an artificial design variable, b_{n+1}, obtaining RSI formulation, as follows,

$$\text{Max} \quad b_{n+1} \tag{11}$$

$$\text{Subject to } b_{n+1} \leq \beta_i(b), \; i=1,..,m$$

$$g_j \leq 0, j = (m+1),..,N$$

where

$$\beta_i(b) = \min_{X} \left\{ | \mathbf{X} - \bar{\mathbf{X}} | \;|\; g_i(\mathbf{X}, b) = 0 \right\} \tag{12}$$

It is convenient to solve the following transformed formulation:

$$\text{Max} \quad b_{n+1} \tag{13}$$

$$\text{Subject to } G_i \leq 0, \; i=1,..,m$$

$$g_j \leq 0, j = (m+1),..,N$$

where

$$G_j = \max_{X\in D} g_j(\mathbf{X},\mathbf{b}),\ j=1,...,m \quad \text{where } D=\{X \mid \|\mathbf{X}-\bar{\mathbf{X}}\| \le b_{n+1}\} \tag{14}$$

This kind of transformation was first developed in 1987 by Lee and Kwak(1987) where the RIA approach is reformulated as a so-called "Performance measure approach (PMA)." Both the RSI formulation and the RIA approach contain a sub-problem of finding the distance to a constraint boundary and hence this transformation can be equally applicable.

The next formulation is using the normalized safety index, v

$$\text{Maximize } \nu = \max_b \min_i \nu_i(\mathrm{b}) \tag{15}$$

A more convenient form of NSI formulation is obtained by introducing an artificial variable,

$$\text{Max} \quad b_{n+1} \tag{16}$$

$$\text{Subject to } b_{n-1} \le \nu_i(\mathrm{b}),\ i=1,..,m$$

$$g_j \le 0,\ j=(m+1),..,N$$

where

$$\nu_i(\mathrm{b}) = -g_i(\bar{\mathbf{X}},\mathrm{b}) \tag{17}$$

5. Shape design of a torque arm

As a practical example of NSI based robust design, a torque arm shape design is shown in Fig. 4. The design variables are five CAD parameters of the arm. Stress constraints on 8 selected critical points on the first boundary and 8 and 6 points for second and third boundary are imposed. The conventional solution is based on reliability index approach (RIA) using a coefficient of variation of 0.1 for the uncertain loads. The results are summarized in Fig. 5. The shape results shown in Fig. 5 are very similar each other, but the computational effort of the NSI is almost the same as a deterministic optimization with about 2000 function calculations while the RIA based method takes 6000 evaluations.

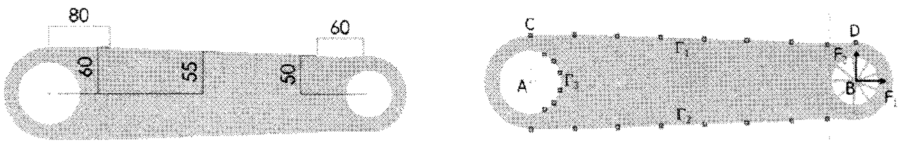
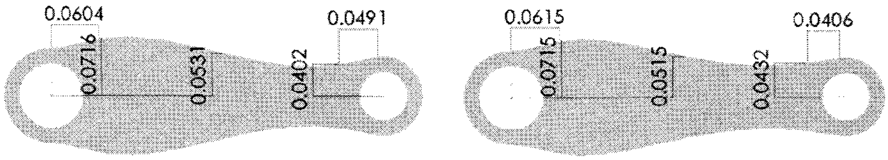


Fig.4. Illustration of Torque arm: Initial design variables and stress check points



(a) Conventional optimization

(b) Proposed optimization

Fig. 5. Robust optimal designs of two different formulation

6. Discussion and conclusion

The concept of allowable load set is introduced and two criteria for the degree of safety are defined. Both the RSI and the NSI provide a sound robust design formulation. RSI requires distance finding while NSI needs only function evaluation. The NSI formulation is very efficient like a deterministic optimization formulation.

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