모델 예측 추적을 이용한 이동 로봇의 경로 추적

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Model Predictive Tracking Control of Wheeled Mobile Robots

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Abstract - This paper presents a model predictive controller for tracking control of the wheeled mobile robots (WMRs) subject to nonholonomic constraint. The input-output feedback-linearization method and the mode transformation are used. The performance of the proposed control algorithm is verified via computer simulation. It is shown that the control strategy is feasible.

1. Instruction

Tracking control of nonholonomic mobile robots aims at controlling robots to tracking a given time varying trajectory. It is a fundamental motion control problem and has been intensively investigated in the robotic domain.

Model Predictive Control (MPC) is one of the frequently used optimization control techniques in industry. It is an online optimization algorithm that predicts system inputs based on current states and system model, finds an open loop control profile by numerical optimization, and applies the first control signal in the optimized control profile to the systems.

Due to the use of MPC to the WMRs, the nonlinear system control stability became one of the main problems. Such as [1], considers the control stability by adding a terminal-state penalty to the cost function and constraining the terminal state to a terminal-state region. Different from them, our control approach presents some particularities. As a result of the using of input-output linearization method [3], two decoupled single-input single-output (SISO) systems are obtained. Then, the discrete model is instead of the continuous one. And the linear model predictive control is applied to the system for the trajectory tracking.

2. Kinematics and dynamics

2.1 Kinematics

Figure.1 presents a geometrical model of the wheeled mobile robot defining the necessary variables to obtain the kinematic model.

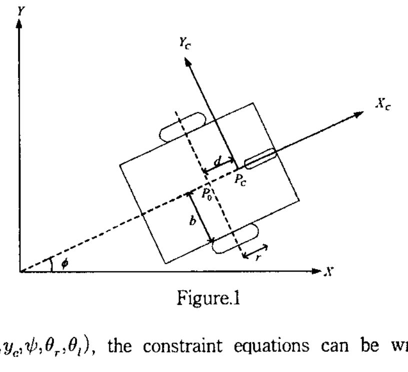

Figure.1

Let $q=(x_c,y_c,\psi,\theta_r,\theta_l)$, the constraint equations can be written in the form

$$A(q)\dot{q}=0 \tag{1}$$

where

$$A(q)=\begin{bmatrix}-\sin\phi & \cos\phi & -d & 0 & 0\\ -\cos\phi & -\sin\phi & -b & r & 0\\ -\cos\phi & -\sin\phi & b & 0 & r\end{bmatrix}. \tag{2}$$

2.1 Dynamics

The Lagrange equations of motion (see[2]) of the robot are given by

$$M(q)\ddot{q}+V(q,\dot{q})\dot{q}=E(q)\tau-A^T(q)\lambda \tag{3}$$

where

$$M(q)=\begin{bmatrix} m_t & 0 & m_t d\sin\phi & 0 & 0\\ 0 & m_t & -m_t d\cos\phi & 0 & 0\\ m_t d\sin\phi & -m_t d\cos\phi & I & 0 & 0\\ 0 & 0 & 0 & I_\omega & 0\\ 0 & 0 & 0 & 0 & I_\omega\end{bmatrix}$$

$$V(q,\dot{q})=\begin{bmatrix} m_t d\dot{\phi}\cos\phi\\ m_t d\dot{\phi}\sin\phi\\ 0\\ 0\\ 0\end{bmatrix},E(q)=\begin{bmatrix}0&0\\0&0\\0&0\\1&0\\0&1\end{bmatrix},\tau=\begin{bmatrix}\tau_r\\ \tau_l\end{bmatrix}$$

2.3 State Space Representation

Now consider the mechanical system given by (2) and (3). Since the constrained velocity is always in the null space of $A(q)$, it is possible to define $n-m$ velocities $\nu(t)=[\nu_1\ \nu_2 \cdots \nu_{n-m}]$ such that

$$\dot{q}=S(q)\nu(t) \tag{4}$$

where $\nu(t)=[\nu_1\ \nu_2]=[\dot{\theta}_r\ \dot{\theta}_l]$.

Using the state space variable $x=[q\ \nu]^T$,we have

$$\dot{x}=\begin{bmatrix}S\nu\\0\end{bmatrix}+\begin{bmatrix}0\\I\end{bmatrix}u=f(x)+g(x)u \tag{5}$$

where $u=(S^TMS)^{-1}S^T\tau+(S^TMS)^{-1}S^T(-M\dot{S}-ES)\nu$.

3. Input-output Feedback Linearization

It is convenient to define a coordinate frame X_c-Y_c at the centre of mass of the robot, with X_c in the forward direction of the robot. We may choose an arbitrary point P_r with respect to the robot coordinate frame X_c-Y_c as a reference point. The mobile robot is to be controlled so that the reference point follows a desired trajectory. Let the reference point be denoted by (x_R,y_R) in the robot frame X_c-Y_c . Then the world coordinates (x_r,y_r) of the reference point are given by

$$x_r=x_c+x_R\cos\phi-y_R\sin\phi \tag{6}$$

$$y_r=y_c+x_R\sin\phi+y_R\cos\phi \tag{7}$$

The appropriate output variables are defined as

$$y=h(q)=[x_r\quad y_r]^T \tag{8}$$

Our nonholonomic system (10) can be rewritten in the condensed form

$$\dot{x}=f(x)+g(x)u$$
$$y=h(x) \tag{9}$$

The necessary and sufficient condition for the system (9) to be input-output linearized and controllable is that $\det(\Phi)\neq 0$ (see[3]), where Φ is the decoupling matrix of the system

$$\Phi=J_h(q)S(q) \tag{10}$$

where

$$J_h(q)=\frac{\partial h}{\partial q}=\begin{bmatrix}1 & 0 & -(x_R\sin\phi+y_R\cos\phi) & 0 & 0\\ 0 & 1 & x_R\cos\phi-y_R\sin\phi & 0 & 0\end{bmatrix} \tag{11}$$

and

$$\Phi=\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ \Phi_{21} & \Phi_{22}\end{bmatrix}$$

$$\Phi_{11}=c((b-y_R)\cos\phi-(d+x_R)\sin\phi)$$

$$\Phi_{12}=c((b+y_R)\cos\phi+(d+x_R)\sin\phi)$$

$$\Phi_{21} = c((b - y_R)\sin\phi + (d + x_R)\cos\phi)$$
$$\Phi_{22} = c((b + y_R)\sin\phi - (d + x_R)\cos\phi)$$

Since the determination of the decoupling matrix is $\det(\Phi) = -\dfrac{r^2(d+x_R)}{2b}$, it is singular if and only if $x_R = d$, which is not impossible. So choose the $x_R \neq d$, we may decouple and linearize the system.

$$u = \Phi^{-1}(q)(\upsilon - \dot{\Phi}(q)v) \tag{12}$$

where the vector represents the external inputs, and is the vector of wheels velocity.

Applying the nonlinear feedback law (12), we get a linearized and decoupled system in the following form:

$$\ddot{y}_1 = \upsilon_1 \tag{13}$$
$$\ddot{y}_2 = \upsilon_2 \tag{14}$$

4. Control Policy Design

After input-output linearization, two decoupled single-input single-output (SISO) system obtained: two second-order systems that represent the position model. Thus, we have, respectively

$$G_{x_r}(s) = \frac{1}{s^2} \tag{15}$$

$$G_{y_r}(s) = \frac{1}{s^2} \tag{16}$$

4.1 Discretization of Position Model

The equivalent temporal representation of (15) is

$$\ddot{y} = u \tag{17}$$

where y is the output and u is the input. Defining the state variables as $x_1 = y$ and $x_2 = \dot{y}$, (17) can be written in state-space as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{18}$$

In compact form

$$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx\end{cases} \tag{19}$$

Discretizing (19) with sampling time h , the equivalent discrete time system is

$$\begin{cases}x_{k+1} = \Phi x_k + \Gamma u_k\\ y_k = Cx_k\end{cases} \tag{20}$$

where $\Phi = e^{Ah}$ and $\Gamma = \int_k^{k+1} e^{A(k+1-\tau)}Bd\tau$.

4.2 MPC design

Now, the MPC algorithm is based on the linear discrete-time prediction model

$$x_{k+1} = \Phi x_k + \Gamma u_k \tag{21}$$

of the open-loop process, where $x(t) \in R^n$ is the state vector at time t, and $u(t) = R^m$ is the vector of manipulated variables to be determined by the controller, and on the solution of the finite-time optimal control problem

$$\begin{aligned}&\min \left\{\frac{1}{2}x'_N P x_N + \frac{1}{2}\sum_{k=0}^{N-1}[x'_k Q x_k + u'_k R u_k]\right\}\\ &s.t.\quad x_{k+1} = \Phi x_k + \Gamma u_k, k = 0,\ldots,N-1\\ &\qquad x_0 = X\\ &\qquad u_{\min} \le u_k \le u_{\max}\\ &\qquad y_{\min} \le Cx_k \le y_{\max}\end{aligned} \tag{22}$$

where N is the prediction horizon, $U = [u'_0 \cdots u'_{N-1}]' \in R^{Nm}$ is the sequence of manipulated variables to be optimized, $Q \ge 0$, $R > 0$ and $P \ge 0$ are weight matrices of appropriate dimensions defining the performance index, $u_{\min}, u_{\max} \in R^m$, $y_{\min}, y_{\max} \in R^p$, $C \in R^{p\times n}$ define constraints on input and state variables, respectively, and " $\le$ " denotes component-wise inequalities.

Then, we consider the problem where the output $y_k \in R^p$ in (20) is required to tracking a time-variable value $y^* \in R^p$. The output errors is regulated

$$e_k \triangleq y_k - y^* = Cx_k - y^* \tag{23}$$

The idea of the moving-horizon control concept is to find the control-variable value that minimize the receding horizon quadratic cost function

$$\min\left\{\frac{1}{2}\sum_{k=0}^{N-1}\left[(y_k - y^*)^T Q_y (y_k - y^*) + u_k^T R u_k\right]\right\} \tag{24}$$

where $Q_y \ge 0$ is the output weighting matrix

At the time k and for the current output y_k , solve the optimal control problem (23) over a fixed future interval $[k, k+N-1]$. Apply u_k , the first step in the resulting optimal control sequence to the system and measure the y_{k+1} . Then, repeat the fixed horizon optimization at time $k+1$ over the future interval $[k+1, k+N-1]$.

5. Simulation

The first reference trajectory in the text is a circle defines as follows:

$$x_r = 2\cos(t)$$
$$y_r = 2\sin(t)$$

The weight parameter of the MPC control can affect the tracking performance. For the persistent excitation trajectories, the parameters are selected as follows:

$$Q = 0.1, R = 1$$

The simulations were tested by MATLAB. The tracking results are shown in Figure.2

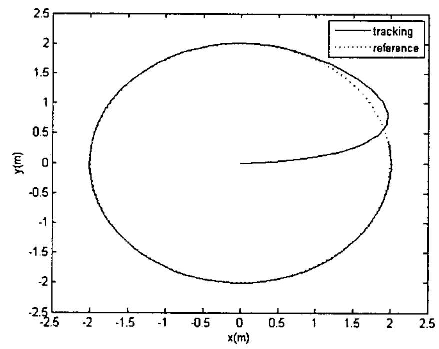

Figure.2

6. Conclusion

In this paper, we have described the nonlinear system architecture of a differential-drive WMR, focusing on its physical characteristic (kinematics and dynamics). The input-output linearization and model discretization are applied to it. And the proposed control law minimizes the quadratic cost function consisting of tracking errors and control effort. Finally, model predictive controller is used to allow a highly accurate path tracking.

The simulation results were presented to illustrate the performance of the controller and prove it is an adapt tracking control method.

[Reference]

[1] Dongbing Gu and Huosheng Hu, "Receding horizon tracking control of wheeled mobile robots", Control Systems Technology, IEEE Transactions on Volume 14, Issue 4, July 2006, pp. 743-749.

[2] I. Kolmanovsky and N.H McClamroch, "Developments in Nonholonomic Control Problems", IEEE Control Systems, Volume 15, Issue 6, 1995, pp. 20-36.

[3] P. Coelho and U. Nunes, "Path-following Control of Mobile Robots in Presence of Uncertainties", IEEE Transactions on Robotics, Volume 21, I ssue 2, April 2005, pp. 252-261.