모델 알고리즘 제어를 이용한 이동 로봇의 경로 추적 제어

장원량, 정길도
전북대학교 전자정보공학부

Path Following Control For Mobile Robots Using Model Algorithm Control

Zhang Yuanliang, Kil To Chong
Division of Electronics and Information Engineering, Chonbuk National University, Jeonju 561756

Abstract - This paper proposed a model algorithm control (MAC) method for trajectory tracking control of the differentially steeredwheeled mobile robots (WMRs) subject to nonholonomic constraint. The dynamic model of the wheeled mobile robot is presented and used as the model to be controlled. The performance of the proposed control algorithm is verified via computer simulations in which the WMR is controlled to track several different reference paths. It is shown that the control strategy is feasible.

1. Introduction

The differentially steered wheeled mobile robots (WMRs) possess the advantages of high mobility, high traction with pneumatic tires, and a simple wheel configuration. In recent years, the research on designing wheeled mobile robots (WMRs) controllers subject to nonholonomic contrains is both extensive and diverse. Chen, *et al.* (2006) developed a visual servo tracking controller for a monocular camera system mounted on an underactuated wheeled mobile robot subject to nonholonomic motion constraints. Paulo and Urbano (2005) presented the implementation of a new control strategy, Kalman-based active observer controller for the path following of wheeled mobile robots subject to nonholonomic constrains. Yang, *et al.* (2005) proposed a robust tracking scheme for nonholonomic wheeled mobile robots with parameter uncertainty, external disturbance and input constraints. Tan and Gu (2005)proposed a control design method for autonomous mobile platforms basing on way point guidance approach combining with model reference trajectory control method. Zhang, *et al.* (2003) discussed dynamic modeling and robust control of a differentially steered mobile robot subject to wheel slip and external loads.

This paper proposed a Model Algorithm Control (MAC) method for tracking control of the wheeled mobile robots. A closed-loop MAC that incorporates process uncertainties by adjusting the discrepancy between the process output and its predicted value is particularly robust against process model errors and disturbances.

2. Mathematical Model of the Wheeled Mobile Robot

Consider a WMR with differentially driven wheels as shown in Fig. 1.

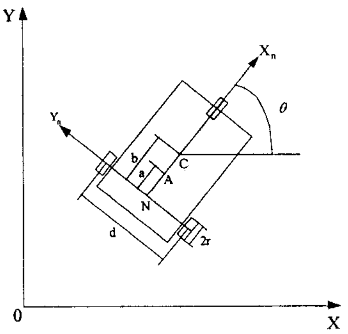

Fig. 1. Model of a nonholonomic wheeled mobile robot.

The dynamic model can be described as follows:

$$M(q)\ddot{q}+V(q,\dot{q})\dot{q}=E(q)\tau-A^T(q)\lambda \qquad (1)$$

where $M(q)\in R^{n\times n}$ is the symmetric and positive definite inertia matrix, $V(q,\dot{q})\in R^{n\times n}$ is the centripetal and Coriolis force Matrix, $A(q)\in R^{m\times n}$ is the matrix associated with the constraints, $\lambda\in R^m$ is the Lagrangian multiplier vector, $E(q)\in R^{n\times r}$ is the input transformation matrix and $\tau\in R^r$ is the torque input vector.

$$\dot{q}=S(q)\eta(t) \qquad (2)$$

$$\bar{M}\dot{\eta}+\bar{V}\eta=\bar{B}\tau \qquad (3)$$

where

$$\bar{M}(q)=\begin{bmatrix} r^2\dfrac{(\frac{md^2}{4}+I)}{d^2}+I_w & r^2\dfrac{(\frac{md^2}{4}-I)}{d^2} \\ r^2\dfrac{(\frac{md^2}{4}-I)}{d^2} & r^2\dfrac{(\frac{md^2}{4}+I)}{d^2}+I_w \end{bmatrix}$$

$$\bar{V}=\begin{bmatrix} 0 & \dfrac{r^2m_cb\dot{\theta}}{d} \\ -\dfrac{r^2m_cb\dot{\theta}}{d} & 0 \end{bmatrix}\ \bar{B}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad (4)$$

and $\tau=[\tau_r \quad \tau_l]^T$ is the torque applied on the right and left wheel, $\eta=[\eta_r \quad \eta_l]^T$ represents the angular velocity of the right and left wheel, $I=m_cb^2+\frac{m_wd^2}{2}+I_c+2I_m$, $m=m_c+2m_w$. Here m_c and m_w are the mass of the mobile robot platform and the mass of one driving wheel with the actuator respectively, I_c, I_w and I_m are the moment of inertia of the platform about the vertical axis through point N, the wheel with the actuator about the wheel axis, and the wheel with the actuator about the wheel diameter respectively.

Assume the linear velocity and the orientation angular velocity of the mobile robot at point N are v and w, therefore we have:

$$\begin{bmatrix}\eta_r\\ \eta_l\end{bmatrix}=\begin{bmatrix}\dfrac{1}{r} & \dfrac{d}{2r}\\ \dfrac{1}{r} & -\dfrac{d}{2r}\end{bmatrix}\begin{bmatrix}v\\ w\end{bmatrix}$$

$$\dot{q}=\begin{bmatrix}\dot{x}_n\\ \dot{y}_n\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ w\end{bmatrix} \qquad (5)$$

Then it is easy to show that the dynamics equation for point A leads to the following:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{v}\\ \dot{w}\end{bmatrix}=\begin{bmatrix}v\cos\theta-aw\sin\theta\\ v\sin\theta+aw\cos\theta\\ w\\ \beta_1w^2\\ \beta_2vw\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \alpha_1 & 0\\ 0 & \alpha_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \qquad (6)$$

where $\beta_1 = \frac{m_c b r^2}{\Theta_u}$, $\beta_2 = -\frac{2 m_c b r^2}{\Theta_w}$, $\alpha_1 = \frac{r}{\Theta_u}$, $\alpha_2 = \frac{rd}{\Theta_w}$, $\Theta_u = mr^2 + 2I_w$, $\Theta_w = 2r^2 I + I_w d^2$, and $u_{1,2} = (\tau_r \pm \tau_l)$.

3. Model Algorithm Control for Nonlinear System

Consider the nonlinear systems described by a discrete-time state-space model in the form:

$$x_m(k+1) = \Phi[x_m(k), u(k)]$$
$$y_m(k) = h[x_m(k)] \tag{6}$$

where x denotes the vector of state variables, u denotes the manipulated input, y represents an output (to be controlled), and the subscript m is added to indicate estimates of x and y obtained in model simulations and differentiate the measured y . $\Phi(x,u)$ is an analytic vector function on $X \times U$, and $h(x)$ is an analytic scalar function on X .

We suppose that system (6) has the relative order r , i.e. r is the smallest number of sampling periods after which the manipulated input $u(k)$ affects the output y .

Online simulation of the model described by Eq. (6) can be used to predict the future changes in the output y as follows:

$$\begin{aligned}
&y_m(k+1) - y_m(k) = h^1[x_m(k)] - h[x_m(k)] \\
&y_m(k+2) - y_m(k) = h^2[x_m(k)] - h[x_m(k)] \\
&\vdots \\
&y_m(k+r-1) - y_m(k) = h^{r-1}[x_m(k)] - h[x_m(k)] \\
&y_m(k+r) - y_m(k) = h^{r-1}[\Phi[x_m(k), u(k)] - h[x_m(k)]
\end{aligned} \tag{7}$$

where r is the relative order of the system, and the following notation will be used:

$$\begin{cases} h^0(x) = h(x) \\ h^l(x) = h^{l-1}\left[\Phi(x,u)\right], \; l = 1,...,r-1 \end{cases} \tag{8}$$

with a finite relative order r , and the algebraic equation exists:

$$h^{r-1}[\Phi(x,u)] = y \tag{9}$$

is locally solvable in u . The corresponding implicit function will be de noted by:

$$u = \Psi_0(x, y) \tag{10}$$

and will be assumed to be well-defined and unique on $X \times h(X)$.

When these predicted changes are added to the measured output signal $y(k)$, one obtains the following closed-loop predictions of the output:

$$\begin{aligned}
&\hat{y}(k+1) = y(k) + h^1[x_m(k)] - h[x_m(k)] \\
&\hat{y}(k+2) = y(k) + h^2[x_m(k)] - h[x_m(k)] \\
&\ldots \\
&\hat{y}(k+r-1) = y(k) + h^{r-1}[x_m(k)] - h[x_m(k)] \\
&\hat{y}(k+r) = y(k) + h^{r-1}[\Phi[x_m(k), u(k)]] - h[x_m(k)]
\end{aligned} \tag{11}$$

where the ^ is used to indicate that $\hat{y}$ represents a prediction of the output.

The question that arises is what should be the choice of $u(k)$ to obtain a desirable output response after r time steps. One can request $\hat{y}(k+r)$ to be in the right direction and cover a fraction of the "distance" between $\hat{y}(k+r-1)$ and the set-point value. In other words, one can define a desirable value y_d of the output at the $(k+r)$th time step by:

$$y_d(k+r) = (1-\alpha) y_{sp} + \alpha \hat{y}(k+r-1) \tag{12}$$

where α is a tunable filter parameter such that $0 < \alpha < 1$.

One can derive a nonlinear MAC controller by requesting that the output prediction match the reference trajectory in the sense of minimizing the performance index of Eq. (12):

$$\min_{u(k)} [y_d(k+r) - \hat{y}(k+r)]^2 \tag{13}$$

Considering Eqs. (12) and (13), this becomes:

$$\begin{aligned}
&\min_{u(k)} \{(1-\alpha)e(k) - h^{r-1}\{\Phi[x_m(k), u(k)]\} \\
&+ \alpha h^{r-1}[x_m(k)] + (1-\alpha) h[x_m(k)]\}^2
\end{aligned} \tag{14}$$

where $e(k) = y_{sp}(k) - y(k)$.

In the absence of input constraints, this minimization problem is trivially solvable. Minimizing $u(k)$ is the solution of the nonlinear algebraic equation:

$$h^{r-1}\{\Phi[x_m(k), u(k)]\} = b(x_m(k), \; e(k)) \tag{15}$$

where $b(x,e) = \alpha h^{r-1}[x] + (1-\alpha)(h[x] + e)$.

Recalling the definition of Ψ_0 (Eq. 10), the solution can be represented as:

$$u(k) = \Psi_0 \left\{ x_m(k), b(x_m(k), e(k)) \right\} \tag{16}$$

Therefore, the derived control law is given by Eq. (27), where $x_m(k)$ is obtained by Eq. (14).

4. Simulation

We simulated the proposed control law to demonstrate its effectiveness. In a simulation program, dynamic model described in Section 2 are used as the mobile robot model. The reference path is a straight line. The tracking performance of the MAC controller for the straight line is shown in Figure 2.

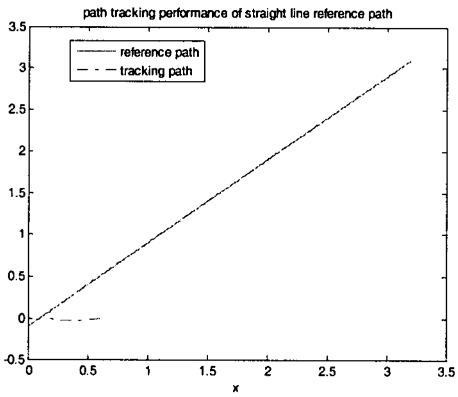

Fig. 2 Path tracking performance of straight line reference path.

5. Conclusion

In this paper we study the path tracking problem of dynamic WMRs subject to nonholonomic constrains. The MAC control method is propos ed for tracking control of the discrete time nonlinear system. Several n umerical simulations are done to show the promise of the proposed M AC control method in terms of tracking performance.

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