정다각형 배열 광마우스를 이용한 최소 자승 속도 추정법에 대한 민감도 분석

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Sensitivity Analysis of Least Squares Velocity Estimation Using a Regular Polygonal Array of Optical Mice

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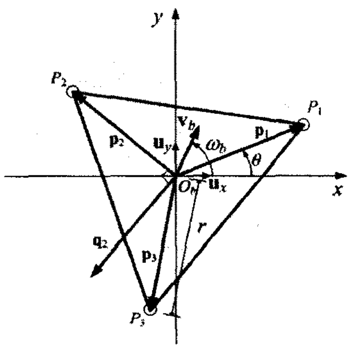
Abstract - This paper presents the sensitivity analysis of the leasst qsuares velocity estimation of an omnidirectional mobile robot using a regular polygonal array of optical mice. First, the velocity kinematics from a mobile robot to an array of optical mice is derived as an overdetermined linear system. Then, for a given set of optical mouse readings, the least squares velocity estimation of a mobile robot is obtained as the simple average. Finally, the sensitivity analysis of the proposed least squares velocity estimation to imprecise installation is made.

1. INTRODUCTION

There have been several attempts to employ the optical mice for the localization of a mobile robot [1-6]. In fact, the optical mouse is an inexpensive but high performance device with sophisticated image processing engine. The velocity estimation of a mobile robot using a set of optical mice can overcome the limitations of typical sensors: wheel slippage in encoders, the line of sight in ultrasonic sensors, and heavy computation in cameras.

For the localization of an omnidirectional mobile robot on the plane, two or more optical mice are required . Most of previous research [1-5] use two optical mice. However, few attempt has been made to use more than two optical mice except [6]. In this paper, we present the sensitivity analysis of the least squares velocity estimation of a mobile robot using a regular polygonal array of optical mice.



<Fig. 1> A regular triangular array of optical mice with $N=3$.

2. VELOCITY ESTIMATION

Assume that N optical mice are installed at the vertices, P_i, $i=1,\cdots,N$, of a regular polygon that is centered at the center, O_b, of a mobile robot traveling on the xy plane. Fig. 1 shows an example of a regular triangular array of optical mice with $N=3$. The position vector, $\mathbf{p}_i=[\,p_{ix}\ \ p_{iy}\,]^t$, $i=1,\cdots,N$, from O_b to P_i, can be expressed as

$$\mathbf{p}_i = \begin{bmatrix} p_{ix} \\ p_{iy} \end{bmatrix} = \begin{bmatrix} r\cos\{\Theta+(i-1)\times\frac{2\pi}{N}\} \\ r\sin\{\Theta+(i-1)\times\frac{2\pi}{N}\} \end{bmatrix} \tag{1}$$

where Θ represents the heading angle of a mobile robot with the forwarding direction aligned with $\mathbf{p}_1$, and r represents the distal distance of each optical mouse. For a regular polygonal arrangement of optical mice, it holds that

$$\sum_{i=1}^{N} p_{ix} = \sum_{i=1}^{N} p_{iy} = 0 \tag{2}$$

regardless of the heading angle Θ.

Let $\mathbf{v}_b=[\,v_{bx}\ \ v_{by}\,]^t$ and w_b be the linear velocity and the angular velocity at the center O_b of a mobile robot, respectively. And, let $\mathbf{v}_i=[\,v_{ix}\ \ v_{iy}\,]^t$, $i=1,\cdots,N$, be the linear velocity at the vertex P_i, which corresponds to the velocity readings of the i th optical mouse. The velocity mapping from a mobile robot to an array of optical mice can be represented as

$$\mathbf{A}\ \dot{\mathbf{x}} = \Theta \tag{3}$$

where

$$\dot{\mathbf{x}} = \begin{bmatrix} v_{bx} \\ v_{by} \\ \omega_b \end{bmatrix} \in \mathbf{R}^{3\times1} \tag{4}$$

$$\Theta = \begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ \vdots \\ v_{Nx} \\ v_{Ny} \end{bmatrix} \in \mathbf{R}^{2N\times1} \tag{5}$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -p_{1y} \\ 0 & 1 & p_{1x} \\ 1 & 0 & -p_{2y} \\ 0 & 1 & p_{2x} \\ \vdots & \vdots & \vdots \\ 1 & 0 & -p_{Ny} \\ 0 & 1 & p_{Nx} \end{bmatrix} \in \mathbf{R}^{2N\times3} \tag{6}$$

From (3), the least squares velocity estimation can be obtained by

$$\dot{\mathbf{x}} = \mathbf{B}\ \Theta \tag{7}$$

where

$$\mathbf{B} = (\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t \in \mathbf{R}^{3\times2N} \tag{8}$$

Finally, for given velocity readings of N optical mice, the linear and angular velocity estimates of a mobile robot can be obtained as

$$\begin{aligned} v_{bx} &= \frac{1}{N}\sum_{i=1}^{N} v_{ix} \\ v_{by} &= \frac{1}{N}\sum_{i=1}^{N} v_{iy} \\ w_b &= \frac{1}{N}\sum_{i=1}^{N} w_i \end{aligned} \tag{9}$$

where

$$w_i = \frac{1}{r}\left[-\sin\left\{\theta+\frac{(i-1)2\pi}{N}\right\}\times v_{ix}+\cos\left\{\theta+\frac{(i-1)2\pi}{N}\right\}\times v_{iy}\right] \tag{10}$$

which represents the angular velocity experienced by the i th optical mouse.

3. SENSITIVITY ANALYSIS

In practice, it may be rather difficult to install optical flow sensors in an exact regular polygonal array symmetric with respect to the center of a mobile robot without installation error. Let us examine the sensitivity of the velocity estimation based on (9) to imprecise installation of optical flow sensors. Suppose that the position error vector, $\delta \mathbf{p}_i,\ i=1,\cdots,n$, of the i th optical flow sensor is described by

$$\delta \mathbf{p}_i = \begin{bmatrix} \delta p_{ix} \\ \delta p_{iy} \end{bmatrix} \tag{11}$$

where δp_{ix} and δp_{iy}, represent the deviation from the vertex of a regular polygon due to imprecise installation.

In the presence of installation error, the velocity kinematics, given by (8), can be expressed as

$$(\ \mathbf{A} + \delta\mathbf{A}\ )(\ \dot{\mathbf{x}} + \delta\dot{\mathbf{x}}\ ) = \boldsymbol{\Theta} \tag{12}$$

where

$$\delta\dot{\mathbf{x}} = \begin{bmatrix} \delta v_{bx} \\ \delta v_{by} \\ \delta \omega_b \end{bmatrix} \in \mathbf{R}^{3\times 1} \tag{13}$$

and

$$\delta\mathbf{A} = \begin{bmatrix} 0 & 0 & -\delta p_{1y} \\ 0 & 0 & \delta p_{1x} \\ 0 & 0 & -\delta p_{2y} \\ 0 & 0 & \delta p_{2x} \\ \vdots & \vdots & \vdots \\ 0 & 0 & -\delta p_{Ny} \\ 0 & 0 & \delta p_{Nx} \end{bmatrix} \in \mathbf{R}^{2N\times 3} \tag{14}$$

While (14) represents the perturbation on $\mathbf{A}$ owing to imprecise installation, (13) represents the resulting error in velocity estimation of a mobile robot.

Premultiplying (12) by $(\ \mathbf{A} + \delta\mathbf{A}\ )^t$, we have

$$(\ \mathbf{A}^t + \delta\mathbf{A}^t\ )(\ \mathbf{A} + \delta\mathbf{A}\ )(\ \dot{\mathbf{x}} + \delta\dot{\mathbf{x}}\ ) = (\ \mathbf{A}^t + \delta\mathbf{A}^t\ )\ \boldsymbol{\Theta} \tag{15}$$

Assuming that the installation error and the resulting estimation error are small enough, we have

$$\delta\mathbf{A}^t\,\delta\mathbf{A} \simeq \mathbf{0}_{2\times 2} \tag{16}$$

and

$$(\ \delta\mathbf{A}^t\,\mathbf{A} + \mathbf{A}^t\,\delta\mathbf{A}\ )\,\delta\dot{\mathbf{x}} \simeq \mathbf{0}_2 \tag{17}$$

Under the assumption of (16) and (17), from (15), we obtain

$$\mathbf{P}\ \delta\dot{\mathbf{x}} + \delta\mathbf{P}\ \ \dot{\mathbf{x}} = \delta\mathbf{A}^t\ \boldsymbol{\Theta} \tag{18}$$

where

$$\mathbf{P} = \mathbf{A}^t\,\mathbf{A} = \begin{bmatrix} N & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & Nr^2 \end{bmatrix} \tag{19}$$

$$\begin{aligned} \delta\mathbf{P} &= \delta\mathbf{A}^t\,\mathbf{A} + \mathbf{A}^t\,\delta\mathbf{A} \\ &= \begin{bmatrix} 0 & 0 & -\sum_{i=1}^{N}\delta p_{iy} \\ 0 & 0 & \sum_{i=1}^{N}\delta p_{ix} \\ -\sum_{i=1}^{N}\delta p_{iy} & \sum_{i=1}^{N}\delta p_{ix} & 2\times\sum_{i=1}^{N}(p_{ix}\delta p_{ix}+p_{iy}\delta p_{iy}) \end{bmatrix} \end{aligned} \tag{20}$$

From (18), the effect on the least squares velocity estimate owing to imprecise installation of optical flow sensors can be approximated by

$$\delta\dot{\mathbf{x}} \simeq \delta\dot{\mathbf{x}}_x + \delta\dot{\mathbf{x}}_\Theta \tag{21}$$

where

$$\delta\dot{\mathbf{x}}_x = -\ \mathbf{P}^{-1}\,\delta\mathbf{P}\ \ \dot{\mathbf{x}} \tag{22}$$

$$\delta\dot{\mathbf{x}}_\Theta = \mathbf{P}^{-1}\,\delta\mathbf{A}^t\,\boldsymbol{\Theta} \tag{23}$$

Finally, for a given set of optical mouse velocity readings and the resulting velocity estimates, the errors in least squares velocity estimation caused by imprecise installation can be obtained as

$$\delta\dot{\mathbf{x}}_x = \begin{bmatrix} \frac{w_b}{N}\sum_{i=1}^{N}\delta p_{iy} \\ -\frac{w_b}{N}\sum_{i=1}^{N}\delta p_{ix} \\ \frac{1}{Nr^2}\left\{v_{bx}\sum_{i=1}^{N}\delta p_{iy} - v_{by}\sum_{i=1}^{N}\delta p_{ix} - 2w_b\sum_{i=1}^{N}(p_{ix}\delta p_{ix}+p_{iy}\delta p_{iy})\right\} \end{bmatrix} \tag{24}$$

$$\delta\dot{\mathbf{x}}_\Theta = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{Nr^2}\sum_{i=1}^{N}(-v_{ix}\delta p_{iy}+v_{iy}\delta p_{1x}) \end{bmatrix} \tag{25}$$

4. CONCLUSION

In this paper, we presented the sensitivity analysis of the least squares velocity estimation of an omnidirectional mobile robot using a regular polygonal array of optical mice. First, the velocity kinematics from a mobile robot to an array of optical mice was derived as an overdetermined system. Then, for a given set of optical mouse readings, the least squares velocity estimate of a mobile robot was obtained as the simple average. Finally, the sensitivity of the proposed least squares velocity estimation to imprecise installation was analyzed.

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