# 자기부상열차 시스템에서 적분형 슬라이딩 모드 제어기를 이용한 부상억제력 제거

Suppression of the Disturbance Force in The Magnetically Levitated Train System Using Integral Sliding Mode Controller

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## Key Words : Magnetically Levitated System (자기부상 시스템), Control System(제어시스템), Integral Sliding Mode Controller(적분형 슬라이딩 모드 제어기), Linear Induction Motor(선형유도 모티)

#### ABSTRACT

In this paper we deal with a design of the integral sliding mode controller to suppress the disturbance force acting on the suspension system of the magnetically levitated train system. One of the important factors that cause the disturbance force acting on the suspension system comes from the low propulsion speed of linear induction motor. In this paper integral sliding mode controller is employed to reject the disturbance force produced by the propulsion system of the linear induction motor. In order to show the effectiveness of the designed controller a dynamic simulation is utilized and the sliding mode controller without integral compensator is compared with the proposed integral sliding mode controller to suppress the disturbance force.

#### 1. Introduction

The magnetically levitated train system can be divided into two parts based on the levitation method: one is a repulsive type using super conductors. A disadvantage of this type of suspension system is needed for operation below the critical speed when the suspended object is stationary. The other type is using ferromagnetic or permanent magnet. This type of electromagnetic suspension system (EMS system) has one significant advantage in that it provides attraction force at zero speed, but such system is inherently unstable (1). In order to overcome the inherent instability a active controller plays a very important role in the electromagnet suspension system to make the stable suspension and to maintain the suspended object within the nominal air gap (2)(3). Especially in the magnetically levitated train system external disturbance force acting on the controller may cause the malfunction of the suspension system. If a *Maglev* train is propelled by the linear induction motor, the suspension controller of the *Maglev* train should have a capability to reject the normal force produced from the linear induction motor. The reason is because the normal force of the linear induction motor acts as the disturbance force on the suspension controller.

In this paper we deal with a design procedure for the integral sliding mode controller to reject the disturbance force acting on the suspension controller (4)(5)(6). First we present a simple mathematical model for the *Maglev* train and then introduce the integral sliding mode controller. Second a mathematical formula for the normal force of the linear induction motor is derived by the curve fitting of the experimental data. Finally we show the effectiveness of the integral sliding mode controller

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to reject the disturbance force by the simulations.

### 2. Mathematical Model

Fig.1 shows a simple schematic diagram for EMS system which has the electromagnets as the suspension actuators, linear induction motor and reaction plate for the vehicle propulsion. As shown in Fig. 1, the passenger vehicle and the bogie can be levitated by the electromagnets attraction force. Once the bogie is levitated, the propulsion system (linear induction motor and reaction plate) is activated to move the passenger vehicle.

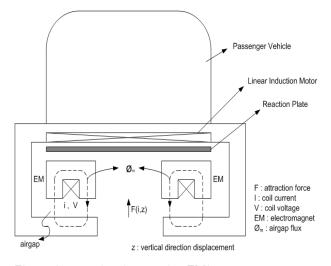


Fig. 1 Schematic diagram for EMS system

The mathematical model of this system is divided into two parts: One is the plant (mechanical) dynamics and the other is the actuator dynamics. The plant dynamics is

$$\ddot{mz} = F(i,z) - mg - f_d \tag{1}$$

where m is the total mass of the controlled object, g is the gravitational acceleration, and  $f_d$  is the external disturbance force acting on the controlled object. In eq. (1) F(i,z) is the electromagnets attraction force which is proportional to the current deviation and inversely proportional to the air gap deviation, expressed such as:

$$F(i,z) = \frac{B^2 A}{\mu_0} = \frac{\mu_0 N^2 A}{4} \left(\frac{i(t)}{z(t)}\right)^2$$
(2)

where B is the flux density of the magnetic core material, A is the cross sectional area of the pole face of the electromagnet,  $\mu_0$  is the permeability in the air space, and N is the number of turns.

Since eq. (2) has high nonlinearity it is necessary that the linear approximation should be carried out for the analysis of eq. (2) with respect to the nominal point  $(i_0, z_0)$ . The Taylor Series Expansion is usually employed and the eq. (2) becomes

$$F(i,z) = k_i i(t) - k_z z(t)$$
(3)

where  $k_i = \frac{\mu_0 N^2 A i_0}{2 z_0^2}$  and  $k_z = \frac{\mu_0 N^2 A i_0^2}{2 z_0^3}$ .

The actuator dynamics is

$$v(t) = Ri(t) + \frac{d}{dt} [L(i,z)i(t)]$$
  
=  $Ri(t) + \frac{\mu_0 N^2 A}{2z(t)} \frac{d}{dt}i(t) - \frac{\mu_0 N^2 A i_0}{2z(t)^2} \frac{d}{dt}z(t)$   
(4)

where v is the coil voltage, R is the coil resistance, and L(i,z) is the magnet inductance which is the function of the air gap displacement such as  $L(i,z) = \frac{\mu_0 N^2 A}{2z(t)}$ . There is a variation of the inductance with respect to the air gap displacement in the second term, and that the third term denotes a voltage which varies with changes in the air gap z(t) and its rate of change similar to back EMF voltage.

By using equations (1), (3), and (4) the state space equations are written in vector matrix form:

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k_z & 0 & \frac{k_i}{m} \\ 0 & \frac{k_z}{k_i} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} v + \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix} f_d \quad (5)$$

where  $f_d$  is the normal force of the linear induction motor. In this paper we ignore the static gravitational force. Eq. (5) can be simply expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef_d(t) \tag{6}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-k_z}{m} & 0 & \frac{k_i}{m} \\ 0 & \frac{k_z}{k_i} & -\frac{R}{L} \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, \ E = \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix}$$

#### 3. Integral Sliding Mode Controller

To enhance the disturbance rejection ability of the control system, we introduce an integrator as a state variable into (5). The integrator output  $z_i$  is expressed as the difference between the integrated reference position r and integrated position z written as:

$$z_i = \int (r-z)dt \tag{7}$$

where r is zero for a nominal design. The block diagram of the proposed control system is shown in Fig. 3.

In order to synthesis the integral sliding mode

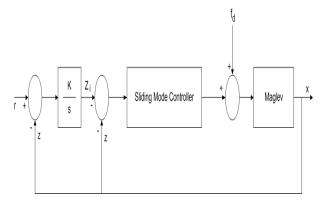


Fig. 3. Block diagram of an integral sliding mode controller for Maglev

controller we write the state variables as  $x = \begin{bmatrix} z_i & z & \dot{z} & i \end{bmatrix}$ , and get the following state space matrices.

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{-k_z}{m} & 0 & \frac{k_i}{m} \\ 0 & 0 & \frac{k_z}{k_i} & \frac{-R}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m} \\ 0 \end{bmatrix}$$
(8)

Eq. (8) is decomposed as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{9}$$

where

$$x_{1} = \begin{bmatrix} z_{i} \\ z \\ z \end{bmatrix}, \ x_{2} = i^{'}$$

$$A_{11} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & \frac{-k_z}{m} & 0 \end{bmatrix}, \ A_{12} = \begin{bmatrix} 0 \\ 0 \\ \frac{k_i}{m} \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} 0 & 0 & \frac{k_z}{k_i} \end{bmatrix}, \ A_{22} = -\frac{R}{L}$$
$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ B_2 = \frac{1}{L}, \ u = v$$
(10)

Let the switching surface be defined as  $\sigma = Sx$  where

$$\sigma = \begin{bmatrix} S_1 & S_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ S_1 \in \mathbb{R}^{1 \times 3}, \ S_2 = \mathbb{R}$$
(11)

If the system dynamics is an ideal sliding surface we have  $\sigma = Sx = 0$ . Using this property we can determine the equivalent system and associated linear control input.  $\sigma = 0$  yields

$$x_2 = -\frac{S_1}{S_2} x_1 \tag{12}$$

Substituting (12) into (9) yields the equivalent system  $\dot{x}_1 = (A_{11} - A_{12}S_2^{-1}S_1)x_1 \quad (13)$ 

Defining  $k = S_2^{-1}S_1$ , we write (13) as

$$x_1 = (A_{11} - A_{12}k)x_1 \tag{14}$$

The location of poles of the resulting system are obtained by selecting k and  $S_2$  as the switching function becomes  $\sigma = Sx$ ,  $S = \begin{bmatrix} S_1 & S_2 \end{bmatrix} = \begin{bmatrix} S_2k & S_2 \end{bmatrix} = S_2 \begin{bmatrix} k & 1 \end{bmatrix}$ . Since  $(A_{11}, A_{12})$  is controllable, a pole placement method is employed to select the gain k in (14). The sliding mode control inputs are separated into the linear and nonlinear components as  $u = u_l + u_{nl}$ . The linear input  $u_l$  can be selected by the following equations:

$$x = Ax + Bu \tag{15}$$

$$\sigma = Sx = 0 \tag{16}$$

From eq. (15) and (16) the equivalent linear control input is

$$u_l = -(SB)^{-1}SAx = -F_lx$$
(17)

The sliding mode reaching condition given by  $\sigma \sigma < 0$ , brings the system dynamics to the sliding surface  $\sigma = 0$ . Choose the nonlinear control as  $u_{nl} = -(SB)^{-1}\rho sgn(\sigma)$  where  $\rho > 0$ . Then, it follows that

$$\dot{\sigma\sigma} = \sigma S(Ax + Bu)$$

$$= \sigma SAx - \sigma SB[(SB)^{-1}SAx + (SB)^{-1}\rho sign(\sigma)]$$

$$= -\rho\sigma sgn(\sigma)$$

$$(18)$$

The control input can then be written as

$$u = -(SB)^{-1}[SAx + \rho sgn(\sigma)]$$
(19)

In practice, a discontinuous control component as  $sgn(\sigma)$  is undesirable because it may cause a chattering problem. The practical control effort is to ensure a neighborhood of  $\sigma = 0$  is reached and maintained. A common choice of a practical nonlinear control input is

$$U_{nl} = -\rho \frac{\sigma}{|\sigma| + \delta} \tag{20}$$

where  $\delta$  is the boundary layer which is selected to reduce the chattering problem and  $\rho$  is a design parameter.

#### 4. Simulations

Table 1 shows the parameters for the EMS type suspension system. Fig. 3 shows the attractive normal force which acts on the suspension system as the external disturbance force is produced at low

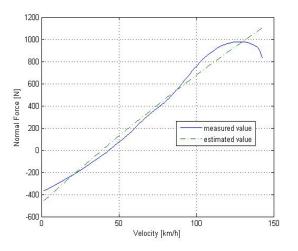


Fig. 3 Estimated force

speed range, that is from 0[km/h] to 40[km/h] (7). The integral sliding mode controller should be designed to reject the external disturbance force which is produced at very low speed. The highest value of the attractive force is produced from 0[km/h] to 10[km/h] as shown in Fig. 3. Thus for the simulations external disturbance force are input to the suspension system at 1[sec] with 1000[N] or 1500[N]. Fig. 4 and Fig. 5 show the simulation results. As shown in Fig. 4 and Fig. 5, even if

1000[N] and 1500[N] are input to the suspension system at 1 [sec], the integral sliding mode controller suppresses well the external disturbance

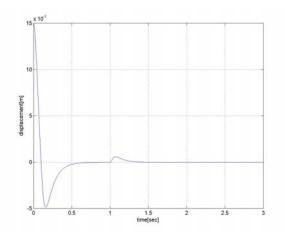


Fig. 4 External disturbance force rejection (1000[N])

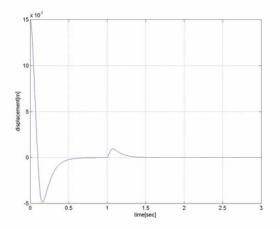


Fig. 5 External disturbance force rejection (1500[N])

force.

### 5. Conclusions

In this paper we dealt with the external disturbance force rejection property of the sliding mode controller in the magnetically levitated train system with the linear induction motor for propulsion system. The external disturbance force is produced from the linear induction motor at low speed.

First, the fundamental mathematical model of the magnetically levitated train was shown and then the force analysis of the linear induction motor with the curve fitting method was presented.

Finally the effective method of the integral sliding mode controller to suppress the external disturbance force was suggested and shown by the simulations.

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