텐서의 비음수 터커 인수분해

김용덕° 최승진
포항공과 대학교 컴퓨터 공학과
{karma13,seungjin}@postech.ac.kr

Nonnegative Tucker Decomposition

Yong-Deok Kim° Seungjin Choi
KoreaDepartment of Computer Science, POSTECH

Subspace analysis or principal component analysis (PCA) is a widely-used linear data model, the task of which is to learn the basis matrix **A** and the encoding variable matrix S which minimizes the Euclidean norm which is also known as Frobenious norm when the argument is a matrix), given a data matrix **X**. A successful spin-off of PCA is independent component analysis (ICA) where the linear data model **X**=**AS** is learned such that row vectors of the encoding variable matrix are as statistically independent as possible, while subspace analysis produces uncorrelated components. In the case where the data matrix contains only nonnegative elements, positive matrix factorization (PMF) or nonnegative matrix factorization (NMF) is known as a useful tool in learning parts-based representation as well as in feature extraction.

Computer vision involves a set of image or video data that can be well represented by 3-way or multiway data array which is known as *tensor*. For example, a vector is a 1-way tensor, a matrix is a 2-way tensor, a cube is a 3-way tensor, and so on. The multiway structure reflects rows, columns, RGB (or HSV) color coordinates, time, and so on. In conventional subspace analysis or matrix factorization methods, 2D image data are converted to 1D image vectors, discarding the spatial structure of the original 2D image data. Recently the 2D extension of subspace analysis methods have been proposed, including 2D-PCA, and 2D-NMF.

A general framework that takes the multiway structure into account, is multilinear algebra involving tensor decomposition. Exemplary tensor decomposition methods include: (1) the Tucker model (also known as multilinear SVD or N-mode SVD) ;(2) the CANDECOMP/PARAFAC model ; (3) nonnegative tensor factorization (NTF) where nonnegativity constraints are incorporated into the CANDECOMP/PARAFAC model or the PARAFAC2 model. In computer vision applications, N-mode SVD was applied to ace image representation, showing that onsidering multiple modes such as different people, expressions, ead poses, and lighting conditions improves the face recognition performance. ultilinear ICA was also applied to face image representation and ecognition. NTF preserves 2D tructure of image data in the factorization, leading to asuperior decomposition in the sense of sparse coding, compared to NMF.

LS error function and KL-divergence are widely used as a discrepancy measure of nonnegativity constrained matrix/tensor factorization models. Recently different error measures such as Csiszar's f-divergences, Amari's alpha-divergences, and Bregman divergences, were considered in the context of NMF. Multiplicative NMF algorithms were proposed in considering Amari's alpha-divergence which is a special instance of Csiszar's f-divergence. The alpha-divergence is a parametric family of divergence functional, including several well-known divergence measures as its special cases.

Existing methods of NTF considered the CANDECOMP/PARAFAC model where a nonnegative tensor is approximated by a linear sum of outer products of nonnegative vectors. In this paper we consider the Tucker model and nonnegativity constraints on the core tensor and mode matrices as well. Then we develop multiplicative updating algorithms for learning a Tucker decomposition of a nonnegative tensor with restricting a core tensor and mode matrices to be nonnegative. The multiplicative updating algorithms iteratively matricize tensor into each mode and then solve NMF problem. The method is referred to as *nonnegative Tucker decomposition* (NTD), in order to distinguish it from previous work, NTF. Developed multiplicative updating algorithms which iteratively minimize the LS-error, KL-divergence, alpha-divergence between nonnegative data tensor and Tucker model respectevley. Our NTD give the unified framework for nonnegativity constrained matrix/tensor factorization models because (1) the NTD includes NMF, nsNMF, 2D-NMF, and NTF as it's special case; (2) alpha-divergence is a parametric divergence measure which contains KL-divergence, Hellinger divergence, Pearson's distance, and so on.