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터미널 슬라이딩 표면을 이용한 혼돈 비선형 시스템의 유한 시간 제어

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Finite Time Control of Chaotic Nonlinear Systems Using Terminal Sliding Surface

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Abstract - In this paper, we design a terminal sliding mode controller for chaotic nonlinear systems. Terminal sliding mode control (TSMC) method can drive the tracking errors to zero within finite time. In addition, TSMC has the advantages such as improved the performance, the robustness, the reliability and the precision by contrast with classical sliding mode control (CSMC). Besides, we can obtain the final time using general formula. Finally, we carry out simulations of some examples, such as Duffing and Lorenz systems, to illustrate the effectiveness of the proposed control.

1. Introduction

Sliding mode control (SMC) is a well-known method which is used to control the linear or nonlinear systems. In addition, SMC systems have been applied to the control of nonlinear systems such as biped robot and spacecraft because of the robustness for the parameter perturbations and the external disturbances [1],[2]. In order to design SMC systems, we define the linear sliding surface which has been widely used to describe desired performances. Its one of representative characteristic is that the convergence of the system states to the equilibrium point is usually asymptotic but not in finite time.

Recently, a new control method called terminal sliding mode control (TSMC) has been developed [3]. TSMC has a nonlinear sliding surface based on the concept of terminal attractor while classical sliding mode control (CSMC) has a linear sliding surface [4]. The tracking error of system with TSMC is driven to zero within finite time but CSMC does not guarantee convergence to origin within finite time. In addition, TSMC has some advantages in comparison with CSMC, such as the improved the performance, the robustness, the reliability and the precision [3].

In this paper, we proposed the control method of nonlinear chaotic system based TSMC for the stability of systems. Computer simulations for simple nonlinear systems can verify the effectiveness of TSMC.

2. Terminal Sliding Mode Control

At terminal sliding surface, the concept of terminal attractor is used. That is important concept in order to overcome the limitations of exponential or asymptotical convergence. TSMC has a nonlinear sliding surface in order to obtain the finite time convergence of the system tracking error. Thus, we defined the terminal sliding surface in order to obtain the finite convergence time of the system as follows:

$$s = \dot{e} + \alpha e^{\gamma} \tag{1}$$

where $\alpha > 0$, $0 < \gamma < 1$ and both denominator and numerator of γ are odd integers.

In TSMC, the system tracking error is determined by terminal sliding surface (1). In addition, the system is infinitely stable in TSM because the surface defines the exponentially stability. However, if we select proper γ, we can get a final time [5]. This fact shows that the system is finitely stable. The convergence time is given as follows:

$$t = -\int_{e(0)}^{0} \frac{de}{\alpha e^{\gamma}} = \frac{|e(0)^{1-\gamma}|}{\alpha(1-\gamma)} \tag{2}$$

where, $e(0)$ is the initial value of e at $t=0$.

From (2), we know that the system tracking error converges to zero within finite time.

3. Simulation Results

3.1 Duffing system

In this subsection, we consider the control of Duffing system. Duffing system is the representative continuous-time chaotic system. The state equation of Duffing system is following form:

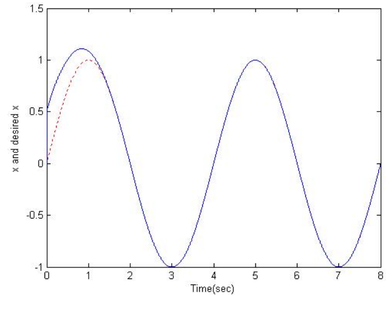
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -p_2 x_2 + p_1 x_1 - p_1^3 + q\cos(\omega t) + u \end{bmatrix} \tag{3}$$

where $p_1 = 1.1$, $p_2 = 0.4$, $q = 2.1$ and $\omega = 1.8$. Also, u is the control input, x is the output of interest.

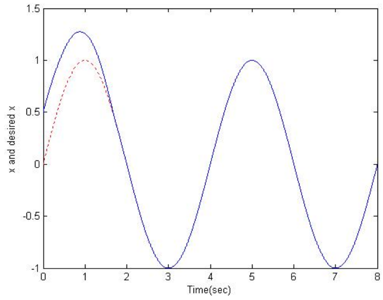
We simulate CSMC and TSMC of chaotic nonlinear system. The results of tracking performance for TSMC and CSMC are shown in Fig. 1 and 2, respectively. Dotted line is appeared desired state, and solid line is appeared real state. Here, we can verify that TSMC has faster convergence speed. In Fig. 3, we show the result of tracking error of TSMC and CSMC. Dotted line is appeared TSMC, and solid line is appeared CSMC. Table 1 compares simulation results of SMC and TSMC, where the mean squre error (MSE) of TSMC is lower than that of CSMC.

<Table 1> The comparison of MSE.

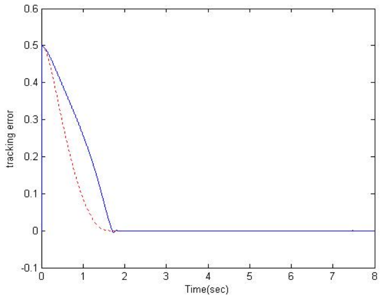
	TSMC	CSMC
MSE	0.0398	0.0608



<Fig. 1> Tracking performance with TSMC



<Fig. 2> Tracking performance with CSMC



<Fig. 3> Tracking error

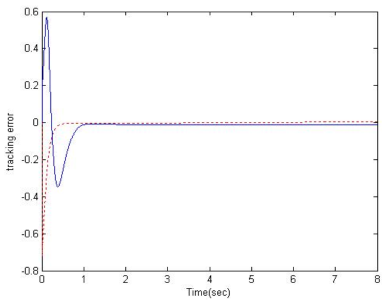
3.2 Lorenz system

In this subsection, we consider the Lorenz system which is the continuous time chaotic nonlinear system. Lorenz's system is based on the Lorenz equations. Those equations can be written as following form[6]:

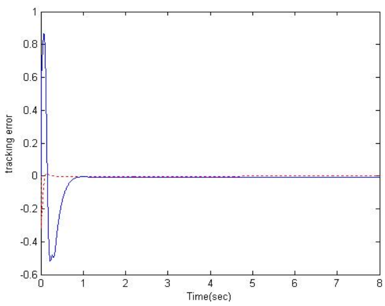
$$\dot{x}_1 = -\sigma x_1 + \sigma x_2$$
$$\dot{x}_2 = r x_1 - x_2 - x_1 x_3$$
$$\dot{x}_3 = x_1 x_2 - b x_3 \quad (4)$$

where $\sigma = 10, \rho = 28$ and $b = 8/3$, respectively.

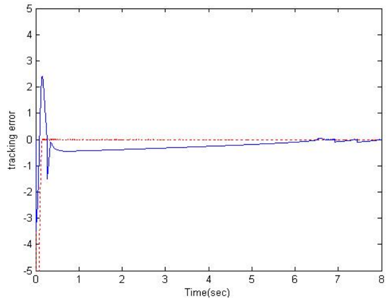
We assume that the control input enters by the state x_3. We want that the trajectory converge into the stable equilibrium points.



<Fig. 4> Tracking error of state x_1



<Fig. 4> Tracking error of state x_2



<Fig. 6> Tracking error of state x_3

<Table 2> The comparison of MSE.

	TSMC	CSMC
MSE of x_1	0.0124	0.0338
MSE of x_2	0.0037	0.0392
MSE of x_3	0.1564	0.2877

The results of tracking error for TSMC and CSMC are shown in Fig. 4, 5 and 6, respectively. Dotted line is appeared TSMC and solid line is appeared CSMC. Here, we can know that TSMC is more efficient and has the faster convergence speed than CSMC. In addition, Table 2 compares simulation results of CSMC and TSMC, where the MSE of TSMC is lower than that of CSMC.

3. Conclusion

In this paper, we have designed a terminal sliding mode controller for chaotic nonlinear system. This terminal sliding mode control technique was used to improve the performance, the robustness, the reliability and the precision by contrast with classical sliding mode control. In addition, we can obtain the finite convergence time using simple method. Finally, the proposed terminal sliding mode controller has been applied to a chaotic nonlinear system, such as Duffing and Lorenz system. In those computer simulation results, we can know some advantages of proposed controller, such as the faster. convergence speed and the better accuracy. That is, we can confirm that TSMC is more efficient than CSMC.

4. Acknowledge

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