

## 시간-주파수 영역 반사파 시스템에서 가중강인최소자승 필터를 이용한 주파수 추정

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### Frequency Estimation for Time-Frequency Domain Reflectometry using Weighted Robust Least Squares Filter

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**Abstract** - In this paper, an experiment of weighted robust least squares frequency estimation for the Gaussian envelope chirp signal which is used in the time-frequency domain reflectometry system was carried out. By incorporating the forgetting factor to the frequency estimator, the weighted robust least squares filter achieved good enough frequency estimation performance for the chirp signal and it can be adopted to implement not only low cost time-frequency domain reflectometry but also real-time time-frequency domain reflectometry implementation.

#### 1. Introduction

As frequency estimation from measured signal has been needed in many real applications such as control, communication, instrument and power systems many researchers have studied the real time direct frequency estimation problem for many years. It is well known that least mean square (LMS) algorithm [1] and conventional weighted least-squares (WLS) algorithm [2] have some advantages comparing with other traditional frequency estimation algorithm such as Pisarenko's harmonic decomposer. It is possible to analyze their convergency character and error variance estimation of direct frequency estimator with mathematical approaches. Although the LMS algorithm has some advantages, it seems that it is not suitable for the sinusoidal signal with dc offset because basically it was set up using the linear prediction model instead of using general state space model of a sinusoidal signal. It has a drawback to guarantee it's performance when it is under noisy environments because unexpected frequency estimation errors can be placed by the uncertain parameter.

A weighted robust least squares (WRLS) algorithm makes the frequency estimation problem interpreted as a robust filtering problem for uncertain linear time varying systems with a stochastic parametric uncertainty in the measurement matrix. We can obtain the weighted robust least squares frequency estimator by compensating the WLS estimation errors. The WRLS algorithm gives not only unbiased frequency estimation but also robustness against parametric uncertainty. Moreover, forgetting factor plays a key role to improve the parameter tracking performance in the non-stationary conditions.

Time-Frequency Domain Reflectometry (TFDR) can detect and localize the fault on a coaxial cable with high resolution comparing conventional reflectometries [3][4]. To provide time and frequency localization the TFDR uses chirp signal with Gaussian envelope. The frequency estimation experiments for the chirp signal which is adopted to TFDR is achieved via WRLS algorithm. The algorithm shows accurate frequency estimation performance and wide range of robustness in the presence of severe sensor measurement noise.

#### 2. Frequency Estimator

##### 2.1 Weighted Robust Least Squares Frequency Estimator

It is shown that the measurement noise generates stochastic parametric uncertainty into the measurement matrix of a linear

time-varying system. Let us consider the following discrete-time observation model of the measured signal,  $d_k$ .

$$d_k = A \sin(\bar{\omega}k + \varphi) + b + \bar{v}_k, \quad \bar{w} \equiv wT, \quad (1)$$

where  $\omega$  is the tone frequency of sinusoids,  $b$  implies the dc offset,  $A$  is the magnitude, and  $\varphi$  means the phase shift.

Through piecewise-constant assumption on the parameters  $\omega$  and  $b$ , we can obtain the linear time-varying uncertain system from (1).

$$\begin{aligned} x_{k+1} &= x_k \\ y_k &= [\widetilde{H}_k - \Delta H_k]x_k + v_k \end{aligned} \quad (2)$$

where

$$\begin{aligned} x_k &\equiv [\cos w \quad b(1 - \cos w)]^T, \quad y_k \equiv d_{k+1} + d_{k-1}, \\ v_k &\equiv \bar{v}_{k+1} + \bar{v}_{k-1}, \quad [\widetilde{H}_k] \equiv [2d_k \quad 2], \text{ and} \\ \Delta H_k &\equiv [2\bar{v}_k \quad 0] \end{aligned}$$

The uncertain matrix  $\Delta H_k$  is not available except its statistical property.

$$W_k \equiv \begin{bmatrix} 4\bar{R}_k & 0 \\ 0 & 0 \end{bmatrix} \quad (3)$$

$$V_k \equiv 0 \quad (4)$$

The recursive form of robust least squares estimator and its statistical behavior was described in [5]. The recursive form of WRLS is similar with RLS's except forgetting factor  $\lambda$  considered to explain the non-stationary case. Derivation of recursive algorithm to calculate the weighted robust least squares estimates is straightforward. Using the definitions of forgetting factor  $\lambda$  and some conditions we have the following recurrent relation about  $P_k$ .

$$P_k^{-1} = \lambda P_{k-1}^{-1} + \widetilde{H}_k^T \overline{H}_k - W_k > 0 \quad (5)$$

$$\begin{aligned} \hat{x}_k^{WRLS} &= (I + P_k W_k) \hat{x}_{k-1}^{WRLS} \\ &\quad + P_k \widetilde{H}_k^T (y_k - \widetilde{H}_k \hat{x}_{k-1}^{WRLS}) \end{aligned} \quad (6)$$

In this experiment,  $\lambda$  is taken as 0.87 for frequency estimation.

#### 3. Time-Frequency Domain Reflectometry

##### 3.1 Implementation of the TFDR

The TFDR system consists of four components : signal generator, signal acquisition, signal distribution and algorithm execution. The TFDR system is shown in Figure 1. The signal generation part makes the input signal which is the linearly modulated chirp signal with the Gaussian envelope. It is accomplished by PXI type Arbitrary Waveform Generator with 200M Samples/sec. rate. The signal acquisition part is implemented by Digital Storage Oscilloscope with 200M Samples/sec. rate. To localize the fault on a cable 10C-FBT with 100m length was used. The parameters of the input signal used in this experiment are shown in table 1.

Parameters	Value
Center Frequency	13.5 [MHz]
Frequency Bandwidth	7 [MHz]
V <sub>pp</sub>	4 [V]

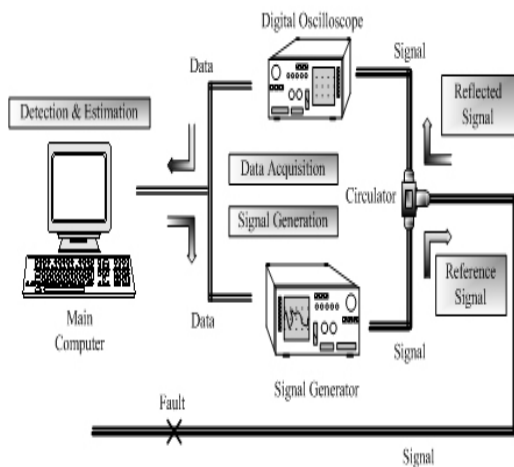
<Table 1> Parameters for The Input Signal

### 3. Results

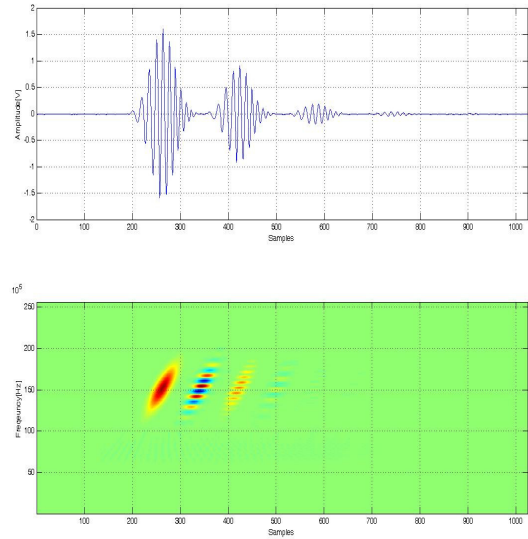
The input and reflected signal used in the TFDR is shown on the top in Fig. 2. The frequency variation of the input and reflected is clearly shown on the bottom in Fig. 2. It was obtained by taking Wigner distribution for the signal [3][4]. All of the 4 marks between 200 and 550 samples on the bottom side in Fig. 2 imply the frequency was linearly increased from 10 to 17 MHz as we expected. The frequency estimation for the same input and reflected signal via the weighted robust least squares filter is shown in Fig. 3. We can also find the four variations between 200 to 850 samples on the bottom in Fig. 3. The first three variations indicate the frequency variation from 10 to 17 MHz,

### 3. Conclusion

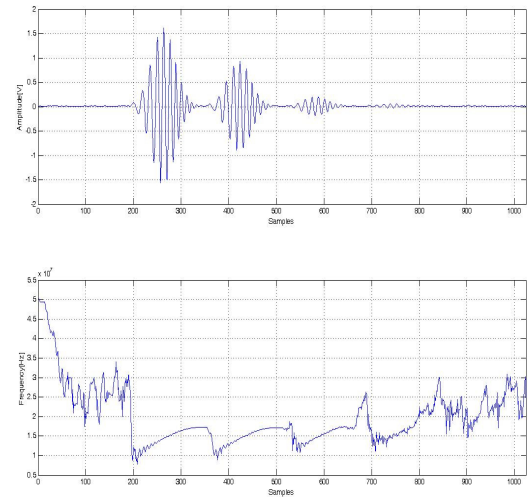
The frequency estimation for the Gaussian envelop chirp signal which is used in TFDR has been carried out using the WRLS filter. By incorporating the forgetting factor to the estimator it shows that WRLS can estimate the frequency of non-stationary signals. It will be very useful to implement TFDR with low performance modules. Moreover, since it requires small amount of computations compared to other existing estimators, it is attractive for real-time implementation.



<Figure 1> Time-Frequency Domain Reflectometry with PXI instrument modules



<Figure 2> Input and Reflected signal and its Wigner Distribution



<Figure 3> Input and Reflected signal and its Frequency Estimation by WRLS Filter

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