

# Analytical Method to Analyze the Effect of Tolerance on the Modal Characteristic of Multi-body Systems in Dynamic Equilibrium

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**Key Words :** Tolerance Analysis( ), Damped Natural Frequency( ), Transfer Function( ), Dynamic Equilibrium Position( ), Sensitivity Analysis( ), Multi-body System( )

## ABSTRACT

Analytical method to analyze the effect of tolerance on the modal characteristic of multi-body systems in dynamic equilibrium position is suggested in this paper. Monte-Carlo Method is conventionally employed to perform the tolerance analysis. However, Monte-Carlo Method spends too much time for analysis and has a greater or less accuracy depending on sample condition. To resolve these problems, an analytical method is suggested in this paper. By employing the sensitivity information of mass, damping and stiffness matrices, the sensitivities of damped natural frequencies and the transfer function can be calculated at the dynamic equilibrium position. The effect of tolerance on the modal characteristic can be analyzed from tolerance analysis method.

1.

Dubowsky <sup>(3)</sup>

. 2005

가

Choi <sup>(4)</sup>

가

Eom <sup>(6)</sup>

(Monte - Carlo Method)

(6)

(Finite Difference Met -

hod)

가

um position)

(dynamic equilibri -

. 1964 Hartenberg <sup>(1)</sup>

<sup>(5,7)</sup>

<sup>(5)</sup>

가

Garrett <sup>(2)</sup>

(FDM)

†

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\*

(Monte - Carlo method)

(5)

2.

$$\mathbf{M}\ddot{\mathbf{x}} + \Phi_{\mathbf{x}}^T \boldsymbol{\lambda} = \mathbf{Q} \quad (1)$$

$\mathbf{M}$  (system inertia matrix),  $\mathbf{Q}$  (generalized force),  $\Phi_{\mathbf{x}}$  (Jacobian matrix),  $\boldsymbol{\lambda}$  (Lagrange multipliers),  $q_i$  (reference body)

$\dot{\mathbf{x}}$  (8) 가

$$\dot{\mathbf{x}} = \mathbf{B}\dot{\mathbf{q}} \quad (2)$$

$\mathbf{B}$

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_D^T & \mathbf{q}_R^T \end{bmatrix}^T \quad (3)$$

$\mathbf{q}_D$ ,  $\mathbf{q}_R$

$\dot{\mathbf{x}}$  가  $\ddot{\mathbf{x}}$

$$\dot{\mathbf{x}} = \mathbf{B}_D \dot{\mathbf{q}}_D + \mathbf{B}_R \dot{\mathbf{q}}_R \quad (4)$$

$$\ddot{\mathbf{x}} = \mathbf{B}_D \ddot{\mathbf{q}}_D + \mathbf{B}_R \ddot{\mathbf{q}}_R + \dot{\mathbf{B}}_D \dot{\mathbf{q}}_D + \dot{\mathbf{B}}_R \dot{\mathbf{q}}_R \quad (5)$$

$\dot{\mathbf{q}}_D$   $\mathbf{B}_D$

$\mathbf{B}_R$   $\mathbf{B}$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_D & \mathbf{B}_R \end{bmatrix} \quad (6)$$

$$\mathbf{M}^* \ddot{\mathbf{q}}_R + \Phi_{\mathbf{q}_R}^C{}^T \boldsymbol{\lambda}^C = \mathbf{Q}^* \quad (7)$$

$\Phi^C$ ,  $\mathbf{M}^*$   $\mathbf{Q}^*$

$$\mathbf{M}^* = \mathbf{B}_R^T \mathbf{M} \mathbf{B}_R \quad (8)$$

$$\mathbf{Q}^* = \mathbf{B}_R^T \mathbf{Q} - \mathbf{B}_R^T (\mathbf{M} \dot{\mathbf{B}}_D \dot{\mathbf{q}}_D + \mathbf{M} \dot{\mathbf{B}}_R \dot{\mathbf{q}}_R) \quad (9)$$

$\mathbf{q}_R$  2

$$\Phi_{\mathbf{q}_R}^C \ddot{\mathbf{q}}_R = \boldsymbol{\gamma}^C \quad (10)$$

$$\boldsymbol{\gamma}^C = -(\Phi_{\mathbf{q}_R}^C \dot{\mathbf{q}}_R)_{\mathbf{q}_R} \dot{\mathbf{q}}_R - 2\Phi_{\mathbf{q}_R t}^C \dot{\mathbf{q}}_R - \Phi_{tt}^C \quad (11)$$

(7) (10)

$$\begin{bmatrix} \mathbf{M}^* & \Phi_{\mathbf{q}_R}^C{}^T \\ \Phi_{\mathbf{q}_R}^C & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_R \\ \boldsymbol{\lambda}^C \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q}^* \\ \boldsymbol{\gamma}^C \end{Bmatrix} \quad (12)$$

(12)

3.

가

$$\ddot{\mathbf{q}}_R = \mathbf{0}, \quad \dot{\mathbf{q}}_R = \mathbf{0}, \quad \ddot{\mathbf{q}}_D = \mathbf{0}$$

(7)

$$\mathbf{B}_R^T (\mathbf{M} \dot{\mathbf{B}}_D \dot{\mathbf{q}}_D - \mathbf{Q}) + \Phi_{\mathbf{q}_R}^C{}^T \boldsymbol{\lambda}^C = \mathbf{0} \quad (13)$$

4.

(7)

$$\mathbf{M}^* \ddot{\mathbf{q}}_R + \Phi_{\mathbf{q}_R}^C{}^T \boldsymbol{\lambda}^C - \mathbf{Q}^* = \mathbf{0} \quad (14)$$

$$\mathbf{q}_R = \begin{bmatrix} \mathbf{u} & \mathbf{v} \\ \mathbf{u}^T & \mathbf{v}^T \end{bmatrix}^T \quad (15)$$

$$\dot{\mathbf{q}}_R = \begin{bmatrix} \dot{\mathbf{u}} & \dot{\mathbf{v}} \\ \dot{\mathbf{u}}^T & \dot{\mathbf{v}}^T \end{bmatrix} \quad (5)$$

$$\dot{\mathbf{q}}_R = \mathbf{R}\dot{\mathbf{v}} \quad (16)$$

$$\mathbf{R} = \begin{bmatrix} -\Phi_u^C & \Phi_v^C \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (17)$$

$$\mathbf{R}^T \quad (14)$$

$$\mathbf{R}^T \mathbf{M}^* \mathbf{R} \ddot{\mathbf{v}} + \mathbf{R}^T \mathbf{M}^* \dot{\mathbf{R}} \dot{\mathbf{v}} - \mathbf{R}^T \mathbf{Q}^* = \mathbf{0} \quad (18)$$

$$(18) \quad q^*$$

$$\hat{\mathbf{M}}^* \delta \dot{\mathbf{v}} + \hat{\mathbf{C}}^* \delta \dot{\mathbf{v}} + \hat{\mathbf{K}}^* \delta \mathbf{v} = \mathbf{0} \quad (19)$$

2

$$\hat{\mathbf{M}}^* = \mathbf{R}^T \mathbf{B}_R^T \mathbf{M} \mathbf{B}_R \mathbf{R} \Big|_{q^*} \quad (20)$$

$$\hat{\mathbf{C}}^* = \frac{\partial}{\partial \dot{\mathbf{v}}} \mathbf{R}^T \mathbf{B}_R^T [\mathbf{M} \dot{\mathbf{B}}_D \dot{\mathbf{q}}_D - \mathbf{Q}] \Big|_{q^*} \quad (21)$$

$$\hat{\mathbf{K}}^* = \frac{\partial}{\partial \mathbf{v}} \mathbf{R}^T \mathbf{B}_R^T [\mathbf{M} \dot{\mathbf{B}}_D \dot{\mathbf{q}}_D - \mathbf{Q}] \Big|_{q^*} \quad (22)$$

5.

5.1

(9)

$$(\lambda_j^2 \hat{\mathbf{M}} + \lambda_j \hat{\mathbf{C}} + \hat{\mathbf{K}}) \phi_j = \mathbf{0} \quad (23)$$

$$\phi_j \quad \lambda_j \quad b \quad (23)$$

$$\phi_j^T \quad (23)$$

$$(\phi_j^T (2\lambda_j \hat{\mathbf{M}} + \hat{\mathbf{C}}) \phi_j = 1)$$

$$\frac{\partial \lambda_j}{\partial b} = -\phi_j^T \left( \lambda_j^2 \frac{\partial \hat{\mathbf{M}}}{\partial b} + \lambda_j \frac{\partial \hat{\mathbf{C}}}{\partial b} + \frac{\partial \hat{\mathbf{K}}}{\partial b} \right) \phi_j \quad (24)$$

5.2  
1

$$|H(j\omega)| = \frac{1}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad (25)$$

b

5.3

$$S = \frac{d\omega_{d,j}}{db} \quad (26)$$

$$S = \frac{d|H(j\omega)|}{db} \quad (27)$$

(sample) (normal distribution) 가 99.73% (4)

$$\sigma^2 = \frac{1}{9} S^2 T^2 \quad (28)$$

T b σ (standard deviation)

6.

6.1

Fig.1

Ω

(bar)

m=3kg, L=1m

(revolute joint)

가 3Ns/rad

θ

$\pi/3$  rad 가  
(virtual rigid body) ,  $\Omega$

Fig.2 가 3%, 6%, 12% 가  
가 3,000 가

2.496rad/s  
3.530 rad/s 3.984 rad/s  
Fig.3 가 3% 가

Fig.4 가 3%, 6%, 12% 가

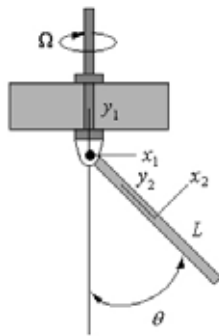


Fig. 1 Rotating simple pendulum

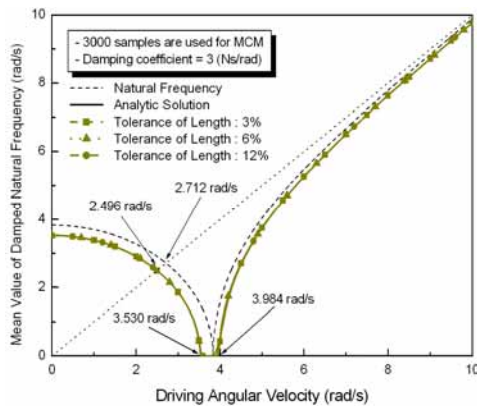


Fig. 2 Mean value of damped natural frequency versus angular velocity

Fig.5 3.835rad/s

3%, 6%, 12% 가 Fig.6 가

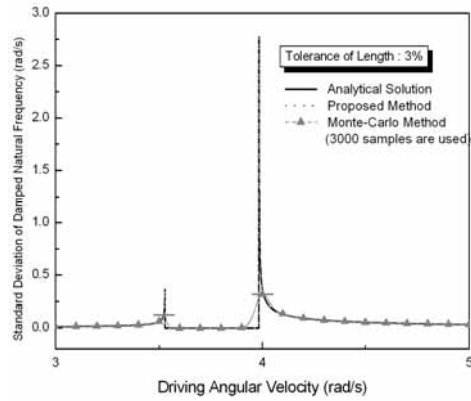


Fig. 3 Comparison of proposed method and Monte-Carlo method

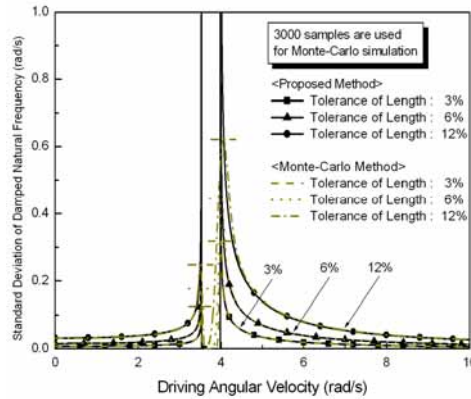


Fig. 4 Comparison of proposed method and Monte-Carlo method

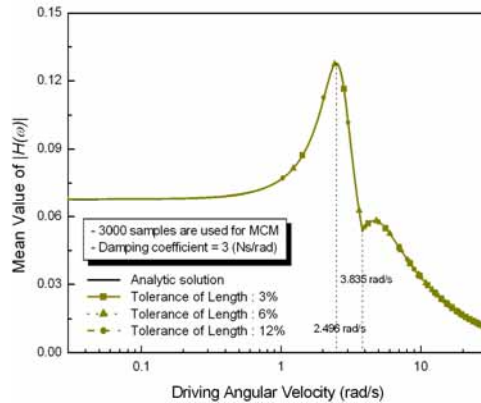


Fig. 5 Mean value of transfer function versus angular velocity

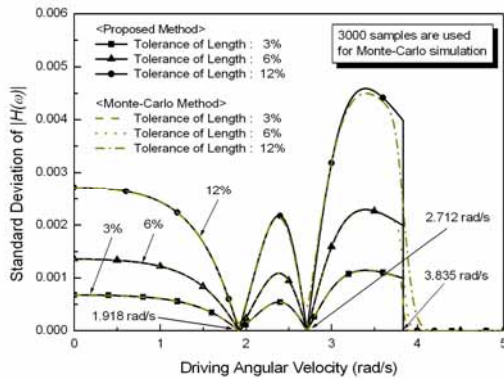


Fig. 6 Comparison of proposed method and Monte-Carlo method

Table 1 Comparison of CPU time between proposed method and Monte-Carlo Method

Method	CPU Time	Ratio
Proposed Method	0.703 sec	1
Monte-Carlo method	448.281 sec	638

Table 1

6.2

Fig.7

$\omega$   
1, 2, 3,  
4 (spindle), 1 (ball),  
2, (collar)  
1  
2, 1 1 3  
, 1 4 ,  
2 4 3 4  
가 0.10922 m  
1 4  
1000 N/m 가 400 Ns/m  
2 0.15 m 가

Fig.8

12% 가 3%, 6%,  
Fig.9

가

가

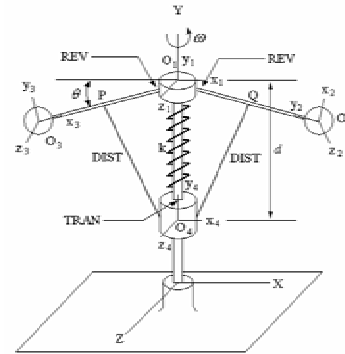


Fig. 7 Governor mechanism

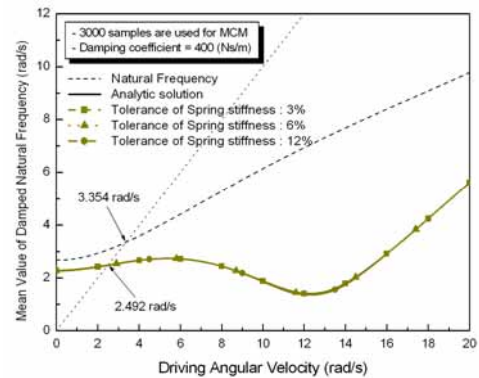


Fig. 8 Mean value of damped natural frequency versus angular velocity

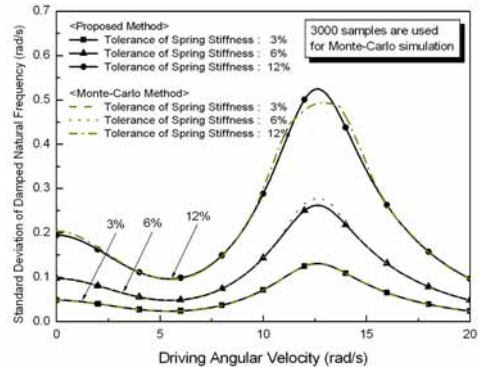


Fig. 9 Comparison of proposed method and Monte-Carlo method

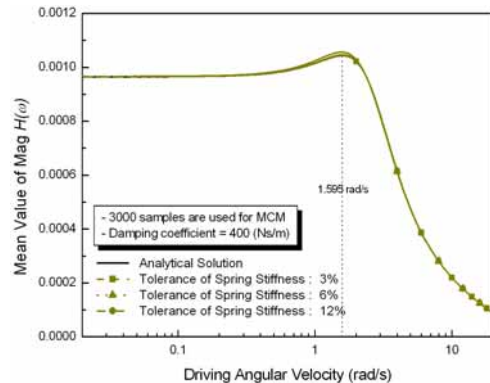


Fig. 10 Mean value of transfer function versus angular velocity

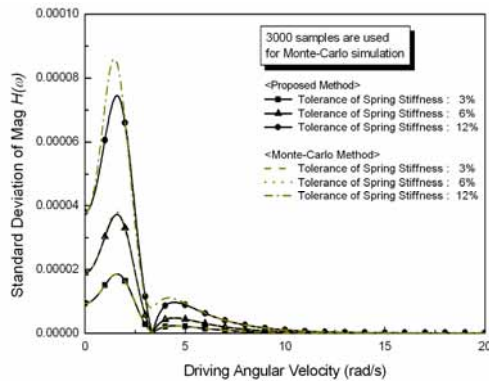


Fig. 11 Comparison of proposed method and Monte-Carlo method

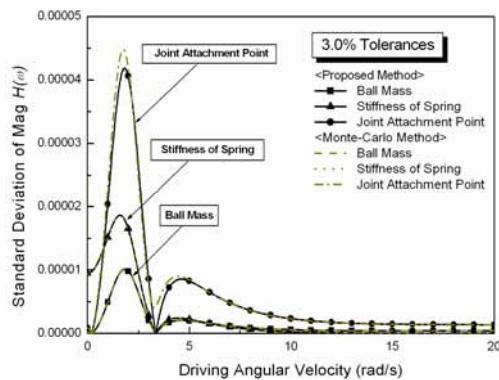


Fig. 12 Parametric studies on the effect of tolerance

Table 3 Comparison of CPU time between proposed method and Monte-Carlo Method

Method	CPU Time	Ratio
Proposed Method	2.188 sec	1
Monte-Carlo method	2525.375 sec	1154

Fig.10

가 . Fig.11 가

(1.595rad/s)

Fig.12

3%

Table 3

7.

가

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