2006년도 대한전자공학회 하계종합학술대회 제29권 제1호

A new algorithm applied in Control-Relevant Discretization of Nonlinear Systems

Qiang Zhang*, Changlei Zhang**, Yu Gao***, Kil To Chong****
Division of Electronics and Information Engineering,
Chongbuk National University

E-mail: *john_zhang19780828@hotmail.com
**demozcl@hotmail.com
***gaoyuelmon@hotmail.com
****kiltochong@hotmail.com

Abstract

In this paper, we propose an attractive novel numerical int egration method that is largely devoid of ill conditioning and is suitable for any nonlinear problem. Since the method is exact for linear problems, it is especially precise for quasi-linear problems, which are frequently encountered in the real w orld. The method is based on a new approach to the computat ion of a matrix exponential. It does not require excessive com putational resources and lends itself to a short and robust piece of software that can be easily inserted into large simulation pac kages.

I. Introduction

Currently, control-relevant systems characterized by time-delay problems are encountered in more and more situations. The reasons for this can be grouped into two major categories. The first is that time delays are becoming increasingly more widespread in control systems because of the convergence of communication and computational systems with traditional control engineering. Controller communication, especially communication over local-area networks (LANs) or wide-area networks (WANs), and complex computations resulting from digital controller implementations result in large time delays. In the case of WANs, such time delays are also time varying. As the communication and computational functions present in embedded control systems increase, the impact of time delays becomes more substantial and cannot be overlooked. The second reason is that control systems with non-negligible time delays exhibit complex behaviors because of their infinite dimensionality. Even a linear time-invariant (LTI) system with a constant time delay in the input or state has infinite dimensionality if expressed in the continuous time domain. As a result, controller design techniques developed over the last several decades for finite dimensional systems are difficult to apply to time-delay systems with any degree of effectiveness. Control system design methods that explicitly account for the presence of time delays are required.

This paper presents a new approach to solve this type of problem. The proposed algorithm is so effective that it can solve both control-relevant systems and some complex chemical problems, neither of which being easy to solve.

II. Algorithm

An autonomous nonlinear ordinary (vector) differential equation for the time function X(t) in explicit form can be denoted as

$$\underline{\dot{X}} = \underline{f}(\underline{X}) \quad (1)$$

with the initial condition X(t0) = X0, where the underlined symbols denote column vectors of length n.

To avoid analytical or numerical annoyances when J is singular or nearly singular, we consider an augmented problem that avoids the matrix inversion altogether and is therefore simpler and more robust. We define the augmented (n + 1)-dimensional vectors

$$\eta \equiv (\frac{\xi}{1}) \qquad \eta_0 \equiv (\frac{0}{1}) \quad (2)$$

with initial condition η(0) = η0. The exact solution of the augmented problem is

$$\underline{\eta}(\tau) = e^{\tau \cdot A} \cdot \underline{\eta}_0 \quad (3)$$

which only requires the computation of a matrix exponential and is also valid if J is singular.

Ⅲ. Examples and Simulations

A simple second order system is examined for this case. The system is modeled as follows:

$$\ddot{x} = \dot{x}(1 - x^2) - x + u \quad (4)$$

We assumed that the initial conditions were x(0) = 0.1, $\dot{x}(0) = 0$, and that the input u was a unit step. First, we rewrote the system in standard state–space representation form. The states of the system were defined as follows:

$$X_1 = x, X_2 = \dot{x} \quad (5)$$

Thus the system form was

$$\dot{X}_1 = f_1(X) + g_1(X)u = X_2 \quad (6)$$

$$\dot{X}_2 = f_2(X) + g_2(X)u = X_2(1 - X) - X_1^2 + u \quad (7)$$

Table 1 shows the results obtained from the proposed method programmed using the Maple language (simulation) and the Matlab solver (reference) for a sampling period of T = 0.05 and an input time delay of D = 0.055.

Table 1 Simulation results of the proposed method

Time step	Simulation x1	Simulation x2	Reference x1	Reference x2
10	0.1874	0.4995	0.1986	0.4772
20	0.5789	1.1915	0.6011	1.1237
30	1.2608	1.2940	1.2368	1.2463
40	1.7208	0.4780	1.6838	0.4815
50	1.8159	–0.0577	1.7681	–0.0623
60	1.7017	–0.3034	1.6802	–0.2550
70	1.5087	-0.3472	1.5324	–0.3259
80	1.4002	–0.3731	1.3599	–0.3610
90	1.2376	–0.3930	1.1737	–0.3817
100	1.0379	–0.4056	0.9804	–0.3889

IV. Conclusion

In the past, we were forced to use the Taylor–Lie series expansion method to solve nonlinear control system problems with time delays. However, our proposed novel method is more attractive because it is largely devoid of ill-conditioning. Since the method is exact for linear problems, it is especially precise for quasi-linear problems. Several relevant simulations have been performed using our proposed algorithm. The method does not require extensive computational resources and has been implemented using a simple and robust Maple-language program (freely available from the authors) that can be easily applied for different real cases.

References

[1]G.R. Gavalas, Nonlinear Differential Equations of Chemically Reacting Systems (Springer–Verlag, Berlin, 1968).

[2] E.H. Kerner, J. Math. Phys. 22, pp. 1366 (1981).

[3] N. Kazantzis and C. Kravaris, “System-theoretic properties of sampled-data representations of nonlinear systems obtained via Taylor–Lie series,” Int. J. Control, 67, pp. 997–1020 (1997).

[4] G.H. Golub and C.F. van Loan, Matrix Computations (Johns Hopkins University Press, Baltimore, 1996).