On the Least Squared Ordered Weighted Averaging (LSOWA) Operator Weights

Byeong Seok Ahn
Department of Business Administration, Hansung University
e-mail: bsahn@hansung.ac.kr

Abstract

The ordered weighted averaging (OWA) operator by Yager has received more and more attention since its appearance. One key point in the OWA operator is to determine its associated weights. Among numerous methods that have appeared in the literature, we notice the maximum entropy OWA (MEOWA) weights that are determined by taking into account two appealing measures characterizing the OWA weights. Instead of maximizing the entropy in the formulation for determining the MEOWA weights, the new method in the article tries to obtain the OWA weights which are evenly spread out around equal weights as much as possible while strictly satisfying the orness value provided in the program. This consideration leads to the least squared OWA (LSOWA) weighting method in which the program tries to obtain the weights that minimize the sum of deviations from the equal weights since entropy is maximized when the weights are equal. Above all, the LSOWA weights display symmetric allocations of weights on the basis of equal weights. The positive or negative allocations of weights from the median as a basis depend on the magnitude of orness specified. Further interval LSOWA weights are constructed when a decision-maker specifies his or her value of orness in uncertain numerical bounds.

1. Introduction

A multiple criteria decision making (MCDM) method under certainty largely consists of two phases: 1) construction of decision problem and information specification, and 2) aggregation and exploitation [1][2]. Among others, synthesizing judgments is an important part of MCDM methods. Yager [14] introduced the ordered weighted averaging (OWA) operator to provide a method for aggregating multiple inputs that lie between the *max* and *min* operators. As the term 'ordered' implies, the OWA operator pursues a nonlinear aggregation of objects considered. The OWA operator is generally composed of the following three steps [20]:
(1) Reorder the input arguments in descending order.
(2) Determine the weights associated with the OWA operator by using a proper method.
(3) Utilize the OWA weights to aggregate these reordered arguments.

In the short time since its first appearance, the OWA operators have been used in an astonishingly wide range of applications in the fields including neural networks [15][16], database systems [17], fuzzy logic controllers [18][19], group decision making problems with linguistic assessments [8][9], data mining [13], location based service (LBS) [12] or more generally geographical information system (GIS) [10][11] and so on..

Appealing point in the OWA operator was the introduction of the concept of *orness* and the definition of an *orness* measure that could establish how 'orlike' a certain operator is, based on the values of its weighting function. Thus the measure can be interpreted as the mode of decision making circumstances by conferring the semantic meaning to the weights used in aggregation process. If an aggregated value is close to the maximum of the ordered objects, the aggregation pursues the 'orlike' aggregation. If an aggregated value is close to the minimum of the ordered objects, on the other hand, the aggregation pursues the 'andlike' aggregation. This concept perfectly coincides with the traditional decision making theory in which *max* decision principle denotes the optimistic decision context and *min* decision principle denotes the pessimistic decision context.

On the other hand, Yager, based on a measure of entropy, proposed a measure of dispersion which gauges the degree of utilization of information in the sense that each of weighting vectors considered can be different to each other by degree of dispersion though they have the same degree of *orness* [14]. One of the first approaches, suggested by O'Hagan [7], determines a special class of OWA operators having maximal entropy of the OWA weights for a given level of *orness*, algorithmically based on the solution of a constrained optimization problem. The resulting weights are called maximum entropy OWA (MEOWA) weights for a given degree of *orness* and analytic forms and property for these weights are further investigated by several researchers [3][4][5][6].

Instead of maximizing the entropy in the formulation for determining the MEOWA weights, the new method in the article tries to obtain the OWA weights which are evenly spread out around the equal weights as much as possible while strictly satisfying the orness value provided in the program. This consideration leads to the least squared OWA (LSOWA) weighting method in which the program tries to obtain the weights that minimize the sum of deviations from the equal weights since entropy is maximized when the weights are equal. Several properties of the LSOWA weights are investigated in detail. Interval LSOWA weights are constructed when a decision-maker specifies his or her value of orness in uncertain numerical bounds and further we present a method, with those uncertain interval LSOWA weights, for prioritizing alternatives that are evaluated by multiple criteria.

2. Determining the LSOWA weights

An OWA operator [14] of dimension n is a mapping f: $R^n \rightarrow R$ that has an associated weighting n vector $W=(w_1,w_2,\cdots,w_n)^T$ such that $w_i \in [0,1]$ for $i \in I=\{1,2,\cdots,n\}$ and $\sum_{i\in I} w_i=1$. Central to this operator is the reordering of the arguments, based upon their values, in particular an argument a_i is not associated with a particular weight w_i but rather a weight w_i is associated with a particular ordered position i of the arguments. The OWA aggregation is a nonlinear aggregation because of the ordering process used.

Yager[3] introduced two characterizing measures associated with the weighting vector W of an OWA operator. The first one, the measure of orness of the aggregation, is defined as

$$orness(W) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$$

and it characterizes the degree to which the aggregation is like an *or* operation. If we consider the special cases of OWA operators,

$W^* = [1, 0, 0, \mathrm{K}, 0]^T$ (maximum operator),

$W_* = [0, 0, 0, \mathrm{K}, 1]^T$ (minimum operator),

$W_{Ave} = [1/n, 1/n, 1/n, \mathrm{K}, 1/n]^T$ (average operator),

then it can easily be shown that

(1) $orness(W^*) = 1$, (2) $orness(W_*) = 0$,

(3) $orness(W_{Ave}) = 0.5$.

The OWA operators with many of the weights near the top will be an 'orlike' operator $(orness(W) \geq 0.5)$, while those operators with most of the weights at the bottom will be 'andlike' operators $(orness(W) \leq 0.5)$. As to the semantics of the OWA's measure of *orness*, Yager suggests that, based on Hurwicz's model, the measure of *orness* can be interpreted as a measure of optimism of the decision making, while the measure of andness is a measure of pessimism.

Yager [14] proposed a measure of dispersion which gauges the degree of utilization of information.

$$disp(W) = -\sum_{i=1}^{n} w_i \ln w_i \ .$$

This measure can be used to gauge the degree to which the information about the individual aggregates is used in the aggregation process. We note that since this dispersion is really a measure of entropy and thus the following properties are valid (1) the dispersion is minimum if w_i=1 for some i and $disp(W)$=0 (2) the dispersion is maximum if w_i=1/n and $disp(W)$=ln n. O'Hagan [7] determined a special class of OWA operators having a maximal entropy of the OWA weights for some prescribed value of *orness*. This approach is based on the solution of the following mathematical programming problem:

$$\text{Maximize} \quad -\sum_{i=1}^{n} w_i \ln w_i \tag{1a}$$

$$\text{subject to} \quad \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i = \Omega, \quad 0 \leq \Omega \leq 1 \tag{1b}$$

$$\sum_{i=1}^{n} w_i = 1, \quad 0 \leq w_i \leq 1, \quad i = 1, \mathrm{K}, n \ . \tag{1c}$$

Filev and Yager [5] provided an analytic solution to the above constrained optimization problem with an aim to use the MEOWA weights among others in dynamic environments, in which the value of Ω changes, without having to solve a new constraint optimization problem.

In the LSOWA method, the program is to obtain the weights that minimize the sum of deviations from the equal weights instead of maximizing entropy itself since it is known that the entropy is maximized when the weights are equal. This consideration can be set forth by the following constrained mathematical program:

$$\text{Mimimize} \quad \sum_{i=1}^{n}(w_i - 1/n)^2 \tag{2a}$$

$$\text{subject to} \quad \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i = \Omega, \quad 0 \leq \Omega \leq 1 \tag{2b}$$

$$\sum_{i=1}^{n} w_i = 1, \quad 0 \leq w_i \leq 1, \quad i = 1, \mathrm{K}, n \tag{2c}$$

The program (2a)-(2c) is a quadratic mathematical program, thus well-known nonlinear software package such as, for example, *Lindo* solver suite can be used to obtain the LSOWA weights. If we omit the nonnegative constraints on w_i in the formulation, we can find a nice analytic solution for determining the LSOWA weights, It will help us to deeply understand the LSOWA weights and simplify the process used for generating the LSOWA weights. In doing so, a composite function can be built such as

$$L(W, \alpha, \beta) = \sum_{i=1}^{n}(w_i - 1/n)^2 + \alpha\left(\sum_{i=1}^{n} w_i - 1\right) + \beta\left(\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i - \Omega\right)$$

which denotes the Lagrange function of constrained optimization problem (2a)-(2c), where α and β are real numbers. Then the partial derivatives of L are computed as

$$\frac{\partial L}{\partial w_j}=2w_j-\frac{2}{n}+\alpha+\beta\frac{n-j}{n-1}=0,\quad \forall j \tag{3a}$$

$$\frac{\partial L}{\partial \alpha}=\sum_{i=1}^{n}w_i-1=0, \tag{3b}$$

$$\frac{\partial L}{\partial \beta}=\frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i-\Omega=0\ . \tag{3c}$$

From Equations 3a and 3b, we obtain, for the w_i's

$$w_i=\frac{1}{n}-\frac{1}{2}\alpha-\frac{n-i}{2(n-1)}\beta,\quad \forall i \tag{4a}$$

$$\sum_{i=1}^{n}w_i=\sum_{i=1}^{n}\left(\frac{1}{n}-\frac{1}{2}\alpha-\frac{n-i}{2(n-1)}\beta\right)=1 \tag{4b}$$

From Equation 4b, we obtain the following relationship between α and β

$$\alpha=-\frac{1}{2}\beta \tag{5}$$

and, substituting this equation into Equation 4a, we get

$$w_i=\frac{1}{n}+\frac{1}{4}\beta-\frac{n-i}{2(n-1)}\beta\ . \tag{6}$$

Finally, by substituting for the w_i's as expressed by Equation 6 into Equation 3c, we obtain an equation relating the specified degree of orness and the Lagrange parameter β:

$$\beta=\frac{24(n-1)(0.5-\Omega)}{n(n+1)}\ . \tag{7}$$

Furthermore Equation 6 can be simplified by using Equation 7

$$w_i=\frac{1}{n}+\frac{6(2i-n-1)}{n(n+1)}(0.5-\Omega),\quad i=1,\ldots,n\ . \tag{8}$$

The analytic solution shown in Equation 8 takes a simple and closed form as compared to the analytic solution for determining the MEOWA weights. Thus the LSOWA weights can be easily determined once the value of orness and the number of objects to be aggregated are specified in advance. As was intended by the mathematical formulation in (2a) – (2c), the LSOWA weights are determined possibly so as to be located around the equal weights (i.e., $1/n$) while satisfying the prescribed value of orness. There exist some peculiar characteristics in the LSOWA weights that are not in the MEOWA weights. We shall show them in several theorems and corollaries below.

THEOREM 1. If the specified value of orness is 0.5, then the LSOWA weights are $W_{ave}=[1/n, 1/n, \ldots, 1/n]$.
Proof. It is obvious from Equation 8. ■

THEOREM 2. For orness $0.5<\Omega\le 1$, the LSOWA weights form a decreasing sequence, $w_i>w_j$ for $i<j$. For orness $0\le\Omega<0.5$, to the contrary, the LSOWA weights form an increasing sequence, $w_i<w_j$ for $i<j$.
Proof.

$$w_i-w_j=\frac{1}{n}+\frac{6(2i-n-1)}{n(n+1)}(0.5-\Omega)-\frac{1}{n}-\frac{6(2j-n-1)}{n(n+1)}(0.5-\Omega)$$
$$=\frac{12(0.5-\Omega)}{n(n+1)}(i-j)$$

Therefore, for $0.5<\Omega\le 1$ and $i<j$, $w_i>w_j$ and for $0\le\Omega<0.5$ and $i<j$, $w_i<w_j$. ■

THEOREM 3. The LSOWA weight at median is $1/n$. That is, $w_{\frac{n+1}{2}}=\frac{1}{n}$ when n is odd and $(w_{\frac{n}{2}}+w_{\frac{n}{2}+1})/2=\frac{1}{n}$ when n is even.
Proof.
When n is odd, $w_{\frac{n+1}{2}}=\frac{1}{n}+\frac{6(2\times(n+1)/2-n-1)}{n(n+1)}(0.5-\Omega)=\frac{1}{n}$.
When n is even,

$$w_{\frac{n}{2}}+w_{\frac{n}{2}+1}=\frac{1}{n}+\frac{6(2\times n/2-n-1)}{n(n+1)}(0.5-\Omega)+\frac{1}{n}+\frac{6(2\times(n/2+1)-n-1)}{n(n+1)}(0.5-\Omega)=\frac{2}{n}.$$

Thus, $(w_{\frac{n}{2}}+w_{\frac{n}{2}+1})/2=\frac{1}{n}$. ■

COROLLARY 1. Let us denote $\Delta_i=w_i-\frac{1}{n}$, i.e., $\Delta_i=\frac{6(2i-n-1)}{n(n+1)}(0.5-\Omega)$. Then,

$$|\Delta_i|=|\Delta_{n-i+1}|,\ i=1,\ldots,n.$$

Proof.

$$\Delta_{n-i+1}=\frac{6(2\times(n-i+1)-n-1)}{n(n+1)}(0.5-\Omega)=-\frac{6(2i-n-1)}{n(n+1)}(0.5-\Omega)\ . \blacksquare$$

The LSOWA weights are rank-based weights and allocate some portion of weights symmetrically on the basis of median. More specifically, if the value of orness is greater than 0.5, positive portion of weight (i.e., Δ_i) is added to the left-sided weights at median and the same portion of weight is subtracted to the right-sided weights at median. On the contrary, if the value of orness is less than 0.5, positive portion of weight is added to the right-sided weights at median and the same portion of weight is subtracted to the left-sided weights at median. According to the Corollary 1, it is obvious that $\sum_{i=1}^{\frac{n+1}{2}}|\Delta_i|=\sum_{i=\frac{n+1}{2}}^{n}|\Delta_i|$, when n is odd and $\sum_{i=1}^{\frac{n}{2}}|\Delta_i|=\sum_{i=\frac{n}{2}+1}^{n}|\Delta_i|$, when n is even.

Example. For n=5 and Ω=0.7, the LSOWA weights from Table I are given

$W(0.7)$=(0.36, 0.28, 0.2, 0.12, 0.04).

It is evident that w_3=0.2 from Theorem 3. Further it holds that $|\Delta_i|=|\Delta_{n-i+1}|$, i=1,2,3 according to Corollary 1 because $\Delta_1=\frac{6(2\times1-5-1)}{5\times6}(0.5-0.7)=0.16$,

$\Delta_2=\frac{6(2\times2-5-1)}{5\times6}(0.5-0.7)=0.08$, $\Delta_3=0$,

$$\Delta_4 = \frac{6(2\times 4-5-1)}{5\times 6}(0.5-0.7) = -0.08\,,$$
$$\Delta_5 = \frac{6(2\times 5-5-1)}{5\times 6}(0.5-0.7) = -0.16\,.$$

Thus, the LSOWA weights can be rewritten as
$W(0.7)$=(0.2+0.16, 0.2+0.08, 0.2, 0.2-0.08, 0.2-0.16).

It is well-known that if a weighting vector W is optimal under some predefined value of orness Ω, then its reverse, denoted by W^R and defined as

$$w_i^R = w_{n-i+1}$$

is also optimal under degree of orness (1- Ω). Indeed, as was shown by Yager[3],we find that

$$disp(W^R)=disp(W) \text{ and}$$
$$orness(W^R)=1-orness(W).$$

It should be noted that Equation 8 for determining the LSOWA weights are derived by omitting the nonnegative conditions on the weights w_i's and thus has some drawbacks. In other words, if the specific LSOWA weights are determined by using the analytic solution and they are nonnegative, the solution is fine, but if the solution results in negative weights, we can not use the LSOWA weights as they are. Instead, we have to solve the nonlinear program by the use of software package. The index of weights and the prescribed value of orness in the analytic solution are two parameters that determine the usefulness of the LSOWA weights.

3. Aggregation multiple objects with interval LSOWA weights

In this section, we deal with a situation in which the LSOWA weights are specified not in the form of exact numerical values but in the form of uncertain forms. This is because when we work with vague or imprecise knowledge, it is difficult to estimate the weights with precision. Then, a more realistic approach may be to use imprecise assessments instead of exact numerical values, that is, by assuming that the parameters which are allowed in the problem are assessed by means of e.g., interval, weak ordinal, or set inclusion. Specifically, in the formulation for determining the LSOWA weights, the orness can be given not in the exact numerical value but in the interval numbers. It seems reasonable that the interval orness renders the LSOWA weights also interval weights. We consider a simple method for deriving such interval weights. This approach is appropriate for a lot of problems, since it allows for the representation of information in a more direct and adequate form if we are unable to express it with precision.

THEOREM 4. For given two values of orness Ω_1 and Ω_2, if $\Omega_1>\Omega_2$, then
when n is odd, $w_i(\Omega_1)>w_i(\Omega_2)$ for $i=1,\ldots,\frac{n+1}{2}-1$
$w_i(\Omega_1)<w_i(\Omega_2)$ for $i=\frac{n+1}{2}+1,\ldots,n$ and
when n is even, $w_i(\Omega_1)>w_i(\Omega_2)$ for $i=1,\ldots,\frac{n}{2}$
$w_i(\Omega_1)<w_i(\Omega_2)$ for $i=\frac{n}{2}+1,\ldots,n$
where $w_i(\Omega_j)$, j =1, 2 denote the ith LSOWA weights at the value of orness Ω_j.

Proof.

$$w_i(\Omega_1)-w_i(\Omega_2) = \frac{1}{n}+\frac{6(2i-n-1)}{n(n+1)}(0.5-\Omega_1)-\frac{1}{n}-\frac{6(2i-n-1)}{n(n+1)}(0.5-\Omega_2)$$
$$=\frac{6(2i-n-1)}{n(n+1)}(\Omega_2-\Omega_1)$$

. For $\Omega_1>\Omega_2$, the sign of $w_i(\Omega_1)-w_i(\Omega_2)$ is determined depending on the index number i as in the statements. ■

Let us denote $Q_k(\Omega)$ as a cumulative LSOWA weight from i=1 to i=k when a value of orness is given as Ω. That is,

$$Q_k(\Omega)=\sum_{i=1}^{k} w_i(\Omega)\,.$$

COROLLARY 2. For given two values of orness Ω_1 and Ω_2, if $\Omega_1>\Omega_2$, then $Q_k(\Omega_1)\geq Q_k(\Omega_2)$ for $k=2,\ldots,n$ where $Q_k(\Omega_1)=\sum_{i=1}^k w_i(\Omega_1)$ and $Q_k(\Omega_2)=\sum_{i=1}^k w_i(\Omega_2)$.

Proof.

$$Q_k(\Omega_1)-Q_k(\Omega_2)=\sum_{i=1}^{k}[w_i(\Omega_1)-w_i(\Omega_2)]=\sum_{i=1}^{k}\frac{6(2i-n-1)}{n(n+1)}(\Omega_2-\Omega_1)$$
$$=\frac{6(\Omega_2-\Omega_1)}{n(n+1)}\sum_{i=1}^{k}(2i-n-1)=\frac{6(\Omega_2-\Omega_1)}{n(n+1)}k(k-n)\geq 0$$

since $\Omega_1>\Omega_2$ and $0<k\leq n$ (equality holds when $k=n$). ■

As was previously mentioned, let us consider a situation in which a decision-maker specifies his or her optimistic value for the aggregation in uncertain ways. If the value of orness is specified in the form of interval, then for $\Omega_1>\Omega_2$, the constraint in Equation 1b should be replaced by Inequality 9.

$$\Omega_2 \leq \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i \leq \Omega_1 \qquad (9)$$

If we solve the constrained optimization problem with an interval orness constraint, the optimal LSOWA weights will be determined while the value of orness is set at not Ω_1 but Ω_2 because the optimal objective value is minimized at less value of orness. Thus, rather than solving the mathematical program as it is, it seems reasonable to think that incomplete orness indicates incomplete weights ranging between the weights regarding Ω_1 and Ω_2 respectively. This consideration leads to the following Corollary 3.

COROLLARY 3. When the value of orness is specified in the form of interval, interval LSOWA weights can be constructed in such a way that
when n is odd, $[w_1(\Omega_2),\ w_1(\Omega_1)],\ldots,[w_{\frac{n+1}{2}-1}(\Omega_2), w_{\frac{n+1}{2}-1}(\Omega_1)]$,

$[1/n, 1/n]$, $[w_{\frac{n+1}{2}+1}(\Omega_1), w_{\frac{n+1}{2}-1}(\Omega_2)]$, $[w_n(\Omega_1), w_n(\Omega_2)]$ and when n is even, $[w_1(\Omega_2), w_1(\Omega_1)], \ldots, [w_{\frac{n}{2}}(\Omega_2), w_{\frac{n}{2}}(\Omega_1)]$, $[w_{\frac{n}{2}+1}(\Omega_1), w_{\frac{n}{2}+1}(\Omega_2)], \ldots, [w_n(\Omega_1), w_n(\Omega_2)]$.

Proof. It directly follows from the results in Theorem 4. ■

It can be easily shown that the sum of lower bounds in interval LSOWA weights is less than one and the sum of upper bounds in interval LSOWA weights is greater than one.

Example. Suppose that a decision-maker specifies his or her orness (i.e., degree of optimism) lie in [0.6, 0.7], then the LSOWA weights can also be specified in interval ones. For n=5 (odd case), interval LSOWA weights are determined by combining these two weights,

$[w_1(0.6), w_2(0.6), w_3(0.6), w_4(0.6), w_5(0.6)]$=[0.28, 0.24, 0.2, 0.16, 0.12] and

$[w_1(0.7), w_2(0.7), w_3(0.7), w_4(0.7), w_5(0.7)]$=[0.36, 0.28, 0.2, 0.12, 0.04].

For n=6 (even case), interval LSOWA weights are determined by combining these two weights,

$[w_1(0.6), w_2(0.6), w_3(0.6), w_4(0.6), w_5(0.6), w_6(0.6)]$=[0.238, 0.210, 0.181, 0.152, 0.124, 0.095] and

$[w_1(0.7), w_2(0.7), w_3(0.7), w_4(0.7), w_5(0.7), w_6(0.7)]$=[0.310, 0.252, 0.195, 0.138, 0.081, 0.024].

4. Concluding remarks

In this article, we present an alternative weighting method, LSOWA for determining the OWA weights. The method is basically in line with the MEOWA weighting method in that it ties to obtain the OWA weights minimizing the variations from the equal weights while satisfying the prescribed value of ones.

When a decision-maker specifies an uncertain interval value of orness, we can construct interval LSOWA weights. Further a method for prioritizing multiple alternatives evaluated by multiple criteria is presented when those interval LSOWA weights are provided.

References

[1] Ahn BS. Extending Malakooti's model for ranking multicriteria alternatives with preference strength and partial information. IEEE Trans Syst Man Cybernet (Part A) 2003; 33: 281-287.

[2] Ahn BS. Multi-attribute decision aid with extended ISMAUT. IEEE Trans Syst Man Cybernet (Part A) (in press).

[3] Ahn BS. On the properties of OWA operator weights functions with constant level of orness. IEEE Trans Fuzzy Syst (in press).

[4] Filev D, Yager RR. On the issue of obtaining OWA operator weights. Fuzzy Set Syst 1998; 94: 157-169.

[5] Filev D, Yager RR. Analytic properties of maximum Entropy OWA operators. Inform Science 1995: 85; 11-27.

[6] Fuller R, Majlender P. An analytic approach for obtaining maximal entropy OWA operator weights. Fuzzy Set. Syst 2001: 124; 53-57.

[7] O'Hagan M. Using maximum entropy-ordered weighted averaging to construct a fuzzy neuron. in Proc. 24th Ann. IEEE Asilomar Conf Signals Syst Comput, Pacific Grove, CA, 1990; 618-623.

[8] Herrera F, Herrera-Viedma E, Verdegay JL. A sequential selection process in group decision making with a linguistic assessment approach. Inform Science 1995: 85; 223-239.

[9] Herrera F, Herrera-Viedma E, Verdegay JL. Model of consensus in group decision making under linguistic assessments. Fuzzy Set Syst 1996: 78; 73-87.

[10] Jiang H, Eastman JR. Application of fuzzy measures in multi-criteria evaluation in GIS. Int J Geogr Inf Sci 2000; 14: 173-184.

[11] Rinner C, Malczewki J. Web-enabled spatial decision analysis using ordered weighted averaging (OWA). J Geogr Syst 2002; 4: 385-403.

[12] Rinner C, Raubal M. Personalized multi-criteria decision strategies in location-based decision support. J Geogr Inf Sci 2004; 10: 149-156

[13] Torra V. OWA operators in data modeling and reidentification. IEEE Trans Fuzzy Syst 2004: 12; 652-660.

[14] Yager RR. On ordered weighted averaging aggregation operators in mulitcriteria decision making. IEEE Trans Syst Man Cybernet 1988; 18: 183-190.

[15] Yager RR. Fuzzy aggregation of modular neural networks with ordered weighted averaging operators. Int J Approx Reason 1995; 13:359-375.

[16] Yager RR. On a semantics for neural networks based on fuzzy quantifiers. Int J Intell Sys 1992; 7: 765-786.

[17] Yager RR. A note on weighted queries in information retrieval systems J Amer Soc Inform Sciences 1987: 28; 23-24.

[18] Yager RR. Connectives and quantifiers in fuzzy sets. Fuzzy Set Syst 1991: 40; 39-75.

[19] Yager RR, Filev DP. Fuzzy logic controllers with flexible structures. in Proc. Second Int. Conf. Fuzzy Sets and Neural Networks, Tizuka, 1992: 317-320.

[20] Xu ZS. An overview of methods for determining OWA weights. Int J Intell Syst 2005; 20: 843-865.