
1. Introduction

Over the last three decades, cellular manufacturing (CM) has been proved a very effective approach for improving the productivity of small to medium-size batch-type manufacturing system.

The fundamental step toward designing CM is to create part families and associated machine cells or vice versa, which has been known as the part-machine grouping (clustering) (PMG) or cell formation (CF) problem in literature. The fundamental objective of PMG is to find independent machine cells with minimum interaction between cells so that a set of part family can be completely produced in a cell.

Given \( m \) part types and \( n \) machine types, the basic input to analysis of PMG is an \( m \times n \) binary part-machine incidence matrix (PMIM) \( A \) where the element \( a_{ij} \) is 1 or 0 depending on whether or not part \( i \) requires processing on machine \( j \). Most of the approaches for solving PMG problem have attempted to find part families and machine cells by transforming its initial PMIM into the block diagonal matrix.

The conventional binary PMG approaches assume that each part-type makes identical demands on each machine type it uses. Obviously, this does not reflect shop floor reality. Since an intermediate operation of a part outside its cell involves two inter-cell moves while the first or last operation requires just one inter-cell moves, a "1" outside the main diagonal block can indicate more than one inter-cell moves depending on the sequence of operation and the volume of that particular part being processed (Nair & Narendran 1998).

Artificial neural network (ANN) model, a recent development in artificial intelligence, is a mathematical model that can be applied to discern patterns in data. Since the problem of transforming a matrix representing the association of parts and machines into a block diagonal form is similar to pattern recognition, it can be applied to PMG for the design of CM system.

Various types of ANNs have been applied to PMG. Among them, Fuzzy ART provides the best results for large-scale PMG problems (Suresh & Kaparthi 1994). It can handle both binary-valued and analogue inputs. However, the conventional ART/Fuzzy ART algorithms tend to produce too many clusters due to category proliferation resulting from the exemplar contraction (Dagli & Huggahalli 1991). Furthermore, the solution quality based on ART/Fuzzy ART algorithms highly depends on the ordering of input vector (Chen & Cheng 1995).

The recent development of Fuzzy ART ANN includes the incorporation of the part operation sequence data into the network (Suresh et al. 1999, Park & Suresh 2003). But existing methods did not incorporate the operation sequences with multiple visits to the same machine and production volumes of part into the network simultaneously by relying on separate binary-valued precedence matrices which...
represent the routing sequence for a part.

In this study, an effective methodology adopting Fuzzy ART neural network is presented to solve the PMG problem considering the operation sequences with multiple visits to the same machine and production volumes of parts. The proposed methodology adopts the non-binary PMIM developed by Won and Lee (2001) so that it can simultaneously capture the real manufacturing characteristics such as the operation sequences with multiple visits to the same machine and production volumes of parts. The proposed approach will be justified on large-size data sets generated with a pseudo-replicated clustering procedure which is a modification over conventional replicated clustering procedure.

2. Methodology for non-binary PMG

2.1 Input representation scheme

In this study, the type I production data-based PMIM in Won and Lee (2001) is employed for input presentation since it reflects the manufacturing characteristics such as the operation sequences with multiple visits to the same machine and production volumes of parts simultaneously, unlike Park and Suresh’s binary precedence matrix (2003) just representing the routing sequences for parts.

Each non-binary element $b_{ij}$ of the type I production data-based PMIM $B$ is given by

$$b_{ij} = \sum_{f=1}^{n} f_{ij} d_{i}$$

where

- $d_{i}$ = production volume of part $i$.
- $R_{ij}$ = set of operation sequence number along which part $i$ visits machine $j$.
- $n_{i}$ = total number of operations by part $i$.
- $f_{ij} = \begin{cases} 1 & \text{if } r=1 \text{ or } n_{i} \\ 2 & \text{if } 1 < r < n_{i} \\ 0 & \text{otherwise}. \end{cases}$

Each element $b_{ij}$ in the type I production data-based PMIM reflects the total amounts of moves incurred by part $i$ with production volume of $d_{i}$ by assigning one inter-cell move to the first or last operation and two inter-cell moves to an intermediate operation.

However, each element of part vectors needs to be converted into the analogue value ranging between 0 and 1 before it is presented into the Fuzzy ART neural network and hence the input vector normalization scheme suitable to feed Fuzzy ART neural network is needed. A typical approach for normalizing input vectors is to find the minimum and maximum values for each attribute of all the input vectors and linearly scale the data(Kamal & Burke 1996). To use this scheme, the whole information on the operation sequences and production volumes of all the parts must be stored in advance before they are presented to the network and this means that such a scaling scheme uses the entire PMIM at the beginning stage of applying the neural network. However, a major advantage from the application of ANN is that the entire PMIM needs not to be stored in memory from the beginning stage of algorithm since only one row is processed at a time(Kaparthi & Suresh 1992).

To avoid exploiting the whole PMIM from the beginning stage of algorithm and process only one row at a time, this study adopts a simple scheme for normalizing input patterns. The proposed scheme normalizes each element $b_{ij}$ of input pattern(part vector) $i$ with its maximum value in pattern $i$ as follows:

$$b_{ij} = \frac{\max \{ b_{ij} \}}{\max \{ b_{ij} \}}$$

2.2 Performance measure

In order to evaluate the goodness of the block diagonal solution to binary PMG, a lot of popular measures have been proposed and used to compare the effectiveness of different solution methods(Sarker & Khan 2001). But the conventional measures of effectiveness of binary PMG can not be used to evaluate the goodness of the non-binary block diagonal solution.

To evaluate the performance of PMG considering the operation sequences of part, in this paper, a simple measure of the goodness of non-binary block diagonal solution, called weighted grouping capability index(WGCI), is proposed. WGCI which is a straightforward extension of grouping capability index(GCI) (Seifoddini & Hsu 1994) does not require the calculation of the similarity coefficients between every pair of parts within part families. WGCI is defined as

$$WGCI = 1 - \frac{\text{the sum of exceptional } b_{ij} }{\text{the sum of all } b_{ij}}$$

(3)

The sum of exceptional $b_{ij}$s in a type I production data-based PMIM represents the actual flows incurred by the operations performed outside the main diagonal blocks and hence WGCI measures the proportion of the actual flows incurred by the operations performed within the main diagonal blocks. Since the sum of exceptional $b_{ij}$s includes all the actual flows incurred by the parts which have non-consecutive multiple operations on a machine, WGCI reflects both the operation sequences and production volumes of parts and is not affected by subjective weighting factor which is arbitrarily assigned by cell designer, unlike conventional performance measure such as grouping efficiency(Sarker & Khan 2001).
2.3 Algorithm

The algorithm for PMG based on type I production data-based PMIM has two major stages: clustering stage and enhancement stage. Clustering stage uses Fuzzy ART neural network to quickly cluster parts into families and then assigns machines to cells. As the row vectors are scanned, the amount of part processing of each machine by each cluster represented by the sum of $b_{ij}$s is calculated and each machine is assigned to the cluster which has the most part processing.

A reassignment procedure adopted in enhancement stage is a modification of reassignment procedure in Chen and Cheng (1995) and Won (2000) applied on the binary PMIM. The proposed reassignment procedure applied on the non-binary PMIM seeks to minimize inter-cell part moves and maximize within-cell machine utilization based on the following weighted maximum density rule which is an extension over the conventional maximum density rule:

**Weighted maximum density rule:**
- For an exceptional part $i$, find its most appropriate part family in which it undergoes the most portions of operations represented by the sum of $b_{ij}$s than any other part family and reassign it to that part family. If ties occur, select the part family in which that part undergoes the most operations. If ties occur again, select the smallest part family.
- For an exceptional machine $j$, find its most appropriate machine cell in which it processes the most portions of operations represented by the sum of $b_{ij}$s than any other machine cell and reassign it to that machine cell. If ties occur, select the machine cell in which that machine processes the most parts. If ties occur again, select the smallest machine cell.

Stopping condition of the algorithm is stated as follows:

**Stopping condition:**
- i) No empty part families exist,
- ii) no singleton part families exist, and
- iii) no parts(machines) are improperly assigned.

The whole algorithm is then described as follows:

**Clustering stage:**
- [Step 0] Use equation (2) to prepare for the input vectors.
- [Step 1] For the specified vigilance threshold $\rho$, choice parameter $\alpha$ and learning parameter $\beta$, apply Fuzzy ART algorithm to cluster parts into families.
- [Step 2] Assign machines to their most appropriate cells.

**Enhancement stage:**
- [Step 3] Apply the weighted maximum density rule to reassign improperly assigned parts and machines to their most appropriate part families and machine cells.
- [Step 4] If stopping condition is satisfied, stop. Otherwise, go to [Step 3] and repeat.

3. Psuedo-replicated clustering procedure

The effectiveness of the proposed algorithm needs to be tested on ill-structured large-size PMG problems. In order to show the robustness and recoverability of PMG algorithms to randomly generated large-size data sets, replicated clustering has often been used (Park & Suresh 2003). In replicated clustering, a known best solution is generated first and randomly reordered, and these scrambled data are presented to an algorithm. The clusters resulting from the algorithm are then compared and evaluated with the known best starting solution. In this study, a psuedo-replicated clustering procedure which is applicable as an alternative for conventional replicated clustering since the procedure starts with the near-best starting solution is proposed to generate large-size data sets including both the operation sequences with multiple visits to the same machine and the production volumes of parts.

The psuedo-replicated clustering procedure proceeds as follows:

- **Psuedo-replicated clustering:**
  - i) An appropriately intermediate-size problem is solved with PMG algorithm and identify the number of clusters and the value of WGC1.
  - ii) Assume that the incumbent solution to that problem is the best one and apply Adil et al.’s data expansion scheme (1997) to replicate row and columns of the original problem. Scramble the order of input presentation at random.
  - iii) Apply PMG algorithm to expanded problem.

4. Experimental results

4.1 Experiments with small-size problem

The proposed algorithm has been applied to the data set in Wu (1998). On this problem, the Fuzzy ART neural network with $\alpha=0.5$, $\beta=0.1$, and the vigilance threshold of 0.95 has been applied. After three iterations of enhancement stage, figure 1 shows the type I solution matrix with WGC1 equal to 94.62%.
4.2 Experiments with large-size problems
The proposed pseudo-replicated clustering procedure has been applied to the data set in Wu with various expansion levels. In our experiments, the expansion levels equal to 2, 5, and 10 have been applied. The target value of WGCI revealing the recoverability of the proposed algorithm to expanded problems is set at 94.62% under the configuration of clusters not less than 6, 15, and 30, respectively, for each expansion level. For each expansion level, 25 problems have been generated and randomly scrambled.

(Table 1) shows the experimental results. The second leftmost whole columns show the values of vigilance threshold, number of clusters, and WGCI found when the original problem is replicated with the expansion level of 2. To compare the solution quality under equal number of clusters, the algorithm has been implemented with the values of vigilance threshold decreasing by 0.01 from the starting value of 0.95 until the six-cluster solution has been found. The experimental result with pseudo-replicated clustering shows minor gap within 1% from the target WGCI value of 94.62% on the average. In 11 problem instances of 25 problem, the proposed algorithm recovers the original problem.

(Table 1) also reports the experimental results with the large-size problems replicated with the expansion levels of 5 and 10. To these large-size problems, the Fuzzy ART network with a vigilance threshold of 0.95 has been implemented to produce the solutions with the clusters not less than 15 and 30, respectively, for each expansion level. The table shows that the proposed algorithm has produced the solutions that have gap within 2% on the average from the target WGCI under the clusters more than the reference numbers on the data sets replicated with the expansion level of 10.

5. Concluding remarks
In this study, effective approach adopting Fuzzy ART neural network has been proposed to solve the non–binary PMG problem which considers real manufacturing factors such as the operation sequences with multiple visits the same machine and production volumes of the parts.

The proposed algorithm seeks to overcome the category proliferation problem that is inherent to most ANN algorithms by implementing supplementary procedure which reassigns improperly assigned parts and machines and finds good–quality solutions. New performance measure for evaluating and comparing the goodness of different non–binary block diagonal solutions has been proposed.

To show the robustness and recoverability of the proposed algorithm on large-size data sets, pseudo-replicated clustering procedure that is a variant of the conventional replicated clustering has been suggested.

The experimental results with pseudo-replicated clustering shows the robustness and recoverability of the proposed algorithm on large-size data sets within minor gap from the target value of the proposed new performance measure.

References


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* denotes the resulting problem size (no. of parts □ no. of machines).

** denotes the vigilance threshold.