상호작용하는 스톡스 유동 내 탄성체의 유한요소해석

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Finite element analysis of elastic solid/stokes flow interaction problem

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Introduction

The distributed Lagrangian multiplier/fictitious domain (DLM/FD) method is one of the ways to solve solid/fluid interaction problems. We apply the DLM/FD formulation of Yu [J. Comput. Phys. **207**(2005) 1] for the flexible body/fluid interactions to simulate the elastic solid/Stokes flow interaction problem, which is based on the formulation of Glowinski et al. [Int. J. Multiphase Flow 25 (1999) 755]. This work is done to find out proper condition of solid/fluid interaction problem that we will use later on. We choose a simple system, a bottom-attached bar in the pipe flow, so we can easily compare the results by changing sets and conditions below. Four different sets were used in the same system by changing two factors. One is Lagrangian multiplier's distribution and the other is solid/fluid boundary condition. In four sets, we checked mesh refinement with changing conditions such as solid/fluid mesh ratio, pseudo-time step scale, etc. We found the effect of each factor and optimum conditions. This result will be used to extend our study to suspended elastic particle systems in the Stokes flow.

Governing equations and numerical scheme

We define a simple system, a bottom-attached elastic bar in pressure driven pipe flow. In this system, solid is linear elastic and fluid is Newtonian. For their interactions we use Lagrangian multipliers.

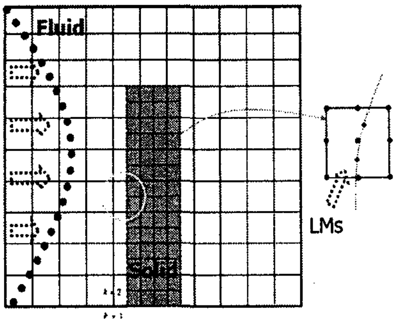

Fig. 1 Computational domain of the elastic solid/Stokes flow interaction problem

The governing equations are as shown below.

$$\text{for fluid: } \nabla \cdot \mathbf{v}_f = 0 \quad \text{on } \Omega \setminus P$$

$$\nabla \cdot \boldsymbol{\sigma}_f = 0, \quad \boldsymbol{\sigma}_f = -p\mathbf{I} + 2\eta\mathbf{D} \quad \text{on } \Omega \setminus P$$

$$\text{for solid: } \nabla \cdot \mathbf{u}_s = 0 \quad \text{on } P$$

$$\nabla \cdot \boldsymbol{\sigma}_s = 0, \quad \boldsymbol{\sigma}_s = -p_s\mathbf{I} + 2\mu\mathbf{D}(\mathbf{u}_s) \quad \text{on } P$$

$$\text{for interface: } \boldsymbol{\sigma}_s \cdot \hat{\mathbf{n}} = \boldsymbol{\sigma}_f \cdot \hat{\mathbf{n}} \quad \text{on } \partial P \text{ (on LM's)}$$

$$\mathbf{v}_f = \frac{\mathbf{u}_s}{\Delta t} \quad \text{on } \partial P \text{ (on LM's)}$$

To find out the effect of Lagrangian-multiplier's distribution and the solid/fluid boundary condition, we choose four different sets.

	LM's distribution	Interfacial condition
ElaSusI21	solid surface	velocity
ElaSusI22	solid domain	velocity
ElaSusI23	solid domain	stress / velocity
ElaSusI24	solid surface	stress / velocity

Table. 1 Four different sets of conditions

In four sets, we check on mesh refinement with changing conditions like solid/fluid mesh ratio, pseudo-time step scale, and deformation limit.

Results and discussion

All four sets show good mesh refinement and there is no big difference among them. They don't show any pseudo time step dependence either. We can verify the robustness of this simulation algorithm through both tests. Just one result of each test is shown below.

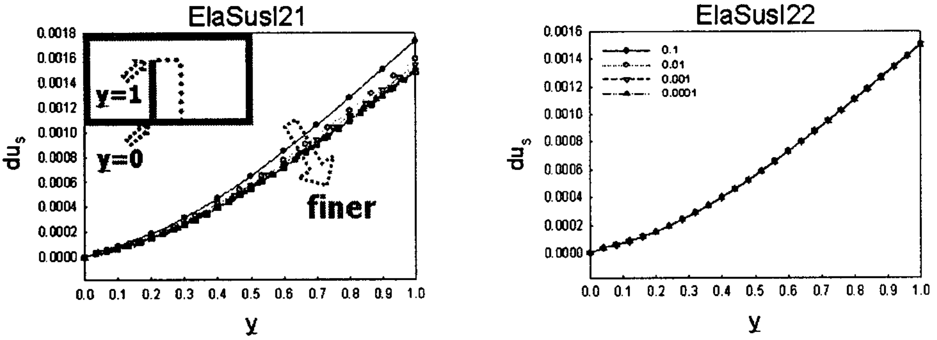

Fig. 2 mesh refinement(left) and pseudo-time step dependence test(right) results

To find out the proper solid/fluid mesh ratio, we change the solid mesh size form 2x10 to 10x50 in a 50x50 fluid mesh size. Note that we use 9-node quadrilateral element mesh for fluid and 4-node quadrilateral element for solid. When the solid mesh size is bigger than or same with fluid mesh size, there is no problem. When solid mesh size become smaller, however, there appear locking problems. We can see that too many collocation points (where Lagrangian multipliers are used) cause a locking. The deformation and strain result of one set are shown below.

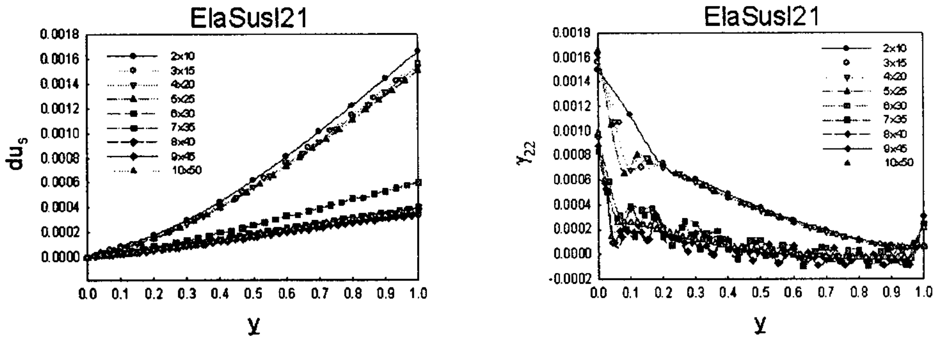

Fig. 3 solid/fluid mesh ratio test result

We check deformation limit by increasing pressure of the pipe flow. Until solid's deformation is about 10% of original bar's bottom length, they show good mesh refinement. Actually they show good mesh refinement even in larger deformation especially ElaSusI21/24 (which Lagrangian multipliers are only on solid boundary), but it's not meaningful result because of linear elasticity of the solid in governing equation.

ElaSusI22/23 (which Lagrangian multipliers are on whole solid domain) show better results of flow shear rate and velocity profile. They show very small velocity near the solid wall, and there are very small portion of fluid which go through solid boundary. We can also see very small vorticity in them. However, ElaSusI21/24 (which Lagrangian multipliers are only on solid boundary) show velocity jump on collocation points. The shear rate and velocity profile results of two sets are shown below.

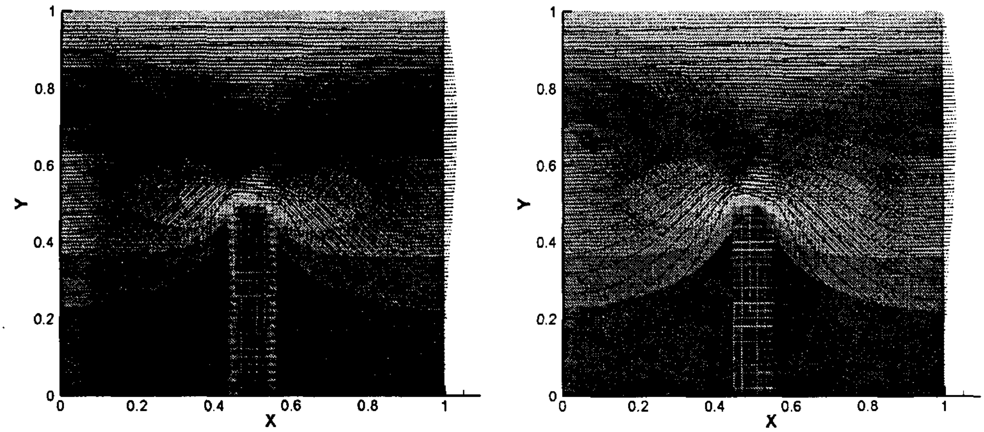

Fig. 4 The shear rate and velocity profile results of ElaSusI21(left) and ElaSusI22(right)

Through our simulation result, we find out that stress boundary condition on solid/fluid interface does not affect to the result, so we get same results between ElaSusI21/24 and between ElaSusI22/23. We may consider just a velocity boundary condition on solid/fluid interface, and it makes work easier when extending to complex problem. However, there are dependency on Lagrangian multipliers' distribution. ElaSusI22/23 show better result, which means that it is better to use Lagrangian multiplier on whole solid domain.

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