대변형 진동 유동 하에서 수직응력의 발생

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Normal stress development under large amplitude oscillatory shear flow

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Introduction

Although linear viscoelasticity is a very useful tool for understanding the relationship between the micro-structure and the rheological properties of complex fluids, it is important to recall that the theory is only valid when the deformation is either quite small or very slow. However, in most processing operations, deformation is usually both large and rapid. Thus it is necessary to investigate nonlinear viscoelastic experiment and modeling. Hyun et al. reported that the rheological behavior of complex fluids could classify by their micro-structure under large amplitude oscillatory shear flow[1]. And he also have suggested a number of methods to analyze the nonlinear behavior of complex fluids[2][3]. Especially the Fourier transformation methods to analyze experimental LAOS data has been developed by many articles[4]. Recently Cho et al. decomposed the nonlinear experimental data into elastic and viscous component clearly[5]. Large amplitude oscillatory shear (LAOS) behavior of complex fluids, which form micro-structures depending on their deformation history, has been investigated by using a network model[6]. Baek has examined various nonlinear viscoelastic constitutive equations under LAOS flow: UCM, Giesekus, FENE-P, Marrucci, Leonov and Pom-pom model[7].

Up to now, it was focused that the shear stress is not sinusoidal any more at nonlinear region and how the shear stress is distorted. However the normal stress is developed and critical to determine the nonlinear properties. The normal stress is also important because it is related with the flow instabilities. Especially Some articles have reported that these edge fracture instabilities are related with the second normal stress difference and showed that the second normal stress difference is responsible for the development of secondary flows in polymer processing[8]. Oscillatory shear is a relatively stable and easy flow to generate.

In this study, we used the nonlinear viscoelastic constitutive equations for the upper convective Maxwell model(UCM) and Giesekus model to show that how the normal stress differences is developed under large amplitude oscillatory shear flow. And we suggested the methods to analyze the normal stress differences.

Theory

The most straightforward way to combine non-linearity and time

dependent effect is to corporate into the simple Maxwell model by replacing the substantial time derivative with the upper convected time derivative.
Upper Convected Maxwell(UCM) model:

$$\boldsymbol{\tau} + \lambda \boldsymbol{\tau}_{(1)} = 2\lambda G \boldsymbol{D}$$

All bold character implies that it is tensor form and subscript (1) denotes upper convective time derivative which is defined like below.

$$\boldsymbol{\tau}_{(1)} = \frac{\partial}{\partial t}\boldsymbol{\tau} + \mathbb{v} \cdot \nabla \boldsymbol{\tau} - (\nabla \mathbb{v})^T \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \nabla \mathbb{v}$$

$\boldsymbol{\tau}$ is stress tensor, $\nabla \mathbb{v}$ is velocity gradient and $\boldsymbol{D}$ is rate of deformation tensor. The UCM model is a simplest nonlinear formula because it contains upper convective time derivative which contains products of velocity gradient and stress tensor.

The Giesekus model differs from the UCM model from the viewpoint of anisotropic property. This model is based on anisotropic drag, which means it is surrounded by other oriented dumbbells. It is started from simple dumbbell theory for dilute solutions, and this model is for concentrated solution or melts. α is an anisotropic parameter, and it ranges from 0 to 1.
Giesekus model:

$$\boldsymbol{\tau} + \lambda \boldsymbol{\tau}_{(1)} + \frac{\alpha}{G}\boldsymbol{\tau} \cdot \boldsymbol{\tau} = 2\lambda G \boldsymbol{D}$$

In small amplitude oscillatory shea(SAOS) regime, the normal stress differences predicted follows by a number of references,

$$N_1(t, w, \gamma_0) = \gamma_0^2[G''(w) - B(w)\cos(2wt) - C(w)\sin(2wt)]$$

- Bird[9] by Lodge rubber like liquid model.

$$N_1 = N_1^d + N_1'\cos 2wt - N_1''\sin 2wt$$

- Ferry[10]

Therefore, empirically the normal stress differences is presented follows

$$N_i = N_{ia} + N_{i0}(\sin(2wt + \delta_N))$$
$$= N_{ia} + N_i'\cos 2wt + N_i''\sin 2wt$$

$$N_i' = N_{i0}\sin\delta_N, \quad N_i'' = N_{i0}\cos\delta_N$$

$$\tan\delta_N = \frac{N_i'}{N_i''}$$

It is similar process to define linear viscoelasticity.

Result and discussion

First, the LAOS result of the UCM model is shown in Fig. 1. The shear stress curve is not distorted even though strain amplitude increases. The first normal stress difference is also sinusoidal pattern and the second normal stress difference is zero in UCM model. G' and G" do not change in all strain range. The UCM model does not show non-linearity and this result is what we expected.

Fig. 2 shows the process of shear stress curve distortion of the Giesekus model(α=0.25, f=1Hz, λ=G=1). All curves are normalized and shifted to show their shape easily. As the strain amplitude increases, the shear stress curve changes from sinusoidal to "sawlike" or "forward tilted" shape[2]. We could find that the non-linearity is increasing and go to equilibrium state. The non-linearity(I_3/I_1) is defined by Fourier transformation of the shear stress curve. And from fig.2, the normal stress differences oscillate double frequency for the input strain. They are also distorted as the strain amplitude increases. As a result of Fourier transformation. the non-linearity for the normal stress differences is increasing and go to equilibrium state. It contains even harmonic of fundamental strain frequency. We found following relationship in the region that the Fourier intensity increases with a power-law function in fig.3 and fig.4,

$$I_n \propto \gamma_0^{\,n} \quad n = 1, 2, 3, \cdots$$

And we are analyzing N', N". We will show the details later.

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Reference

1. Hyun K. et al., *J. Non-Newtonian Fluid Mech.* 107, 51-65 (2002)
2. Hyun K. et al., *Korea-Australia Rheology Journal,* 15(2), 97-105 (2003)
3. Hyun K. et al., *Rheologica Acta,* 46, 239-249 (2006)
4. Wilhelm M. *Macromol. Mater. Eng.* 287, 83-105 (2002)
5. Cho K. S. et al., *Journal of Rheology,* 49(3), 747-758 (2005)
6. Sim H. G. et al., *J. Non-Newtonian Fluid Mech.* 112, 237-250 (2003)
7. Baek. E. S., Master thesis, Seoul National University, (2006)
8. Zarraga I.E., *Journal of Rheology,* 44(2), 185-220 (2000)
9. Bird R. B. et al, Dynamics of polymeric liquids, Wiley Interscience, New York(1987)
10. Ferry J. D., Viscoelastic properties of polymers, John Wiley & Sons (1980)

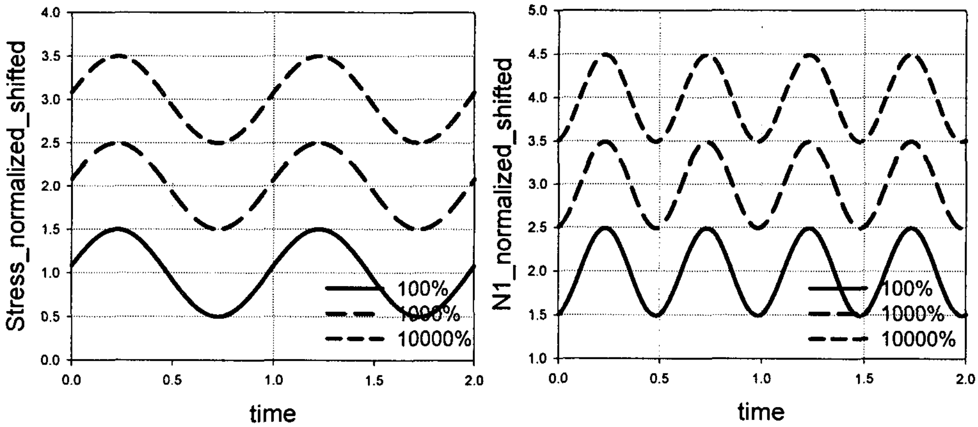

Figure 1. The shear stress curve and first normal stress difference curve of UCM model

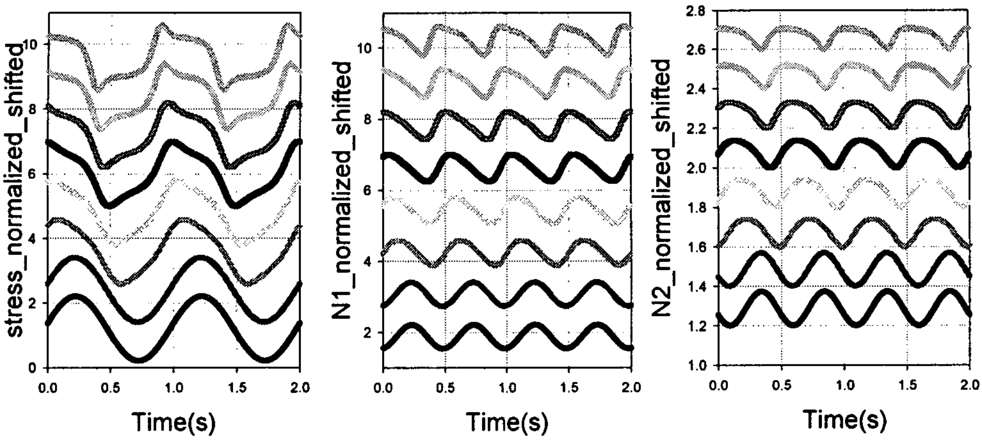

Figure 2. The shear stress curve and first & second normal stress difference curve of Giesekus model

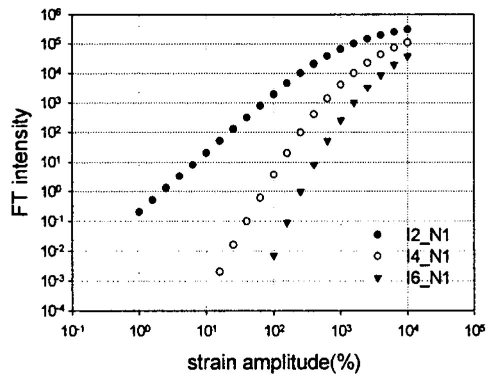

Figure 3. The result of Fourier transformation of the N1

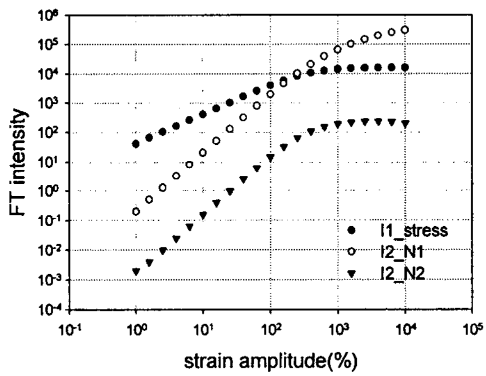

Figure 4. FT intensity of shear stress for first order harmonic and normal stress differences for second order harmonic