Parallel Property of Pressure Equation Solver with Variable Order Multigrid Method for Incompressible Turbulent Flow Simulations

Hidetoshi NISHIDA* and Toshiyuki MIYANO

Department of Mechanical and System Engineering
Kyoto Institute of Technology (KIT)
Matsugasaki, Sakyo-ku, Kyoto 606-8585, Japan
E-mail: nishida@kit.ac.jp

Key Words: Parallelized Multi Grid Method, Incompressible Turbulent Flow Simulation

ABSTRACT

The incompressible flow simulations are usually based on the incompressible Navier-Stokes equations. In the incompressible Navier-Stokes equations, we have to solve not only momentum equations but also elliptic partial differential equation (PDE) for the pressure, stream function and so on. The elliptic PDE solvers consume the large part of total computational time, because we have to obtain the converged solution of this elliptic PDE at every time step. Then, for the incompressible flow simulations, especially the large-scale simulations, the efficient elliptic PDE solver is very important key technique.

In the parallel computations, the parallel performance of elliptic PDE solver is not usually high in comparison with the momentum equation solver. When the parallel efficiency of elliptic PDE solver is 90% on 2 processor elements (PE)s, that is, the speedup based on 1PE is 1.8, the speedup on 128PEs is about 61 times of 1PE. This shows that we use only a half platform capability. On the other hand, the momentum equation solver shows almost theoretical speedup [1,2]. Therefore, it is very urgent problem to improve the parallel efficiency of elliptic PDE solver.

In this paper, the parallel property of elliptic PDE solver, i.e., the pressure equation solver, with variable order multigrid method [3] is presented. Also, the improvement of parallel efficiency is proposed. The present elliptic PDE solver is applied to the direct numerical simulation (DNS) of 3D turbulent channel flows. The message passing interface (MPI) library is applied to make the computational codes. These MPI codes are implemented on PRIME POWER system with SPARC 64V (1.3GHz) processors at Japan Atomic Energy Research Institute (JAERI).

The incompressible Navier-Stokes equations in the Cartesian coordinates can be written by

$$\frac{\partial u_i}{\partial x_i} = 0 \ , \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \ , \tag{2}$$

where u_i (i=1,2,3) denotes the velocity, p the pressure and ν the kinematic viscosity. The pressure equation can be formally written by

$$\frac{\partial^2 p}{\partial x_i \partial x_i} = \nabla^2 p = f \ , \tag{3}$$

where f is the source term.

The pressure equation (3) is solved by the variable order multigrid method with Neumann

boundary conditions. As the relaxation scheme, the checkerboard SOR method and RRK scheme [4] are adopted. In the RRK scheme, the unsteady term in pseudo-time is added to Eq.(3).

$$\frac{\partial p}{\partial \tau} = \nabla^2 p - f. \tag{4}$$

For the parabolic PDE in pseudo-time, Eq.(4), the RRK scheme can be written by

$$\begin{aligned} p^{m+1} &= p^m + \frac{2\mathbf{g}_1(\mathbf{g}_1,\mathbf{g}_3) - \mathbf{g}_3(\mathbf{g}_1,\mathbf{g}_1)}{(\mathbf{g}_3,\mathbf{g}_3)}, \\ \mathbf{g}_1 &= \Delta\tau \mathbf{Q}(p^m), \quad \mathbf{g}_2 = \Delta\tau \mathbf{Q}(p^m + c_2 \mathbf{g}_1), \\ \mathbf{g}_3 &= b_1\mathbf{g}_1 + b_2\mathbf{g}_2, \quad \mathbf{Q}(p) = \nabla^2 p - f, \end{aligned} \tag{5}$$

where **Q** denotes the spatial discretization operator, $\Delta\tau$ is the pseudo-time step and the operators such as ($\mathbf{g}_1$,$\mathbf{g}_3$) denote inner product of vectors $\mathbf{g}_1$ and $\mathbf{g}_3$. The coefficients b_1, b_2, and c_2 satisfy the relations $b_1+b_2=1$, $c_2=-1/2$.

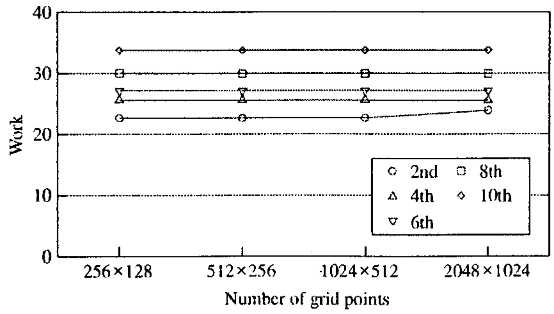
First, the 2D case with the source term $f=-5cos(x)cos(2y)$ and the Neumann boundary conditions $\partial p/\partial x_i=0$ is considered. The work unit until convergence is shown in Fig. 1. In both relaxation schemes, the multigrid convergence can be obtained and the RRK scheme has the independent property of spatial accuracy. Figure 2 shows the parallel efficiency defined by

$$Efficiency = \frac{T_{single}}{N \cdot T_{paralel}} \times 100\,(\%) , \tag{6}$$

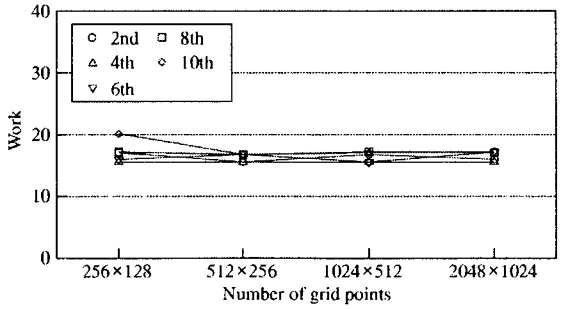
where N denotes number of PEs, T_{single} and $T_{parallel}$ are the CPU time on single PE and NPEs, respectively. In Fig.2, the RRK scheme shows the higher efficiency than the checkerboard SOR method. In order to improve parallel efficiency, we consider the restriction of PE on coarser multigrid level. Figure 3 shows the parallel efficiency with the restriction of PE on coarser than 64x32 grid level. In Fig.3, version 1 and version 2 denote the parallel efficiency without and with restriction of PE on coarser multigrid level. It is clear that the parallel efficiency can be improved.

Next, the DNS of 3D turbulent channel flows is performed by the variable order method of lines [2]. The order of spatial accuracy is the 2nd order and the number of grid points are 32x64x32 and 64x64x64 in the x, y and z directions. The numerical results with Reynolds number Re_τ=150 are compared with the reference database of Kasagi *et al.* [5]. Figures 4-6 show the mean streamwise velocity, velocity fluctuation, Reynolds shear stress profiles, respectively. The present DNS results are in very good agreement with the reference spectral solution.

Table 1 shows the parallel efficiency of 2nd order multigrid method in the DNS of 3D turbulent channel flow. As the number of grid points is larger, the parallel efficiency becomes higher. In comparison with the checkerboard SOR method, the RRK scheme has the higher parallel efficiency in version 1 and version 2. Also, the multigrid method with version 2 shows higher parallel efficiency.



(a) Checkerboard SOR method



(b) RRK scheme

Fig. 1 Multigrid convergence for 2D problem.

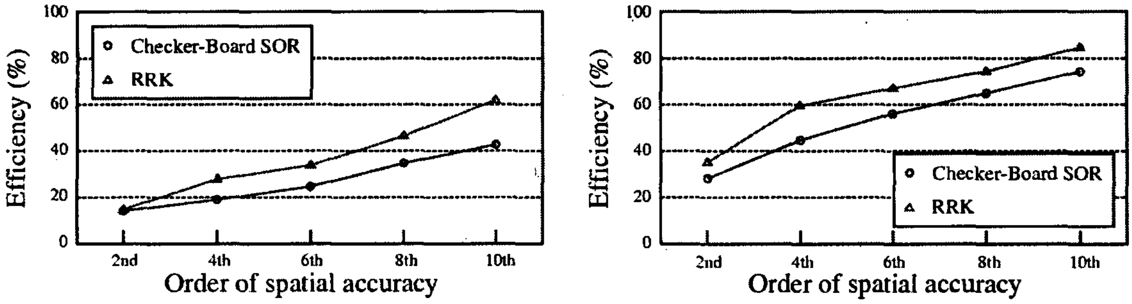


Fig. 2 Parallel efficiency for 2D problem.

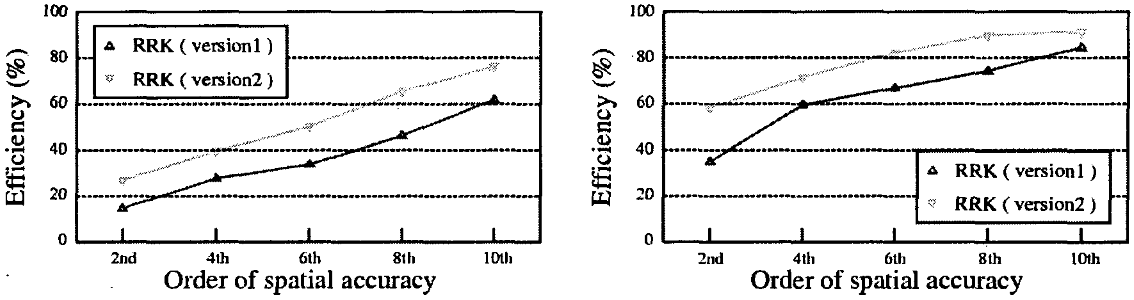


Fig. 3 Improvement of parallel efficiency.

Then, it is concluded that the present variable order multigrid method with RRK scheme has the high parallel efficiency and the present parallel approach is very hopeful to simulate the large-scale incompressible turbulent flows.

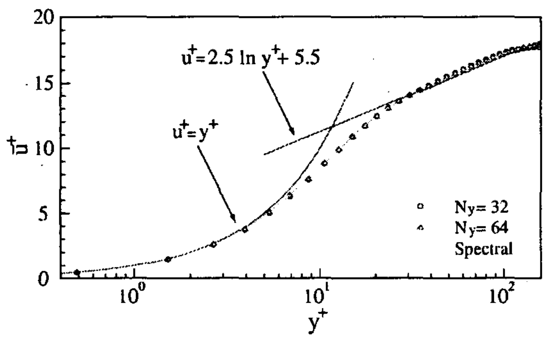


Fig. 4 Mean streamwise velocity.

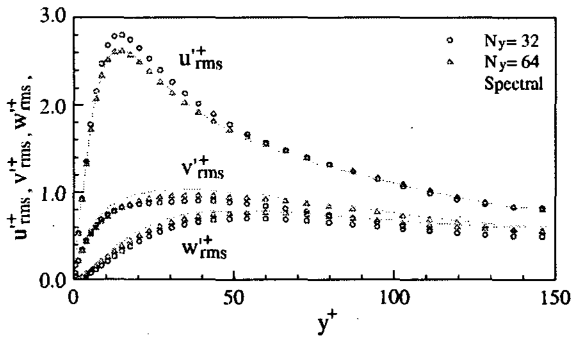


Fig. 5 Velocity fluctuation.

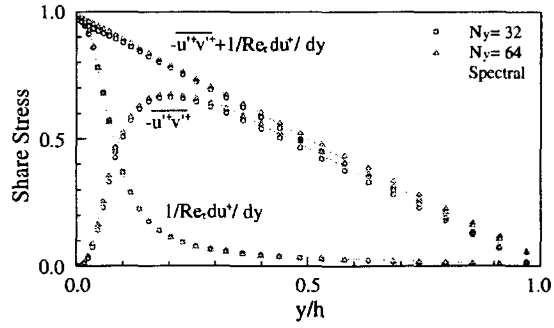


Fig. 6 Reynolds shear stress.

Table 1 Parallel efficiency for 3D turbulent channel flow.

	Checkerboard SOR method		RRK scheme	
32×64×32	Time(sec./step)	Efficiency(%)	Time(sec./step)	Efficiency(%)
Single	0.711	-	1.113	-
Parallel (version 1)	0.147	60.459	0.207	67.210
Parallel (version 2)	0.143	62.150	0.188	74.003
64×64×64	Time(sec./step)	Efficiency(%)	Time(sec./step)	Efficiency(%)
Single	3.711	-	7.731	-
Parallel (version 1)	0.628	73.865	1.242	77.812
Parallel (version 2)	0.582	79.704	1.120	86.307

REFERENCES

[1] H. Nishida, H. Yoshioka and M. Hatta (2005). "Higher Order Parallel DNS of Incompressible Turbulence Using Compressible Navier-Stokes Equations", Parallel Computational Fluid Dynamics - Multidisciplinary Applications (Elsevier), pp.121-128.

[2] H. Nishida, S. Nakai and N. Satofuka, (2003). "Parallel Efficiency of a Variable Order Method of Lines", Parallel Computational Fluid Dynamics - New Frontiers and Multi-Disciplinary Applications (Elsevier), pp.321-328.

[3] H. Nishida and N. Satofuka, (1987). "Automatic Higher-Order Multi Grid Method and Its Application", Memoirs of the Faculty of Engineering and Design, Kyoto Institute of Technology, 36, pp.24-35.

[4] A. Wambecq, (1978). "Rational Runge-Kutta Method for Solving System of Ordinary Differential Equations", Computing, 20, pp.333-342.

[5] N. Kasagi, Y. Tomita and A. Kuroda, (1992), "Direct Numerical Simulation of Passive Scalar Field in a Turbulent Channel Flow", Trans. ASME, J. Heat Transf., 114, pp.598-606.