strict-feedback 비선형 시스템의 출력궤환 적응 신경망 제어기

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Adaptive Output-feedback Neural Control for Strict-feedback Nonlinear Systems

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ABSTRACT

An adaptive output-feedback neural control problem of SISO strict-feedback nonlinear system is considered in this paper. The main contribution of the proposed method is that it is shown that the output-feedback control of the strict-feedback system can be viewed as that of the system in the normal form. As a result, proposed output-feedback control algorithm is much simpler than the previous backstepping-based controllers. Depending heavily on the universal approximation property of the neural network (NN) only one NN is employed to approximate lumped uncertain nonlinearity in the controlled system.

1. Introduction

In order to cope with a nonlinear system with nonlinearly parameterized or unstructured uncertainties, control approaches using universal approximation properties of fuzzy logic system (FLS) and neural network (NN) have been extensively studied [1-8]. Beside, backstepping contorl scheme has been a powerful method for synthesizing adaptive controller for the class of nonlnlinear systems with linearly parameterized uncertainty. Recently, to broaden the class of nonlinear systems that can be dealt with, some researchers have been tried to combine adaptive backstepping scheme with the approximator-based contollers. In [2, 10-16], several adaptive backstepping approaches for strict- and pure-feedback nonlinear systems based on universal approximators have been proposed. In those algorithms, adaptive backstepping design provides a systematic method for the design of controller for the system of the form:

$$\begin{aligned} x_i &= f_i(\mathrm{x}_i)+g_i(\mathrm{x}_i)x_{i+1},\ i=1,\cdots,n-1 \\ x_n &= f_n(\mathrm{x}_n)+g_n(\mathrm{x}_n)u \\ y &= x_1 \end{aligned} \tag{1}$$

where $\mathrm{x}_i=[x_1,\cdots,x_n]^T\in R^i$, $i=1,\cdots,n$ and $u\in R$ and $y\in R$ are the state vector and the system input and output, respectively; $f_i(\cdot)$ and $g_i(\cdot)$, $i=1,\cdots,n$ are unknown smooth functions.

However, there are some problems in the previous adaptive approximator-based controllers based on backstepping design method. First of all, some very tedious and complex analysis is needed to determine regression matrices, virtual controls and their derivatives. The complexity is inherited to the approximator-based controller. Moreover, the complexity grows in geometrical progression as the order of the controlled system increases. For the practical implementation, this complexity must be avoided. Another problem is that, since the time-derivatives of the virtual control term are also unavailable, they must be the part of the inputs to the approximators. This results in the severe increments of the dimensions of the approximators.

In this paper, we propose an adaptive output-feedback neural controller for (1) which is not based on backstepping scheme. To the author's knowledge, there is no results available in the literature to control the uncertain strict-feedback system (1) whose output is the only measurable variable. The key point of the proposed method is that the state-feedback control problem of the strict-feedback system is viewed as the output-feedback one of the nonlinear system in the normal form. Based on this fact, it is shown that controller design and stability analysis is much simpler than the previous backstepping-based algorithms. Only one radial-basis function network (RBFN) with $n+1$ input variables is employed to approximate unknown lumped nonlinearity, which sets the simplicity of our proposed control scheme off very well.

2. Problem Formulation

In this paper, we consider the system (1) with the measurable state vector $\mathrm{x}=\mathrm{x}_n$ With regard to controllability, the following assumption must be made.

Assumption 1 : The signs of the g_is are all known. Without loss of generality, we assume that $g_i(\mathrm{x}_i)>0$ for all $\mathrm{x}_i \in R^i$, $i=1,\cdots,n$.

The control objectives are that output y tracks the desired output y_d and all the signals involved are bounded.

By induction, if we define $\alpha_1=f_1$ and $\beta_1=g_1$, the following is satisfied for $i=2,\cdots,n$

$$\begin{aligned} z_i &\equiv \alpha_{i-1}(\mathrm{x}_{i-1})+\beta_{i-1}(\mathrm{x}_{i-1})x_i \\ \dot{z}_i &= \alpha_i(\mathrm{x}_i)+\beta_i(\mathrm{x}_i)x_{i+1} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \alpha_i(\mathrm{x}_i) &\equiv \sum_{j=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_j}+\frac{\partial \beta_{i-1}}{\partial x_j}x_i\right)(f_j+g_jx_{j+1}) \\ &\quad +b_{i-1}f_i \\ \beta_i(\mathrm{x}_i) &\equiv \beta_{i-1}g_i=\prod_{j=1}^{i}g_j. \end{aligned} \tag{3}$$

As a result, the strict-feedback system (1) can be redescribed as the following normal form with respect to the newly defined state variables z_is:

$$\begin{aligned} \dot{z}_i &= z_{i+1},\ i=1,\cdots,n-1 \\ \dot{z}_n &= \alpha_n(\mathrm{x})+\beta_n(\mathrm{x})u \\ y &= z_1 \end{aligned} \tag{4}$$

Also, it is easy to show that there exist a vector function $\mathbf{r}(\mathbf{z})=\mathbf{x}$ where

$$\mathbf{r}(\mathbf{z})=[z_1\ \ r_2(\mathbf{z}_2)\ \ r_3(\mathbf{z}_3)\cdots r_n(\mathbf{z})]^T \tag{5}$$

Substituting this into (4) yields

$$\begin{aligned} \dot{z}_i &= z_{i+1},\ i=1,\cdots,n-1 \\ \dot{z}_n &= \alpha_n(\mathbf{r}(\mathbf{x}))+\beta_n(\mathbf{r}(\mathbf{x}))u \\ &\equiv a(\mathbf{z})+b(\mathbf{z})u \\ y &= z_1 \end{aligned} \tag{6}$$

It should be noted that apart form the fact that functions $a(\cdot)$ and $b(\cdot)$ are functions of $\mathbf{x}$ they are totally unknown. From assumption 1, it is also noted that a constant $b_0>0$ exists such that $b(\mathrm{x})\geq b_0$, $\forall\, \mathrm{x}\in R^n$. This assumption poses a controllable condition on the system (4).

3. Controller Design

3.1 Higher-order observer and lumped uncertanty

As can be observed in (4) and (6), the z_is are incomputable since the a_is and b_is are unknown functions. Thus, we employ a high-gain observer (HGO) to estimate z_i, $i=2,\cdots,n$ as the following lemma.

Lemma 1: Suppose the function $y(t)$ and its first $n-1$ derivatives are bounded. Consider the following linear system:

$$\begin{aligned} \varepsilon\dot{\xi}_1 &= \xi_2 \\ \varepsilon\dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \varepsilon\dot{\xi}_n &= -d_1\xi_n-d_2\xi_{n-1}-\cdots-d_{n-1}\xi_2-\xi_1+y(t) \end{aligned} \tag{7}$$

where ε is a small design constant and parameters, d_1 to d_{n-1} are chosen such that the polynomial $s^n+d_1s^{n-1}+\cdots+d_{n-1}s+1$ is Hurwitz. Then, there exist positive constants h and t^* such that $\forall\, t>t^*$ we have

$$\begin{aligned} &|\hat{\mathrm{z}}-\mathrm{z}|\leq \varepsilon h \\ &\hat{\mathrm{z}}=\left[z_1,\frac{\xi_2}{\varepsilon},\frac{\xi_3}{\varepsilon^2},\cdots,\frac{\xi_n}{\varepsilon^{n-1}}\right]^T. \end{aligned} \tag{8}$$

The proof of Lemma 1 can be found in [12].

The vector y_d e and a filtered tracking error s are then defined as follows:

$$\begin{aligned} \mathrm{y}_d &= [y_d,\ \dot{y}_d,\cdots,y_d^{(n-1)}]^T \\ \mathrm{e} &= \mathrm{z}-\mathrm{y}_d \\ s &= \left(\frac{d}{dt}+\lambda\right)^{n-1}e=[\Lambda^T\ \ 1]\mathrm{e} \\ e &= y-y_d=z_1-y_d \end{aligned} \tag{9}$$

where $\Lambda=[\lambda^{n-1},(n-1)\lambda^{n-2},\cdots,(n-1)\lambda]^T$ with $\lambda>0$. The estimations of e and s using (9) are denoted as follows:

$$\begin{aligned} \hat{\mathrm{e}} &= \hat{\mathrm{z}}-\mathrm{y}_d \\ \hat{s} &= [\Lambda^T\ \ 1]\hat{\mathrm{e}}. \end{aligned} \tag{10}$$

Lemma 2 : Considering that (1) satisfies Assumption 1, if the ideal control with HGO (7) is designed as

$$u^* = -k\hat{s}-u_{ad}^*(\mathbf{z},\hat{v}) \tag{11}$$

$$u_{ad}^*(\mathbf{z},\hat{v}) = \frac{a(\mathbf{z})+\hat{v}}{b(\mathbf{z})} \tag{12}$$

where $k>0$ is a design constant, then the filtered tracking error s is uniformly ultimately bounded.

In the proposed control scheme, only one RBFN is employed to estimate the following unknown function (12). The input vector to the RBFN is denoted by $\mathbf{x}_{in}=[\mathbf{z}\ \ \hat{v}]^T$ that will be replaced by the estimated vector $\hat{\mathbf{x}}_{in}=[\hat{\mathbf{z}}\ \ \hat{v}]^T\in R^{n+1}$ later.

3.2 Brief description of RBFN

In this paper, one RBFN is employed to capture the unknown nonlinearity (10) of the system. In general, the output of the multi-input single-output RBFN is described by

$$\hat{u}_{ad}(\mathrm{x}_{in}) = \mathrm{w}^T \Phi(\mathrm{x}_{in}). \tag{13}$$

Here, $\mathrm{x}_{in}=[\mathrm{x}^T \ \hat{v}]^T \in R^{n+1}$ is the input vector to the RBFN; $\hat{u}_{ad} \in R$ the RBFN output; $\mathrm{w} \in R^L$, the adjustable parameter vector; $\Phi(\cdot): R^{n+1} \to R^L$, a nonlinear vector function of the inputs; L the number of RBFs. The ith element of w w_i, $i=1,\cdots,L$, is the synaptic weight between the ith neuron in the hidden layer, and output neuron and $\Phi_i(\mathrm{x}_{in})$ is a Gaussian function in the form of

$$\Phi_i(\mathrm{x}_{in}) = \exp\left(-\frac{|\mathrm{x}_{in} - \mathrm{m}_i|}{2\sigma_i^2}\right) \tag{14}$$

where m_i is a $(n+1)$-dimensional vector representing the center of the ith basis function, and σ_i is the variance representing the spread of the basis function. The primary advantage of RBFN is that it has the capability to approximate nonlinear mappings to any degree of accuracy.

3.3 Control and adaptive laws and stability analysis

Substituting the unavailable u_{ad}^* into $\hat{u}_{ad}$ in (13), we determine the control input as follows:

$$u = -k\hat{s} - \hat{\mathrm{w}}^T \Phi(\hat{\mathrm{x}}_{in}). \tag{15}$$

The adaptive law for the $\hat{\mathrm{w}}$ is chosen as the following lemma.

Lemma 3 : The update law for $\hat{\mathrm{w}}$ is determined as

$$\dot{\hat{\mathrm{w}}} = \gamma(\hat{s}\Phi(\hat{\mathrm{x}}_{in}) - \sigma_s(\hat{\mathrm{w}})|\hat{s}|\hat{\mathrm{w}}) \tag{16}$$

where γ is the positive learning rate and

$$\sigma_s(\hat{\mathrm{w}}) = \begin{cases} \frac{c_\Phi}{\varepsilon_w} & \text{if } |\hat{\mathrm{w}}| > \varepsilon \\ 0 & otherwise \end{cases} \tag{17}$$

with ε_w being a design constant and $|\Phi| \le c_\Phi$. Then, $|\hat{\mathrm{w}}| \le \varepsilon_w$.

Theorem 1 : Consider the adaptive system comprising (1) under assumption 1, controller (15) with HGO (7) and adaptive law (16). The filtered error s is semi-globally uniformly ultimately bounded.

The proof is omitted.

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References

[1] M. Kristic, I. Kanellakopoulos, and P. Kokotovic, Nonlinear and Adaptive Control Design, A Wiley-Interscience publication, 1995.

[2] J.-H. Park and G.-T. Park, "Robust adaptive fuzzy controller for nonaffine nonlinear systems with dynamic rule activation,'' \emph{Int. J. Robust and Nonliner Control," vol. 13, no. 2, pp. 117-139, 2003.

[3] J.-H. Park, S.-H Huh, S.-H. Kim, S.-J. Seo, G.-T. Park, "Direct adaptive controller for nonaffine nonlinear systems using self-structuring neural networks," IEEE Trans. Neural Networks, vol. 16, no. 2, pp. 414-422, 2005.

[4] S. S. Ge and C. C. Hang and T. Zhang, "Adaptive neural network control of nonlinear systems by state and output feedback," IEEE Trans. Systems, Man and Cybernetics-Part B:Cybernetics, vol. 29, no. 6, pp. 818-828, 1999.

[5] M. U. Polycarpou and M. J. Mears,"Stable adaptive tracking of uncertain systems using nonliearly parameterized on-line approximators," Int. J. Control, vol. 70, no. 3, pp. 363-384, 1998.

[6] Y. Li, S. Qiang, X. Zhuang, O. Kaynak,"Robust and adptive backstepping control for nonlinaer systems using rbf neural networks," IEEE Trans. Neural Networks, vol. 15, no. 3, pp. 693-7001, 2004.

[7] D. Wang, J. Huang, "Adaptive neural network control for a class of uncertain nonlinear systems in pure-feedback form," Automatica, vol. 38, pp. 1365-1372, 2002.

[8] S. S. Ge, C. Wang,"Direct adaptive nn control of a class of nonlinear systems," IEEE Trans. Neural Networks, vol. 13, no. 1, pp. 214-221, 2002.

[9] S. S. Ge, C. Wang,"Adaptive NN control of uncertain nonlinear pure-feedback systems," Automatica, vol. 38, pp. 671-682, 2002.

[10] J. Q. Gong, B. Yao,"Neural network adaptive robust control of nonlinear systems in semi-strict feedback form," Automatica, vol. 37, pp. 1149-1160, 2001.

[11] D. M. Dawson, J. J. Carroll, and M. Schneider,"Integrator backstepping control of a brush dc motor turning a rogotic load," IEEE Trans. System, Man, and Cybernetics - Part B:Cybernetaics, vol. 2, pp. 233-244, 1994.

[12] S. Behatsh, "Robust output tracking for nonlinear systems," Int. J. Control, vol. 51, no. 6, pp. 1381-1407, 1990.