순궤환 비선형 시스템의 적응 신경망 제어기

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Adaptive Neural Control for Pure-feedback Nonlinear Systems

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ABSTRACT

Adaptive neural state-feedback controllers for the fully nonaffine pure-feedback nonlinear system are presented in this paper. By reformulating the original pure-feedback system to a standard normal form with respect to newly defined state variables, the proposed controllers require no backstepping design procedures. Avoiding backstepping makes the controller structure and stability analysis considerably to be simplified. The proposed controllers employ only one neural network to approximate unknown ideal controllers, which highlights the simplicity of the proposed neural controller.

1. Introduction

The following fully nonaffine nonlinear pure-feedback system is considered in this paper.

$$\begin{aligned} x_i &= f_i(\mathbf{x}_i,\ x_{i+1}), i=1,\cdots,n-1 \\ x_n &= f_n(\mathbf{x},\ u) \\ y &= x_1 \end{aligned} \quad (1)$$

where $\mathbf{x}_i=[x_{1,}\cdots,x_n]^T\in R^i$, $i=1,\cdots,n$ and $u\in R$ and $y\in R$ are the state vector and the system input and output, respectively; $f_i(\cdot)$s, $i=1,\cdots,n$ are unknown smooth functions. Apart from the fact that f_is are smooth functions of $\mathbf{x}_{i+1}$, it should be noted that they are *totally unknown*. Thus, it is assumed that there exists no information on whether or not they contain useful nonlinearities for stability that do not require to be cancelled. By employing an NN approximator, the controlled system (1) needs no linear-in-the-parameters condition, i.e., functions f_i may not be linearly parameterized.

It should be noted that only few results have been published on the adaptive approximator-based control of this cloass of nonlinear system [1-3]. Although the same class of system is considered in [3] its control algorithm is based on adaptive backstepping algorithmthat has a heavy computational burden as mentioned earlier. The proposed adaptive neural controller requires no backstepping, which makes the control law and stability analysis considerably simple. Only one radial-basis function network (RBFN) with $n+2$ inputs is employed to approximate unknown ideal controllers, which highlights the simplicity of the proposed controller.

2. Problem Formulation

For the controllability issue, the following assumption must be made.

Assumption 1: The absolute values of $\partial f_i/\partial x_{i+1}$, $i=1,\cdots,n-1$ and $\partial f_n/\partial u$ are nonzero and bounded, that is, the following inequalities hold:

$$\begin{aligned} &0<\left|\frac{\partial f_i(\mathbf{x}_i,x_{i+1})}{\partial x_{i+1}}\right|<\infty, i=1,\cdots,n-1 \\ &0<\left|\frac{\partial f_n(\mathbf{x}_n,u)}{u}\right|<\infty, i=1,\cdots,n-1 \end{aligned} \quad (2)$$

Without loss of generality, instead of requiring respective partial derivatives are positive as in [23-25], it is assumed that the product of them is positive and bounded. That is, the following inequality holds

$$0<\left(\Pi_{i=1}^{n-1}\frac{\partial f_i(\mathbf{x}_i,x_{i+1})}{\partial x_{i+1}}\right)\frac{\partial f_n(\mathbf{x},u)}{u}<\infty \quad (3)$$

for all $(\mathbf{x},u)\in R^{n+1}$. Only with this mild assumption, the controller is designed in what follows. The aim of this paper is to design an adaptive neural controller for nonaffine pure-feedback nonlinear SISO systems under plant uncertainties which guarantees uniform ultimate boundedness of all the estimated variables of the closed-loop system and the output tracking of a given reference output $y_d(t)$.

By induction, if we define $a_1=0$ and $b_1=f_1$, the following is satisfied for $i=2,\cdots,n$

$$z_i \equiv a_{i-1}(x_{i-1})+b_{i-1}(x_i) \qquad (4)$$
$$\dot{z}_i = a_i(x_i)+b_i(x_{i+1})$$

where

$$a_i(x_i)=\sum_{j=1}^{i-1}\frac{\partial}{\partial x_j}(a_{i-1}(x_{i-1})+b_{i-1}(x_i))f_j(x_{j+1})$$
$$b_i(x_i)=\frac{\partial b_{i-1}(x_i)}{\partial x_i}\dot{x}_i \qquad (5)$$
$$=\left(\Pi_{j=1}^{i-1}\frac{\partial f_j(x_{j+1})}{\partial x_{j+1}}\right)f_i(x_{i+1})$$

with $x_{n+1}=[x_n\ u]^T$. As a result, the original pure-feedback system can be redescribed as the following normal form with respect to the newly defined state variables z_is:

$$\dot{z}_i = z_{i+1},\ i=1,\cdots,n-1$$
$$\dot{z}_n = a_n(x)+b_n(x,u) \qquad (6)$$
$$y = z_1$$

The above description gives the insight that the state-feedback control problem of the original pure-feedback system can be viewed as that of the standard normal system, which make the controller to avoid backstepping design procedure.

3. Controller Design

3.1 Higher-order observer and lumped uncertanty

The vector y_d, e, and a filtered tracking error s are then defined as follows:

$$y_d = [y_d, \dot{y}_d, \cdots, y_d^{(n-1)}]^T$$
$$e = z - y_d$$
$$s=\left(\frac{d}{dt}+\lambda\right)^{n-1}e=[\Lambda^T\ \ 1]e \qquad (7)$$
$$e = y - y_d = z_1 - y_d$$

where $\Lambda=[\lambda^{n-1},(n-1)\lambda^{n-2},\cdots,(n-1)\lambda]^T$ with $\lambda>0$.

Lemma 1 : If the assumption 1 holds, there exists a unique function $u^*(x,v,s)$ that satisfies the following equality:

$$a_n(x)+b_n(x,u^*(x,v,s))+v=-ks \qquad (8)$$

where $k>0$ is a input gain at designer's disposal.

From lemma 1, it is straightforward to show that Lyaounov function $L_s = s^2/2$ is asymptotically stable for this ideal controller. In this paper, we do not try to approximate the ideal controller that satisfies eq. directly. Instead, the u^* is rearranged as

$$u^*=-ks-(-ks-u^*(h)) \qquad (9)$$
$$@-ks-u_{ad}^*(h)$$

and one RBFN with its input being η is employed to approximate unknown function $u_{ad}^*(\eta)$. It should be noted that since the s and v are unavailable, they will be substituted by estimated values later. For the time being, the input vector to the RBFN is assumed to be η.

3.2 Brief description of RBFN

In this paper, one RBFN is employed to capture the unknown nonlinearity (10) of the system. In general, the output of the multi-input single-output RBFN is described by

$$\hat{u}_{ad}(x_{in})=w^T\Phi(x_{in}). \qquad (10)$$

Here, $x_{in}=[x^T\ \ \hat{v}]^T\in R^{n+1}$ is the input vector to the RBFN; $\hat{u}_{ad}\in R$, the RBFN output; $w\in R^L$, the adjustable parameter vector; $\Phi(\cdot): R^{n+1}\to R^L$, a nonlinear vector function of the inputs; L, the number of RBFs. The ith element of w, $w_i, i=1,\cdots,L$, is the synaptic weight between the ith neuron in the hidden layer, and output neuron and $\Phi_i(x_{in})$ is a Gaussian function in the form of

$$\Phi_i(x_{in})=\exp\left(-\frac{|x_{in}-m_i|}{2\sigma_i^2}\right) \qquad (11)$$

where m_i is a $(n+1)$-dimensional vector representing the center of the ith basis function, and σ_i is the variance representing the spread of the basis function. The primary advantage of RBFN is that it has the capability to approximate nonlinear mappings to any degree of accuracy.

3.2 Introduction of high-gain observer and filtered tracking error

For the actual control action, the time-derivatives of the system output y is required. However, only the state variables of are assumed to be available, the following high-gain observer (HGO) is adopted:

$$\epsilon\dot{\xi}_1 = \xi_2$$
$$\epsilon\dot{\xi}_2 = \xi_3$$
$$\vdots \qquad (12)$$
$$\epsilon\dot{\xi}_n = -d_1\xi_n - d_2\xi_{n-1} - \cdots - d_{n-1}\xi_2 - \xi_1 + y(t)$$

where ϵ is a small design constant and parameters, d_1 to d_{n-1} are chosen such that the polynomial $s^n+d_1s^{n-1}+\cdots+d_{n-1}s+1$ is Hurwitz. Then, there exist positive constants h and t^* such that $\forall t>t^*$ we have

$$|\hat{z}-z| \le \epsilon h$$
$$\hat{z}= \left[z_1, \frac{\xi_2}{\epsilon}, \frac{\xi_3}{\epsilon^2}, \cdots, \frac{\xi_n}{\epsilon^{n-1}}\right]^T. \tag{13}$$

The estimations of e and s using (5) are denoted as follows:

$$\hat{e} = \hat{z} - y_d$$
$$\hat{s} = [\Lambda^T \ \ 1]\hat{e}. \tag{14}$$

Using these definitions, the actual inputs to the RBFN is determined as $\hat{\eta} = [x^T \ \hat{v} \ \hat{s}]^T$ where $\hat{v} = -y_d^{(n)} + [0 \ \Lambda^T]\hat{e}$.

3.4 Control and adaptive laws

The actual control input is proposed as:

$$u = -k\hat{s} - \hat{w}^T\Phi(\hat{\eta}). \tag{15}$$

The adaptive law for the $\hat{w}$ is chosen as the following lemma.

Lemma 2 : The update law for $\hat{w}$ is determined as

$$\dot{\hat{w}} = \gamma\left(\hat{s}\Phi(\hat{\eta}) - \sigma_s(\hat{w})|\hat{s}|\hat{w}\right) \tag{16}$$

where γ is the positive learning rate and

$$\sigma_s(\hat{w}) = \begin{cases} \frac{c_\Phi}{\epsilon_w} & \text{if} \ \ |\hat{w}| > \epsilon_w \\ 0 & otherwise. \end{cases} \tag{17}$$

with ϵ_w being a design constant and $|\Phi| \le c_\Phi$. Then, $|\hat{w}| \le \epsilon_w$.

The proof of Lemma 2 is omitted.

Theorem 1 : Consider the adaptive system consisting of the plant under assumption 1, controller (15), and adaptive law for RBFN weight vector (16). Then, the filterted trancking error s is uniformaly ultametely bounded.

The proof is omitted.

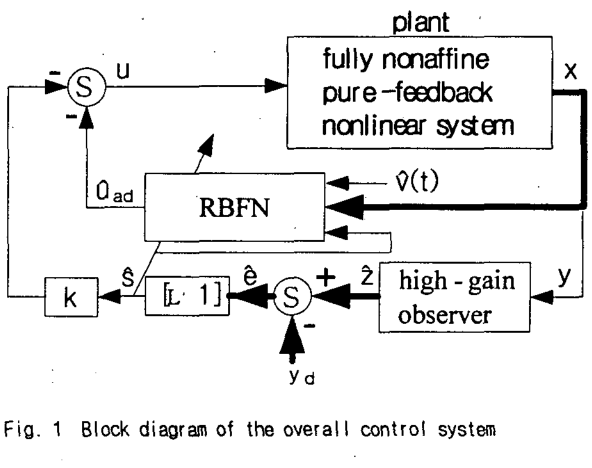

Fig. 1 Block diagram of the overall control system

3. Conclusions

Adaptive neural controllers for SISO fully nonaffine pure-feedback nonlinear system are presented in this paper. By reformulating the original pure-feedback system to a standard normal form with respect to the output and its $n-1$ time derivatives, the proposed controller requires no backstepping design procedure. That is, it is shown that the original control problemof the considered system can be reformulated as the output-feedback control of standard normal system.Avoiding backstepping makes the controller structure and stability analysis considerably simplified. The proposed controllers employ only one neural network to approximate unknown ideal controllers, which highlights the simplicity of the proposedneural controllers.

This work was financially supported by MOCIE through the EIRC program.

References

[1] S. S. Ge, C. Wang, "Direct adaptive NN control of a class of nonlinear systems," IEEE Trans. Neural Networks, vol. 13, no. 1, pp. 214-221, 2002.

[2] J. Q. Gong, B. Yao,"Neural network adaptive robust control of nonlinear systems in semi-strict feedback form," Automatica, vol. 37, pp. 1149-1160, 2001.

[3] Y. Yang, G. Geng, J. Ren, "A combined backstepping and small-gain approach to robust adaptive fuzzy control for strict-feedback nonlinear systems,'" IEEE Trans. System, Man, and Cybernetics - Part A, vol. 34, no. 3, pp. 406-420, 2004.