구조물의 동특성 개선을 위한 모드 매개변수의 민감도 해석

The Sensitivity Analysis for Structure Modification using Partial Differentiation

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Key Words : Sensitivity Analysis(민감도해석), Modal parameter(모드매개변수), Eigenvalue(고유값), Crankshaft(크랭크샤프트)

ABSTRACT

This study predicts the modified structure of eigenvectors and eigenvalues due to the changes in the mass and the stiffness of the structure. The sensitivity method of natural frequency using partial differential are derived with respect to the physical parameter to calculate the structure modification. The method are applied to the 3 degrees of freedom뵐lumped mass model by modeling the mass and stiffness, and then applies the method to a real crankshaft system. The position, direction of parameter change and modified value were predicted for modification. Finally the predicted value is used to investigate the magnitude of vibration and we found that the effect of modification results to reduce the level of magnitude vibration is satisfactory.

1. Introduction

Recently, improvement in design structure is performed through experiment and using the accumulative experienced of engineers. Therefore, it is wasteful in consuming time, expenses and resources. Especially, when considering the trend towards reduced weight in manufactured products, it is important to understand well the dynamic characteristics of the product precisely before production commences. One of the steps is the sensitivity analysis which requires a numerical skill for the optimum design. Also, the sensitivity analysis may predicts in advance the influence of some effects of the total structure system due to the modification of the design. For instance, Nelson(1) presented direct method of sensitivity analysis and is found splendidly practical in use by many researchers. Mills-Curran(2) had performed the possibility of iterative eigenvalue application but in practical use, this method had a problem. Rudisill(3) determined the changes rates for the first eigenvalue with eigenvector and the changes rates for the second eigenvalue. Fox and Kapoor(4) illustrated linear combination for non-modified structure amount changes of an eigenvector and determining the sensitivity coefficient from the normalized mass and orthogonal condition and calculated the changes rates for the first eigenvector. Wang(5) applied mode summation technique of accelerated mode to the sensitivity analysis and improved the estimation accuracy. Min, Hyun-Gi et. al.(6) used direct differential method and studied the kinematics automobile suspension system with the sensitivity analysis.

The sensitivity analysis explained above are widely used by most of the researchers; however none of them considered the sensitivity analysis to determine the position, and the modified value but in analysis it was found as a key problem. Once the modification changes, the structure correspondently change the natural frequency and the results is not the optimum value. For instance, if we intend to change the original structure but with a less modified points and the value which the changes of the dynamic parameters can achieve the desired value, what should we do?

To overcome such problems, this study propose a sensitivity analysis method to determine the most sensitivity point, direction of parameter change and modified value for modification. In this study, the sensitivity method of natural frequency using partial differential are derived with respect to physical parameter to calculate the structure modification. The method is applied to the 3 degrees of freedom뵐lumped mass model by modeling the mass and stiffness and applies the method to a real crankshaft system. The position, direction of parameter change and modified value is predicted for modification crankshaft system and the result is used to investigate the level of vibration magnitude.

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2. Theory

2.1 Dynamic Characteristics of structure changes

The equation of motion in a general vibration system with harmonic excitation is expressed as follows

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\} \quad (1)$$

where,
$[M]$ is a mass matrix,
$[C]$ is a damping matrix,
$[K]$ is a stiffness matrix,
$\{F(t)\}$ is a force vector,
$\{x\}, \{\dot{x}\}, \{\ddot{x}\}$ is displacement, velocity and acceleration.

When the structure changes, the mass, damping and stiffness change correspondently. The equation of changing structure can be written as

$$[M+\Delta M]\{\ddot{x}\}+[C+\Delta C]\{\dot{x}\}+[K+\Delta K]\{x\}=\{F(t)\} \quad (2)$$

here, ΔM is the mass increment value, ΔC is the damping increment value and ΔK is the stiffness increment value.

Taking the Laplace transforms for equation (2) we find

$$\left(s^2[M+\Delta M]+s[C+\Delta C]+[K+\Delta K]\right)\{X(s)\}=\{F(s)\} \quad (3)$$

Considering, the linear transformation $\{X(s)\}=\{Y(s)\}[\phi]$ and premultiplication both side by $[\phi]^T$, then

$$\left(s^2[M^m+\Delta M^m]+s[C^m+\Delta C^m]+[K^m+\Delta K^m]\right)\{Y(s)\} = [\phi]^T\{F(s)\} \quad (4)$$

where,

$$[\Delta M^m]=[\phi]^T[\Delta M][\phi]$$
$$[\Delta C^m]=[\phi]^T[\Delta C][\phi]$$
$$[\Delta K^m]=[\phi]^T[\Delta K][\phi]$$

A linear relationship exists in the equation (4) and

$$\{Y(s)\}=[\phi^*]\{Z(s)\} \quad (5)$$

where,

$$[\phi^*]^T[M^m+\Delta M^m][\phi^*]=[\overline{M}^m]$$
$$[\phi^*]^T[C^m+\Delta C^m][\phi^*]=[\overline{C}^m]$$
$$[\phi^*]^T[K^m+\Delta K^m][\phi^*]=[\overline{K}^m]$$

From equation (3) and (5) we get

$$\left(s^2[\overline{M}^m]+s[\overline{C}^m]+[\overline{K}^m]\right)\{Z(s)\}=[\phi^*]^T[\phi]^T\{F(s)\}$$

or (6)

$$\left(s^2[\overline{M}^m]+s[\overline{C}^m]+[\overline{K}^m]\right)\{Z(s)\}=[\overline{\phi}]^T\{F(s)\}$$

Here, $[\overline{M}^m]$, $[\overline{C}^m]$ and $[\overline{K}^m]$ are diagonal matrices.

Equation (6) is a new modal model. The new mode shape matrix of a new structure as

$$[\overline{\phi}]=\left[[\phi^*]^T[\phi]^T\right]^T=[\phi\phi^*] \quad (7)$$

Modal parameters $[M^m]$, $[C^m]$, $[K^m]$ and $[\phi]$ can be obtain from the modal analysis of the original structure, then build a new modal model after modification. A linear transformation by new structure modal matrix

$$\{X(s)\}=[\overline{\phi}]\{Z(s)\} \quad (8)$$

Substituting (6) into (8), yields

$$H_{pq}(s)=\frac{X_q(s)}{F_p(s)} = \sum_r^N \frac{\left([\overline{\phi}]_p[\overline{\phi}]_q\right)_r}{\left(s^2[\overline{M}^m]_r+s[\overline{C}^m]_r+[\overline{K}^m]_r\right)} \quad (9)$$

where,
$H_{pq}(s)$: frequency response between point p and q.

2.2 Sensitivity of natural frequency

The relationship between natural frequency and modal mass M^m, modal stiffness K^m is

$$[\omega_n^2][M^m]=[K^m] \quad (10)$$

where ω_n^2, M^m and K^m are diagonal, and

$$[M^m]=\{\phi\}^T[M]\{\phi\}$$
$$[K^m]=\{\phi\}^T[K]\{\phi\} \quad (11)$$

The sensitivity of natural frequency ω_n^2 with respect to structure parameters P_m is the partial differential of ω_n^2 with respect to P_m.

2.3 Sensitivity of natural frequency with respect to mass

From equation (10) we know

$$\frac{\partial[\omega_n]}{\partial M_i}=\frac{1}{2}[\omega_n]^{-1}[K^m]\frac{\partial[M^m]^{-1}}{\partial M_i} \quad (12)$$

From equation (11)

$$\frac{\partial[M^m]}{\partial M_i} = [\phi]^T \frac{\partial[M]}{\partial M_i}[\phi] = [\phi_{ni}^2] \qquad (13)$$

Where $[\phi_{ni}^2]$ is diagonal matrix composed of mode of the point i .

The inverse matrix differentiations are

$$\frac{\partial[M^m]^{-1}}{\partial M_i} = -[M^m]^{-1}\frac{\partial[M^m]}{\partial M_i}[M^m]^{-1}$$

or

$$\frac{\partial f_n}{\partial M_i} = -\frac{1}{4\pi}\omega_n\phi_{ni}^2 \qquad (14)$$

The sensitivity of i^{th} natural frequency with respect to mass of i^{th} point is related with the ω_n and i^{th} mode only. Therefore, the mode accuracy of modified point is important.

2.4 Sensitivity of natural frequency with respect to stiffness

From equation (10) and equation (11)

$$\frac{\partial[\omega_n]}{\partial K_{ij}} = \frac{1}{2}[\omega_n]^{-1}\frac{\partial[K^m]}{\partial K_{ij}}[M^m]^{-1} = \frac{1}{2}[\omega_n]^{-1}[\phi]^T\frac{\partial[K]}{\partial K_{ij}}[\phi][M^m]^{-1} \qquad (15)$$

For $i=j$ matrix

$$\frac{\partial K}{\partial K_{ij}} = \begin{bmatrix} \ddots & & \\ & 1 & \\ & & \ddots \end{bmatrix} \qquad (16)$$

Substitute (16) to (15)

$$\frac{\partial K}{\partial K_{ij}} = \frac{1}{2\omega_n}[\phi_{ni}^2] \qquad (17)$$

2.5 Frequency response sensitivity

We know that

$$[H(j\omega)] = [Z(j\omega)]^{-1}$$
$$[Z(j\omega)] = (j\omega)^2[M] + j\omega[C] + [K] \qquad (18)$$

here, $[H(j\omega)]$ is the frequency response matrix and $[Z(j\omega)]$ is the impedance matrix. Therefore the frequency response sensitivity with in terms of partial differential equation or with respect to physical parameters is $\frac{\partial[H(j\omega)]}{\partial P_m}$.

From equation (18), we obtained as follows

$$\frac{\partial[H(j\omega)]}{\partial P_m} = -[H]\frac{\partial[Z]}{\partial P_m}[H] \qquad (19)$$

3. Application

Fig. 1 is a simple model of 3 DOFs lumped mass used for the application of the above theory.

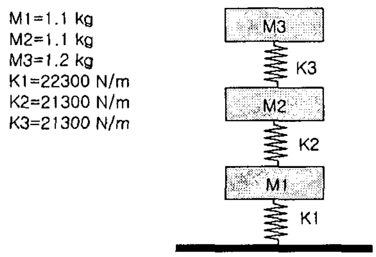

Fig. 1 Simple 3DOFs modeling

Five cases were investigated. Case 1, the mass at point 1 was added with 0.398kg, case 2 the mass at point 2 was added with 0.398kg, case 3 the stiffness was added at point 3 with 1.1N/m, case 4 the mass are added at point 3 with 0.398 kg and 0.191 kg respectively and for case 5 the stiffness was added at point 2 with 1.79 N/m. The sensitivity of natural frequencies was determined at each cases.

Fig. 2 shows the model of a crankshaft system to be used in the proposed method. The process of determining the natural frequencies sensitivity is performed as in previous procedures.

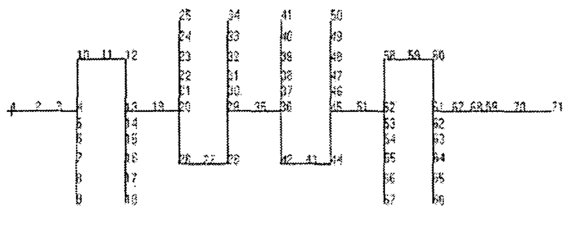

Fig. 2 Crankshaft modeling

Case 1, an additive mass was added to point 35 with 0.5kg. Case 2, a 1 kg mass was added to point 1. Case 3 a 1 kg mass was added to point 71. Case 4 and 5 a 1.5 kg mass and 2x1010N/m was added to point 71 respectively. The increments of masses and stiffness were added vertically in y-axis. The selected point in each case at the position manner is to gain the instructive effects of flywheel, middle뭘journal bearing and pulley at point 1, 35 and 71 respectively. Also, a mass sensitivity curve is calculated to identify the quantity value and the position of the mass modification. Finally the results are satisfied through the magnitude of the frequency response.

4. Results and discussions

4.1 Sensitivity of natural frequency

Table 1 shows the results obtained from the simulation analysis. The table shows that the effects of the parameter changes on different mode are not the same. For example if the mass is increase, the natural frequencies value of the all modes decreases but if the stiffness is increase there is no changes to the natural frequency. All cases show that the natural frequencies are not identical each other when the structure has been modified. Note also the system is much more sensitive to a change in the mass.

Table 1 Comparisons of natural frequency for simple 3DOFs modal model

Cases	Mode	Original	Modified	sensitivity
Case 1 (M1+0.398 kg)	1	9.731	9.567	-0.008
	2	25.030	25.030	-0.855
	3	39.841	38.070	-0.449
Case 2 (M2+0.398 kg)	1	9.731	9.196	-0.202
	2	25.030	26.754	-0.263
	3	39.841	37.060	-0.188
Case 3 (K3+1.1 N/m)	1	9.731	9.3164	0.0038
	2	25.030	26.789	0.0007
	3	39.841	39. 603	0.0001
Case 4 (M3+0.191 kg)	1	9.731	8.316	-0.331
	2	25.030	26.789	-0.472
	3	39.841	39.603	-0.164
Case 5 (K2+1.79 N/m)	1	9.731	9.317	0.0006
	2	25.030	26.790	0.0014
	3	39.841	39.613	0.0007

Table 2 shows the sensitivity for the first five of natural frequencies obtained results for crankshaft system. The experiment data set were obtained from the modal testing by dividing 32 points on the crankshaft and exciting the crankshaft with an impact hammer at impact points and measuring the responses at measuring points, by means of the FFT analyzer.

As being obtained in the above simple 3DOFs modal model it was confirmed that the increasing of mass, shows that the natural frequencies mode increases and all cases show the natural frequencies are not identical each other once the structure is modified, and the sensitivity for the both cases.

Also, it was found that the errors original and the modified natural frequencies respect to the experimental results were increased as the mass increases. To contrast the effect of the additive mass, each cases of errors results were shown in the type of bar chart as in the Fig. 3. Errors are obviously increased for the first three modes and these modes are identified as prevailing bending in-plane and out-plane modes in the modal analysis.

Table 2 Comparison Comparisons of natural frequency for crankshaft system

Cases	Mode	Ori.	Mod.	Exp.	Sensitivity
Case 1 (M35 + 0.5 kg)	1	312.8	314.7	335	-2.84e-6
	2	364.3	317.9	460	-0.001
	3	725.5	700.1	750	-1.127e-6
	4	824.4	784.5	805	-5.676e-6
	5	1007.1	988.4	1080	-3.576e-4
Case 2 (M1 + 1 kg)	1	312.8	301.3	335	-0.002
	2	364.3	314.7	460	-2.855e-9
	3	725.5	672.5	750	-0.003
	4	824.4	774.4	805	-8.246e-4
	5	1007.1	977.4	1080	-0.001
Case 3 (M71 + 1 kg)	1	312.8	280.6	335	-0.004
	2	364.3	314.7	460	-0.005
	3	725.5	620.3	750	-0.007
	4	824.4	769.9	805	-5.653e-4
	5	1007.1	968.1	1080	-0.001
Case 4 (M71 + 1.5 kg)	1	312.8	268.7	335	-0.003
	2	364.3	314.7	460	-2.018e-7
	3	725.5	603.5	750	-0.004
	4	824.4	768.7	805	-2.899e-4
	5	1007.1	964.7	1080	-8.304e-4
Case 5 (K71+2x10^{10}N/m)	1	312.8	312.9	335	1.061e-7
	2	364.3	364.2	460	1.600e-37
	3	725.5	718.9	750	6.828e-8
	4	824.4	824.5	805	7.9251e-9
	5	1007.1	1007.1	1080	8.882e-36

4.2 Mass curve sensitivity

Fig. 4 shows the mass at point 35 and mass at point 71 sensitivity curves. In the figure, the x-axis is represented as an increment of the mass while y-axis is presented as a peak value of the frequency response. Here we found that, there are two intersection points of curve sensitivity.

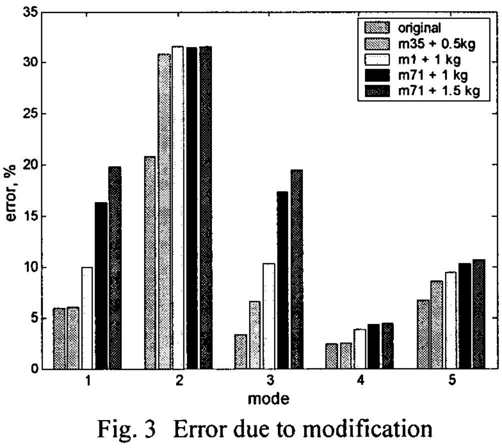

Fig. 3 Error due to modification

The first intersection point is located in between 550 gram to 600 gram and the second one is located in between 950 gram to 1000 gram. Since, the additive mass could not be simply added at any location, so the minimum possible mass is the better. Therefore the first intersection point (567 gram) is the best value to be selected and point 71 is to be preferred. The point 71 is preferred since it was the most effected on the sensitivity curve and the mass sensitivity curve is rapidly decreasing in peak level as the mass increases in the intersection of point mass range.

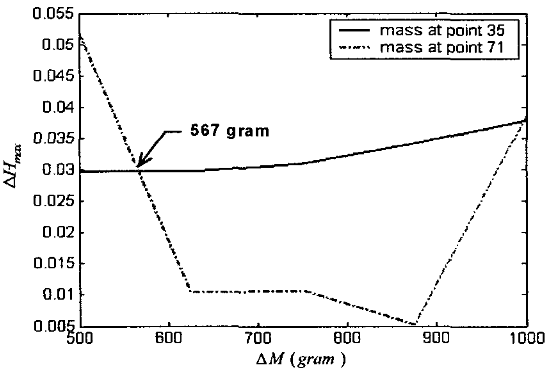

Fig. 4 M35 and M7 sensitivity curves

4.3 Investigation of the amplitude level

Once the quantity value has been identified, the value is used to check the effects of the amplitude vibrations before and after modification of crankshaft structure. When we modify M35=M71=567 gram, the original and the modified curve of frequency response are shown in the Fig. 5 and Fig. 6. It was confirmed that the peak values for both in-plane and out-plane of frequency responses were reduced. This also means that the selected point and the quantity value are suitable, correctable and available to be determined in the proposed method. At in-plane mode, not only the amplitude level reduces but shifting the peaks to a low frequency domain. The phenomena was not happened to the out-plane mode because the response is measured when the additive mass is added vertically in y-axis. However, if the additive mass is added in z-axis instead of y-axis and measure the response, definitely the same phenomena will be obtained as in-plane mode. In contrast, we found the proposed method was not only given an interactive knowledge but also we had learned the dynamic behaviors of the system itself.

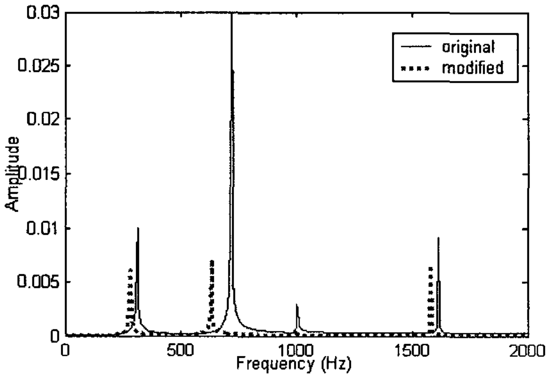

Fig. 5 Comparison of frequency response in in-plane mode

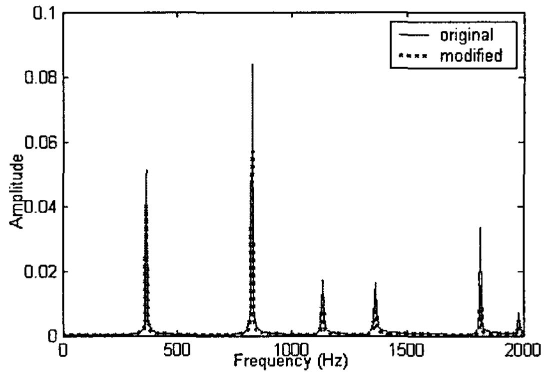

Fig. 6 Comparison of frequency response in out-plane mode

5. Conclusions

Partial differential sensitivity analysis using a newly proposed 3 degree of freedom lump퉫 mass and crankshaft model has been presented and the conclusions can be drawn as follows;

1) The application of modal model is good for structure modification and also a useful technique which can be applied for the complex structure.
2) The accuracy of the mode shape determine the accuracy of whole process whereby the accuracy of parameter is the key identification of direction and quantity value.
3) The proposed technique is sufficient and convenient to use in personal computer for reduction of the vibration level.

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