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A New Approach of State Estimation based on Particle Filter

파티클 필터에 기반한 새로운 상태 예측 방법

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Abstract

A particle filter is one of the most famous filters.The reason why the particle filter is widely used is that particle filter deals with the state estimation problem for not only linear models with Gaussian noise but also the non-linear models with non-Gaussian noise and it receives great attention from many engineering fields.In the point of view state estimator, particle filter is feedforward observer. According to the characteristic of dynamic system, the feedforward observer can estimate real state. However, the speed of convergence of feedforward observer between the actual state and the estimated state cannot be satisfied. Since the particle filter is a sort of feedforward observer, the convergence speed of particle filter is slow, and the particle filter cannot estimate actual state like particle collapse problem.

In order to overcome the limitation of particle filter as a kind of feedforward estimator, we propose a new particle filter which has feedback term, called particle filter with feedback. Our proposed method is analyzed theoretically and studied by computer simulation. Comparisons are made with other filtering mehod.

KEYWORD ***particle filter, feedback estimator, state observer,***

1. INTRODUCTION

To estimate the latent variables of the dynamics, several filtering methods have been reported, for example, Kalman filter (KF) [1] and grid based filter [2]. In these filters, the posterior density probability was assumed to be Gaussian. Unfortunaly, however in many real problems the posterior density is not Gaussian but is multimodal and its performance is not as good as expected.

These days, many researches have developedthe nonlinear filtering methods such as extended Kalman filter (EKF) and the approximation grid based filter [2] to overcome this problem. One of the alternative methods is the particle filter [2,3]. In the particle filter, any assumption on the functional form of the posterior is not made. Instead, the posterior probability density is approximated as a set of particles. When the particles are properly placed, weighted and propagated, posteriors can be estimated sequentially over time. The density of particles represents the probability of posterior function. By using some finitely many particles, we can estimate almost any kind of system dynamics; even nonlinear system with non-Gaussian, or multimodal distributions.

However, since the particle filter is a kind of feedforward state estimator, it has some limitation. The speed of convergence is not satisfied us, and the particles cannot converge around actual state, so called particle collapse.

In order to overcome these drawbacks, we propose a new particle filter, called the particle filter with feedback (PFF). As feedback concept is added in standard particle filter, the performance of particle filter can be improved. Theproposed filter is analyzed theoretically and studied by computer simulation. Comparisons are made with the Kalman filter, and standard particle filter.

The organization of this paper is as follows; Section 2 discusses the basic filters, especially Bayes filter, and the particle filter, and section 3 shows algorithm of particle filter with feedback. Simulation results are presented, which is to estimate the tracking problem in Section 4. Finally, conclusion remarks are given in Section 5.

2. PARTICLE FILTER

Particle filters are a sample based variant of Bayes filters, which represent the belief $Bel(x_k)$ by a set of S_k of N weighted samples distributed according to $Bel(x_k)$

$$\text{Set of Particle at } k^{th} \text{ time } (S_k) = \{(x_k^m, w_k^m) \mid m = 1, \ldots, N\}$$

Here, each x_k^m is a state, and the w_k^m is nonnegative numerical factors called importance weights. The basic form of the particle filter realizes the recursive Bayes filter according to a sampling procedure, often reffered to as sequential importance with resampling (SIR). A time update of the basic particle filter algorithm is outlined in Table 1.

The particle filter algorithm consists of three steps: sampling, calculation of the importance weight and the resampling. In the sampling step, samples are generated according to $p(x_t \mid u_t, x_{t-1}^m)$. In the next step, the importance weight is computed for each particle. In the third step, the samples of the sampling step are chosen again with replacement according to theirimportance weights. In resampling step, the particles which have large weight have large possibility to select.

Algorithm Standard PF
$[\{x_k^m, w_k^m\}_{m=1}^N] = \mathrm{PF}[\{x_{k-1}^m, w_{k-1}^m\}_{m=1}^N, z_k]$
For $m = 1:N$
Draw $x_k^m \sim p(x_k \mid x_{k-1}^m, u_k)$
Assign the particle a weight w_k^m according to $p(z_k \mid x_k^m)$
Resampling x_k^m according to w_k^m
End

Table 1. Stanadard Particle Filter

3. PARTICLE FILTER WITH FEEDBACK

The particle filter deals with a particle filter deals with the state estimation problem for not only linear models with Gaussian noise but also the non-linear models with non-Gaussian noise.

However, the particle filter has some fatal drawbacks, since the standard particle filter is the feedforward filter, as we mention Chapter 1. The feedforward filter would converge actual state slowly, so particles go so far as to be drawn are drawn properly, and this situation may lead to "particle collapse," where all particles occupy the same point in the state space, giving a poor representation of the posterior density. "Particle collapse" may lead to miss actual state, so we cannot estimate actual state.

In order to overcome the drawback of particle filter, we add feedback observer concept into standard particle filter, PFF.In this chapter, we assume that the system which we want to estimate is linear system. But, in the case of tracking problem, we can know only measurement data z_k. The measurement data z_k contain not only actual state information, x_k, but also process noise, w_k, and measurement noise, w_k.

So, whole system equation is expressed by below.

$$x_k = Fx_{k-1} + Gw_k \quad (1.1)$$

$$z_k = Hx_k + v_k \quad (1.2)$$

The goal of state estimation problem is to estimate using noisy measurement data z_k. In order to improve the estimation performance, we apply to linear system theory into standard particle filter. Contrary to the sampling stage of standard particle filter case, the sampling stage of proposed method can be expressed by equation (2),

$$x_k^m = Fx_{k-1}^m + L(z_{k-1} - Hx_{k-1}^m) \quad (2.1)$$

$$\hat{z}_k = Hx_k^m \quad (2.2)$$

where x_k^m is m^{th} particle at k^{th} time and F,G,z_k is same as equation (1) and L is observer gain matrix.

Next, Subtracting equation (7) from (6), we can get

$$x_k - x_k^m = F(x_{k-1} - x_{k-1}^m) - L(z_{k-1} - Hx_{k-1}^m) + Gw_k \quad (8.1)$$

$$z_k - \widehat{z_k} = H(x_k - x_k^m) + v_k \quad (8.2)$$

Substituting the equation (8.2) into the state equation (8.1), we obtain the state equation for the error between the estimated state vector and the actual state vector:

$$(x_k - x_k^m) = (F - LH)(x_{k-1} - x_{k-1}^m) + Gw_k - Lv_k \quad (9)$$

Let $e_k^m = (x_k - x_k^m)$, we have

$$e_k^m = (F - LH)e_k^m - Gw_k - Lv_k \quad (10)$$

The noise values, Gw_k and Lv_k are boundeda specific sum of range. So, by digital control theory, if the absolute values of the all eigenvalues of matrix $F-LH$ are smaller than 1, the estimated state vector error, e_k^m, will decay to very small value. So using this sampling strategy, we can estimate state variables more accurately.

The algorithm of proposed method is shown in Table (2).

Algorithm PFF
$[\{x_k^m, w_k^m\}_{m=1}^N] = \text{PFF}[\{x_{k-1}^m, w_{k-1}^m\}_{m=1}^N, z_k, z_{k-1}]$
For $m = 1:N$
Sampling x_k^m using equation (7.1)
Assign the particle a weight w_k^m according to $p(z_k \mid x_k^m)$
Resampling x_k^m according to w_k^m
End

Table 2.
Particle Filter with Feedback

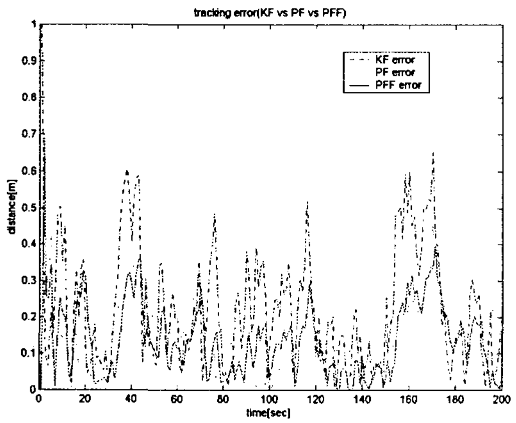
4. EXPERIMENTAL RESULT

The PFF is found to provide better filtering performance on various dynamic systems compared with the standard particle filter, and Kalman filter. Here we show the simulation results in a standard dynamic system. In this simulation, the target moves with in a one-dimensional plane. Its trajectories are generated according to below dynamic model.

$$x_k = Fx_{k-1} + Gw_k \qquad (11.1)$$

$$z_k = Hx_k + v_k \qquad (11.2)$$

where $x_k = [x \ \ \dot{x} \ \ \ddot{x}]_k^T$, $F = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$, $G = \begin{bmatrix} \frac{T^2}{2} & T & 1 \end{bmatrix}^T$, and $H = [1 \ \ 0 \ \ 0]$. w_k is a zero-mean Gaussian process with $N(0, 0.1^2)$ v_k is a zero-mean Gaussian measurement noise with $N(0, 1^2)$, and T is radar scan time. The goal of this simulation is to estimate the state of the preceding target from the sensor measurement $z(k)$. Shown in Figs 1 are the estimation error of the preceding target. We use 500 samples in each filter.



<Fig 1 . Absolute error, Kalman filter (dash-dot line),conventional particle filter (dot line) and proposed particle filter (solid line) >

As you can see in figure 1, the proposed particle filter, PFF, shows better performance than standard particle filter and Kalman filter. The estimated error of proposed method is smaller than other methods of all state.

5. CONCLUSION

In this paper, we have developed a new state estimation method called PFF. The PFF outperforms the particle filter by overcoming the convergence speed of particle filter and "particle collapse." The algorithm was applied to simulated data and we can get more improved tracking performance compared with the conventional PF and Kalman filter

However, since PFF is motivated in standard PF, computational complexity can cause limitation of application of PFF. Futureworks should be done towards decreasing the extra computational burden while maintaining the estimation performance.

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