비국소조건을 가지는 준선형퍼지적분미분방정식에 대한 제어가능성 모델링과 퍼지 제어에 관한 연구

Controllability for the Semilinear Fuzzy Integrodifferential Equations with Nonlocal Conditions

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Abstract

In this paper. we study the controllability for the semilinear fuzzy integrodifferential control system with nonlocal condition in E_N by using the concept of fuzzy number whose values are normal, convex, upper semicontinuous and compactly supported interval in E_N.

Key Words : controllability, fuzzy number, integrodifferential equation, nonlocal

1. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva ([3]) studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala ([6]) proved the existence and uniqueness of fuzzy solution for the following equation:

$$\dot{x}(t) = f(t, x(t)),\ x(0) = x_0,$$

where f is a continuous mapping from $R^+ \times R$ into R and x_0 is a fuzzy number in E^1. Diamond and Kloeden ([2]) proved the fuzzy optimal control for the following system:

$$\dot{x}(t) = a(t)x(t) + u(t),\ x(0) = x_0$$

where $x(\cdot), u(\cdot)$ are nonempty compact interval-valued functions on E^1. Kwun and Park ([4]) proved the existence of fuzzy optimal control for the nonlinear fuzzy differential system with nonlocal initial condition in E_N^1 using by Kuhn-Tucker theorems. Recently, Balasubramaniam and Muralisankar([1]) proved the existence and uniqueness of fuzzy solutions for the following semilinear fuzzy integrodifferential equation($u(t) = 0$) with nonlocal initial condition:

$$\frac{dx(t)}{dt} = A\left[x(t) + \int_0^t G(t-s)x(s)ds\right] + f(t,x) + u(t),\ t \in I = [0, T], \quad (1)$$

$$x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)) = x_0 \in E_N, \quad (2)$$

where $A : I \rightarrow E_N$ is a fuzzy coefficient, E_N is the set of all upper semicontinuous convex normal fuzzy numbers with bounded

α-level intervals, $f: I \times E_N \to E_N$ is a nonlinear continuous function, $G(t)$ is $n \times n$ continuous matrix such that $\frac{dG(t)x}{dt}$ is continuous for $x \in E_N$ and $t \in I$ with $\| G(t) \| \le k$, $k > 0$, $u: I \to E_N$ is control function and $g: I^p \times E_N \to E_N$ is a nonlinear continuous function. In the place of $\cdot$ we can replace the elements of the set $\{t_1, t_2, \cdots, t_p\}$, $0 < t_1 < t_2 < \cdots < t_p \le T$, $p \in N$, the set of all natural numbers.

In this paper, we find the sufficient conditions of controllability for the control system (1)-(2).

2. Preliminaries

A fuzzy number a in real line R is a fuzzy set characterized by a membership function m_a as $m_a : R \to [0,1]$. A fuzzy number a is expressed as

$$a = \int_{x \in R} m_a(x)/x \,,$$

with the understanding that $m_a(x) \in [0,1]$ represent the grade of membership of x in a and $\int$ denotes the union of $m_a(x)/x$'s.([5])

Let E_N be the set of all upper semicontinuous convex normal fuzzy number with bounded α-level intervals. This means that if $a \in E_N$ then the α-level set

$[a]^\alpha = \{x \in R \mid m_a(x) \ge \alpha,\ 0 < \alpha \le 1\}$

is a closed bounded interval which we denote by

$$[a]^\alpha = [a_l^\alpha,\ a_r^\alpha]$$

and there exists a $t_0 \in R$ such that $a(t_0) = 1$.([4])

The support Γ_a of a fuzzy number a is defined, as a special case of level set, by the following

$$\Gamma_a = \{x \in R \mid m_a(x) > 0\}.$$

Two fuzzy numbers a and b are called equal $a = b$, if $m_a(x) = m_b(x)$ for all $x \in R$. It follows that

$$a = b \leftrightarrow [a]^\alpha = [b]^\alpha \text{ for all } \alpha \in (0,1].$$

We denote the suprimum metric d_∞ on E^n and the suprimum metric H_1 on $C(I : E^n)$.

Definition 1. Let $a, b \in E^n$.

$d_\infty(a, b) = \sup\{d_H([a]^\alpha, [b]^\alpha) : \alpha \in (0,1]\}$

where d_H is the Hausdorff distance.

Definition 2. Let $x, y \in C(I : E^n)$

$H_1(x, y) = \sup\{d_\infty(x(t), y(t)) : t \in I\}$.

Let I be a real interval. A mapping $x : I \to E_N$ is called a fuzzy process. We denote

$[x(t)]^\alpha = [x_l^\alpha(t), x_r^\alpha(t)],\ t \in I,\ 0 < \alpha \le 1.$

The derivative $x'(t)$ of a fuzzy process x is defined by

$[x'(t)]^\alpha = [(x_l^\alpha)'(t), (x_r^\alpha)'(t)],\ 0 < \alpha \le 1.$

provided that is equation defines a fuzzy $x'(t) \in E_N$.

The fuzzy integral

$$\int_a^b x(t)dt, \qquad a, b \in I$$

is defined by

$$[\int_a^b x(t)dt\,]^\alpha = [\int_a^b x_l^\alpha(t)dt,\ \int_a^b x_r^\alpha(t)dt\,]$$

provided that the Lebesgue integrals on the right exist.

Definition 3. ([1]) The fuzzy process $x: I \rightarrow E_N$ is a solution of equations (1)-(2) without the inhomogeneous term if and only if

$$(\dot{x}_l^\alpha)(t) = \min\{A_l^\alpha(t)[x_j^\alpha(t) + \int_0^t G(t-s) x_j^\alpha(s)ds],\ i,j = l,r\},$$

$$(\dot{x}_r^\alpha)(t) = \max\{A_r^\alpha(t)[x_j^\alpha(t) + \int_0^t G(t-s) x_j^\alpha(s)ds],\ i,j = l,r\},$$

and

$$(x_l^\alpha)(0) = x_{0l}^\alpha - g_l^\alpha(t_1,\ t_2,\ \cdots,\ t_p,\ x(\cdot)),$$

$$(x_r^\alpha)(0) = x_{0r}^\alpha - g_r^\alpha(t_1,\ t_2,\ \cdots,\ t_p,\ x(\cdot)).$$

Next hypotheses and existence result are Balasubramaniam and Muralisakar's results.(see [1])

(H1) The nonlinear function $g: I^p \times E_N \rightarrow E_N$ is a continuous function and satisfies the inequary

$$d_H([g(t_1, t_2 \cdots, t_p, x(\cdot))]^\alpha, [g(t_1, t_2, \cdots, t_p, y(\cdot))]^\alpha) \le c_1 d_H([x(\cdot)]^\alpha, [y(\cdot)]^\alpha)$$

for all $x(\cdot), y(\cdot) \in E_N$, c_1 is a finite positive constant.

(H2) The inhomogeneous term $f: I \times E_N \rightarrow E_N$ is a continuous function and satisfies a global Lipschitz condition

$$d_H([f(s, x(s))]^\alpha, [f(s, y(s))]^\alpha \le c_2 d_H([x(s)]^\alpha, [y(s)]^\alpha),$$

for all $x(\cdot), y(\cdot) \in E_N$, and a finite positive constant $c_2 > 0$.

(H3) $S(t)$ is a fuzzy number satisfying for $y \in E_N, S'(t)y \in C^1(I: E_N) \cap C(I: E_N)$ the equation

$$\frac{d}{dt}S(t)y = A[S(t)y + \int_0^t G(t-s)S(s)yds]$$

$$= S(t)Ay + \int_0^t S(t-s)AG(s)yds, \quad t \in I,$$

such that $[S(t)]^\alpha = [S_l^\alpha(t), S_r^\alpha(t)]$, and $S_i^\alpha(t)(i = l, r)$ is continuous. That is, there exists a constant $c > 0$ such that $|S_i^\alpha(t)| \le c$ for all $t \in I$.

Theorem 1. ([1]) Let $T > 0$, and hypotheses (H1)-(H3) hold. Then for every $x_0, g \in E_N$, the fuzzy initial value problem (1)-(2) without control function has a unique solution $x \in C(I: E_N)$.

3. Nonlocal controllability

In this section, we show the nonlocal controllability for the control system (1)-(2).

The control system (1)-(2) is related to the following fuzzy integral system:

$$x(t) = S(t)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot))) + \int_0^t S(t-s)f(s, x(s))ds + \int_0^t S(t-s)u(s)ds$$

where $S(t)$ is satisfy (H3).

Definition 4. The equation (3) is nonlocal controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (3) satisfies $x(T) = x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot))(i.e.,$

$$[x(T)]^\alpha = [x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot))]^\alpha)$$

where x^1 is target set.

Assume that the following hypotheses:

(H4) Linear system of equation(3)($f = 0$) is nonlocal controllable.

(H5) $(1 + 2c)c_1 + 2cc_2 T < 1$.

Theorem 2. Suppose that hypotheses (H1)

-(H5) are satisfied. Then the equation(3) is nonlocal controllable.

4. Example

Consider the semilinear one-dimensional heat equation on a connected domain $(0,1)$ for a material with memory, boundary conditions $x(t,0)=x(t,1)=0$ and with initial condition $x(0,z)+\sum_{k=1}^{p} c_k x(t_k, z)$ $= x_0(z)$ where $x_0(z) \in E_N$. Let $x(t,z)$ be the internal energy and $f(t,x(t,z)) = \tilde{2}tx$ $(t,z)^2$ be the external heat. Let $A=\tilde{2}\frac{\partial^2}{\partial z^2}$, $\sum_{k=1}^{p} c_k x(t_k z) = g(t_1, t_2, \cdots, t_p, x(\cdot))$ and $G(t-s)=e^{-(t-s)}$ then the balace equation becomes

$$\frac{dx(t)}{dt} = \tilde{2}[x(t) - \int_0^t e^{-(t-s)}x(s)ds] +$$

$$\tilde{2}tx(t)^2 + u(t), \qquad (5)$$

$$x(0)+g(t_1,t_2,\cdots,t_p,x(\cdot)) = x_0 \qquad (6)$$

The α-level set of fuzzy number $\tilde{2}$ is

$$[\tilde{2}]^\alpha = [\alpha+1, 3-\alpha] \quad \text{for all} \quad \alpha \in [0,1].$$

Then α-level set of $f(t,x(t))$ is

$$[f(t,x(t)]^\alpha = t[(\alpha+1)(x_l^\alpha(t))^2 (3-\alpha)$$

$$(x^{\alpha_r}(t))^2]. \qquad (7)$$

Further,

$$d_H([f(t,x(t))]^\alpha, [f(t,y(t))]^\alpha)$$

$$= d_H(t[(\alpha+1)(x_l^\alpha(t))^2, (3-\alpha)(x_r^\alpha(t))^2]$$

$$, t[(\alpha+1)(y_l^\alpha(t))^2 (3-\alpha)(y_r^\alpha(t))^2])$$

$$= t\max\{(\alpha+1)|(x_l^\alpha(t))^2-(y_l^\alpha(t))^2|,$$

$$(3-\alpha)|(x_r^\alpha(t))^2-(y_r^\alpha(t))^2|\}$$

$$\le 3T|x_r^\alpha(t)+y_r^\alpha(t)]|\max\{|x_l^\alpha(t)-y_l^\alpha(t)|,$$

$$|x_r^\alpha(t)-y_r^\alpha(t)|\} = c_2 d_H([x(t)]^\alpha, [y(t)]^\alpha)$$

where c_2 is satisfies the inequality in hypothesis (H5), and also

$$d_H([g(t_1,t_2,\cdots,t_p,x(\cdot))]^\alpha, [g(t_1,t_2,\cdots,t_p,$$

$$y(\cdot))]^\alpha) = d_H(\sum_{k=1}^{p} c_k[x(t_k)]\sum_{k=1}^{p} c_k[y(t_k)]^\alpha)$$

$$\le |\sum_{k=1}^{p} c_k| \, max_k d_H([x(t_k)]^\alpha, [y(t_k)]^\alpha)$$

$$= c_1 d_H([x(\cdot)]^\alpha, [y(\cdot)]^\alpha)$$

where c_1 is satisfies the inequality in hypothesis (H5).

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