Correlation coefficient between generalized intuitionistic fuzzy sets

일반화된 직관적 퍼지집합들의 상관계수

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Abstract

Based on the geometrical representation of a generalized intuitionistic fuzzy set, we take into account all three parameters describing generalized intuitionistic fuzzy set, propose a method to calculate the correlation coefficient for generalized intuitionistic fuzzy sets in finite set and probability space, respectively, and discuss some properties of correlation and correlation coefficient of generalized intuitionistic fuzzy sets.

Key Words: Generalized intuitionistic fuzzy sets, correlation coefficients, hesitancy degree.

1. Introduction

The notion of an intuitionistic fuzzy set was suggested by Atanassov [1,2] as a generalization of a Zadeh fuzzy set.

Definition 1. An intuitionistic fuzzy set A in a nonempty crisp set X is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote membership function and non-membership function, respectively, of A and satisfy

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \text{ for any } x \in X.$$

Note, obviously, that a Zadeh fuzzy set, written down as an intuitionistic one, is of the form $A = \{(x, \mu_A(x), 1 - \mu_A(x)) : x \in X\}$.

Obviously, for an intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, it is observed that $\mu_A(x) + \nu_A(x) \leq 1$ for any $x \in X$ and hence $\min\{\mu_A(x), \nu_A(x)\} \not> 0.5$ for any $x \in X$. As for examples, the attributes (i) 'beauty' and 'fatty', (ii) 'attentiveness' and 'dullness', (iii) 'interior' and 'frontier' etc. are such that both the attributes are not significant simultaneously, but sum of their degrees may exceed 1.

Having motivated from the observation, Mondal and Samanta [7] defined a generalized intuitionistic fuzzy set as follows:

Definition 2. A generalized intuitionistic fuzzy set (GIF set, for short) A in a non-empty crisp set X is an object having the form

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote membership function and non-membership function, respectively, of A and satisfy

$$\min\{\mu_A(x), \nu_A(x)\} \leq 0.5 \text{ for any } x \in X.$$

Correlation is a term that describes the relationship between GIF sets. As an important content in fuzzy mathematics, correlation between GIF sets will have gained much attentions for their wide applications in real world, such as pattern recognition, decision making and market prediction.

Park [8] defined the correlation between GIF sets

A and B in finite set $X=\{x_1, x_2, \cdots, x_n\}$ as follows:

$$C_1(A,B)=\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)),$$

and the correlation coefficient between A and B was given by

$$k_1(A,B)=\frac{C_1(A,B)}{\sqrt{C_1(A,A)\cdot C_1(B,B)}}.$$

Also he defined the correlation between GIF sets A and B in a probability space (X,B,P) as follows:

$$C_2(A,B)=\int_X(\mu_A\mu_B+\nu_A\nu_B)\,dP,$$

and the correlation coefficient between A and B was given by

$$k_2(A,B)=\frac{C_2(A,B)}{\sqrt{C_2(A,A)\cdot C_2(B,B)}}.$$

Note that if $X=\{x_1, x_2, \cdots, x_n\}$ and a probability P is given by $P(A)=|A|/n$, where $|A|$ is the cardinality of A, then k_1 is exactly k_2. As he stated, the values of k_1 and k_2 lie in the interval $[0,1]$.

In this paper, we give a geometrical representation of the GIF set and take into account all three parameters describing the GIF set. Thus, based on the kind of geometrical background, we discuss the concepts of correlation and correlation coefficient of GIF sets in finite set and probability space, respectively, by adding the term of hesitancy to Park's formula.

2. Geometrical interpretation of GIF sets

For a given GIF set $A=\{(x, \mu_A(x), \nu_A(x)) : x\in X\}$ in X, we call

$$\phi_A(x)=1-\mu_A(x)-\nu_A(x)$$

the generalized intuitionistic index of x in A. It is a hesitancy degree of x to A, and it is obvious that $-0.5\le\phi_A(x)\le 1$ for each $x\in X$. For example, let A be a GIF set with membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$, respectively. If $\mu_A(x)=0.8$ and $\nu_A(x)=0.4$, then we have $\mathrm{phi}_A(x)=1-0.7-0.4=-0.2$. It can be interpreted as the degree that the object x belongs to the GIF set A is 0.8, the degree that the object x does not belong to the GIF set A is 0.4 and the degree of hesitancy is -0.2. Hence, GIF set A in X can be expressed as $A=\{(\mu_A(x), \nu_A(x), \phi_A(x)) : x\in X\}$. If A is an intuitionistic fuzzy set, then $0\le\phi_A(x)\le 1$ for each $x\in X$. It means that third parameter $\phi_A(x)$ can not be casually omitted if A is a GIF set, not an intuitionistic fuzzy set. Thus we can give a convenient representation of a GIF set as $A=\{(\mu_A(x), \nu_A(x), \phi_A(x)) : x\in X\}$.

One of convenient geometrical interpretations of the GIF sets is shown in Fig. 1. Mondal and Samanta [7] consider a universe X and subset Y in the Euclidean plane with Cartesian coordinates.

For a fixed GIF set A, a function f_A from X to Y can be constructed, such that if $x\in X$, then $p=f_A(x)\in Y$, and the point $p\in Y$ has the coordinates $(\mu_A(x), \nu_A(x))$ for which

$0\le\mu_A(x), \nu_A(x)\le 1$ and

$\min\{\mu_A(x), \nu_A(x)\}\le 0.5$.

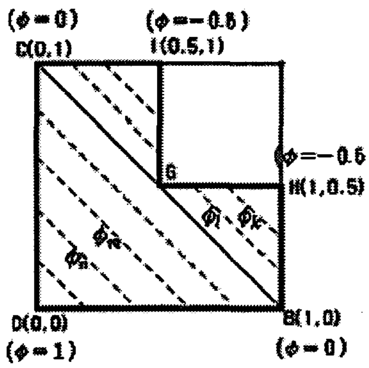

Fig. 1. An orthogonal projection of the real (three-dimension) representation (triangle BCD in Fig. 2) of a GIF set.

The above geometrical interpretation can be used as an example when considering a situation at beginning of negotiations - cf. Fig. 1 (applications of generalized intuitionistic fuzzy sets for group decision making, negotiations and other real situations are presented in real life).

A line parallel to a segment BC describes a set of experts with the same level of hesitancy. For example, in Fig. 1, four sets presented with generalized intuitionistic indices equal to ϕ_n, ϕ_m, ϕ_l and ϕ_k, where $-0.5<\phi_k<\phi_l<\phi_m<\phi_n<1$.

In other words, Fig. 1 (the polygon DBHGIC) is an orthogonal projection of the real situation (the polygon ABEGFC) presented in Fig. 2.

An element of a GIF set has three coordinates (μ_i, ν_i, ϕ_i), hence the most natural representation of a GIF set is to draw a cuboid (with width 1, length 1 and height 1.5), and the polygon ABEGFC (Fig. 2) represents a GIF set. As before (Fig. 1), the polygon DBHGIC is the orthogonal projection of the polygon ABEGFC.

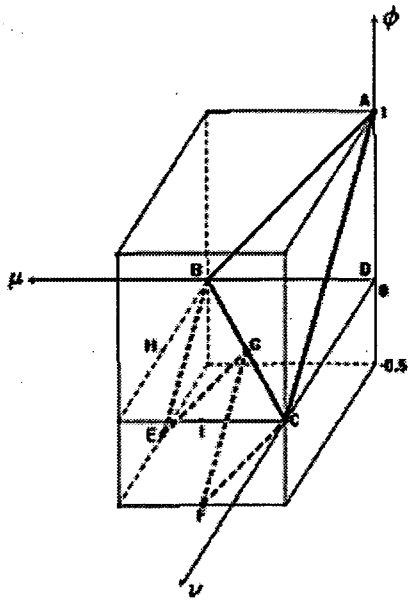

Fig. 2. A three-dimension representation of a GIF set.

Therefore, this representation of a GIF set will be a point of departure for considering the our method in calculating correlation coefficient between GIF sets.

3. Correlation coefficient in finite set

Although in most cases the correlation coefficient k_1 gives intuitively satisfying results, there are some cases in which this is not true. The following example shows one such case.

Example 1. Let A and B be GIF sets in $X=\{x_1, x_2, x_3\}$ given by

$\mu_A(x_i) = \nu_A(x_i) = 0.5$,

$\mu_B(x_i) = \nu_B(x_i) = 0.25,\ i=1,2,3.$

Then we can easily check that $k_1(A,B)=1$ but $A \neq B$.

In order to solve this problem, we propose new method to calculate the correlation coefficient between GIF sets by adding the term of hesitancy to Park's formula.

Definition 3. For GIF sets A and B in a finite set $X=\{x_1, x_2, \cdots, x_n\}$, we define correlation coefficient, $\rho_1(A,B)$, between A and B as follows:

$$\rho_1(A,B) = \frac{C_1(A,B)}{\sqrt{C_1(A,A) \cdot C_1(B,B)}},$$

where $C_1(A,B) = \frac{1}{n}\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \phi_A(x_i)\phi_B(x_i))$ is correlation between A and B in X.

Proposition 1. For GIF sets A, B in a finite set X, the following hold:

(a) $\rho_1(A,B) = \rho_1(B,A)$;

(b) If $A=B$, then $\rho_1(A,B)=1$.

Theorem 1. For GIF sets A, B in a finite set X, we have

(a) $|\rho_1(A,B)| \le 1$.

(b) $\rho_1(A,B)=1$ if and only if $A=B$.

In finite set X, A will called a nonfuzzy set if $\mu_A(x)=1$ or $\nu_A(x)=1$ or $\phi_A(x)=1$.

Theorem 2. Let A, B be nonfuzzy sets in a finite set $X=\{x_1, x_2, \cdots, x_n\}$. If $C_1(A,B)=0$, then A and B satisfy one of the following conditions: for any $x \in X$,

(a) $\mu_A(x_i) + \mu_B(x_i) = 1$;

(b) $\nu_A(x_i) + \nu_B(x_i) = 1$;

(c) $\phi_A(x_i) + \phi_B(x_i) = 1$.

4. Correlation coefficient in probability space

Let (X, B, P) be a probability space. A GIF set A in a probability space X can be represented by $A = \{(\mu_A(x), \nu_A(x), \phi_A(x)) : x \in X\}$, where $\mu_A, \nu_A : X \to [0,1]$ are, respectively, Borel measurable functions satisfying $\min\{\mu_A(x), \nu_A(x)\} \le 0.5$ for any $x \in X$.

Note that $\phi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is also Borel measurable function.

Definition 4. For GIF sets A and B in X, we define correlation coefficient, $\rho_2(A,B)$, between A and B as follows:

$$\rho_2(A,B)=\frac{C_2(A,B)}{\sqrt{C_2(A,A)\cdot C_2(B,B)}},$$

where, $C_2(A,B)=\int_X(\mu_A\mu_B+\nu_A\nu_B+\phi_A\phi_b)dP$ is correlation between A and B.

Remark 1. (a) If $X=\{x_1,x_2,\cdots,x_n\}$ and a probability P is given by $P(A)=|A|/n$, where $|A|$ is cardinality of A, then $\rho_2(A,B)$ is exactly $\rho_1(A,B)$.

(b) If A and B are ordinary fuzzy sets satisfying the condition $\mu_A(x)+\nu_A(x)=1$, then $k_1(AA,B)=\rho_1(A,B)$ and $k_2(A,B)=\rho_2(A,B)$.

Proposition 2. For GIF sets A,B in a probability space X, the following hold:

(a) $\rho_2(A,B)=\rho_2(B,A)$;

(b) If $A=B$, then $\rho_2(A,B)=1$.

If A,B are GIF sets satisfying the condition $P\{\mu_A=\mu_{B,}\ \nu_A=\nu_B\}=1$, we denote by $A=B$ a.e.

Theorem 3. For GIF sets A,B in a probability space X, we have

(a) $|\rho_2(A,B)|\le 1$.

(b) $\rho_2(A,B)=1$ if and only if $A=B$ a.e.

The following examples are used for comparing the proposed method with Park's method.

Example 2. Let A,B be the two GIF sets given in Example 1. By Definition 3, we find $\rho_1(A,B)=0.5774$ and thus we obtain the intuitively satisfying result $\rho_1(A,B)\neq 1$.

Example 3. Let A,B be GIF sets in $X=\{x_1,x_2,x_3\}$ defined by

$A=\{(x_1,0,0.25),(x_2,0,0.2),(x_3,0,0.3)\}$

$B=\{(x_1,1,0.5),(x_2,0.5,1),(x_3,1,0.3)\}$.

Then $\rho_1(A,B)=-0.2030$ but $k_1(A,B)=0.4992$.

5. Conclusions

From the statistical point of view, the range for correlation coefficient should lie in [-1,1]. Our proposed method attains this requirement. The correlation coefficients computed in this paper, which lie in [-1,1], give us more information than the correlation coefficients by [8], which lie in [0,1].

The correlation coefficients ρ_1 and ρ_2 tell us not only the degree of the relationship between two GIF sets but also whether two GIF sets are positively or negatively related. The correlation coefficients k_1 and k_2 from Park [8] only represent the strength of the relation. Thus, the proposed methods are more reasonable than Park's method because the term of hesitancy can not casually omitted to express a GIF set.

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