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Making Change in School Mathematics: Lessons About the Interactions Among Research, Policy, and Practice from the PROM/SE Project

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Introduction

The United States context for mathematics education in the K-12 setting has shifted in the past decade. The leadership provided by the national professional organizations, such as the National Council of Teachers of Mathematics, has been expanded to include substantial involvement and interest from professional mathematicians and the business community. The constitutional responsibility of individual states for the education of their students remains, but stronger federal guidance and accountability have been introduced with the *No Child Left Behind Act* of 2001. Here I present some features of the current context, and then describe a major research and development effort funded by the U.S. National Science Foundation (the PROM/SE project) that is addressing the confluence of the standards-assessment policy context, the need to improve teachers knowledge and skills for teaching mathematics, the importance of coherent and challenging curriculum, and the capacity of school districts to commit to serious improvement of mathematics achievement. I will provide three examples of how the work of PROM/SE is relating research to policy and practice, together with discussion of implications for further work.

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The U.S. Context for Mathematics Education Reform: A Brief Description

In the United States, efforts to reform mathematics teaching and learning in K-12 schools have been guided by the standards movement since the 1989 release by the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The subsequent revision, *Principles and Standards for School Mathematics* (NCTM, 2000) furthered these efforts, at a time when the interest of professional mathematicians in influencing the content and pedagogy of school mathematics was also increasing. The math wars in the US have been documented in Wilson (2003), and have included vigorous debate about what the appropriate content for school mathematics should be, about how it should be taught, and about the purposes of school mathematics. Some debate centers on the perceived tensions of educating all students to high standards in mathematics and also ensuring that adequate numbers of students are prepared to enter the science, technology, engineering, and mathematics (STEM) workforce. An effort to reach common ground is reported in Ball et al., 2005.

In 2002, the *No Child Left Behind*(NCLB) Act became federal law in the US. One aspect that is particularly relevant to mathematics is the requirement that all states develop grade-by-grade standards specifying what mathematics should be taught in each grade, and that each state should assess mathematics annually at each of the grades three through eight, and at least once between grades 10-12. The standards are to be challenging, "contain coherent and rigorous content, and encourage the teaching of advanced skills."(U.S. Department of Education [USDE], 2002, p. 1445). The result has been extensive activity in the past few years at the state level to produce standards documents that provide the specificity needed to guide this assessment. According to Reys et al. (2006), new state level standards or expectations documents have been developed in 39 states since 2002.

The assessments tied to these standards are high-stakes in that there are consequences for schools and districts who fail to make "adequate yearly progress" in their core assessment areas. These include public designation of "failing schools"and loss of federal funds. Schools that fail to make adequate yearly progress for two consecutive years are labeled as "in need of improvement" and required to submit a school improvement plan. Their status as "failing schools" must be publicly declared. Schools that become targeted for "corrective action" must restructure their internal organization and abide by externally made decisions about staffing and curriculum (USDE, 2002, pp. 1482-1484).

An analysis of the current state documents has been undertaken by the Center for the Study of Mathematics Curriculum (Reys et al., 2006). Their basic findings indicate that there is a lack of consensus across states in what is expected of students in mathematics at particular grade levels. For example, the total number of grade level expectations in mathematics for Grade 4 ranged from 32 to 89 (see Table 1).

Table 1. Number of fourth grade learning expectations per state by content strand (from Reys, Dingman, McNaught, Regis, & Togashi, submitted for publication, p. 21)

	Number & Operations	Geometry	Measurement	Algebra	Data Analysis, Prob & Stat	Total Number of LEs
California	16	11	4	7	5	43
Texas	15	7	3	4	3	32
New York	27	8	10	5	6	56
Florida	31	11	17	10	20	89
Ohio	15	8	6	6	13	48
Michigan	37	5	11	0	3	56
New Jersey	21	10	8	6	11	56
North Carolinal	14	3	2	3	4	26
Georgia	23	10	5	3	4	45
Virginia	17	8	11	2	3	41

The implications of the current policy context for the continued improvement of school mathematics are unclear. State and district curriculum leaders are consumed with providing support to teachers and schools to ensure that prospects for reaching adequate yearly progress are good. The reform directions provided by the NCTM now coexist with the need to focus on very specific expectations and assessment outcomes. (NCTM will soon release a new document, called Curriculum Focal Points, which provides more focused and practical guidance about the central and foundational ideas of school mathematics.)

An Opportunity for Systemic Improvement in K-12 Mathematics Education: The PROM/SE Project

The PROM/SE initiative[2] is a five-year comprehensive research and intervention effort to improve mathematics and science teaching and achievement. The project involves about 60 school districts from across the states of Michigan and Ohio, and demographically represents a microcosm of the US. Structurally,the project comprises a set of six partners, including Michigan State University and five consortia of K-12 school districts. The sixty districts include 586 school buildings and approximately 5000 teachers of mathematics and science across grades K-12. One mathematics teacher and one science teacher per building have been identified as PROM/SE Associates. Associates are serving in a teacher-leader capacity to transmit the ideas of the project to their colleagues.

In 2003-3004, the initial year of the project, a major data-gathering effort was mounted to determine baseline information across several areas. Instruments were developed to assess and survey student achievement in mathematics and science, teachers time teaching particular content, and district features and curricular emphases. Analyses of state and local standards also were conducted, using techniques developed in the curriculum analysis in the Third International Mathematics and Science Study. This rich and detailed database is serving as a main foundation for all ongoing aspects of the project, including as a tool for identifying key focal areas of the discipline-specific professional development for teachers, and for contributing guidance in the development and analysis of state standards. The project is committed both to supporting state and local policy that aims at coherent curriculum, together with professional development of teachers of mathematics to build their understanding of, and capacity to implement, coherent and high expectations in mathematics for all students. This approach integrates the view that curriculum standards and systems for teacher education and professional development both need to be improved.

The three major goals of PROM/SE are:

- establish higher expectations for mathematics and science achievement at all levels;

2) PROM/SE (Promoting Rigorous Outcomes in Mathematics and Science Education) is funded by the National Science Foundation, Mathematics and Science Partnership program, Cooperative Agreement No. EHR-0314866. Co-PIs are Joan Ferrini-Mundy, William Schmidt, George Leroi, Peter Bates, and Terry Joyner.

- establish greater coherence in the curricula and instruction for mathematics and science; and
- establish data-based decision making as an essential component of improving mathematics and science achievement.

In the remainder of this paper I will describe how the PROM/SE project is addressing these goals in ways that illustrate how policy and practice are informed by the research underway in the project. The policy that has been influenced here is state standards, and the practices that are addressed are professional development activities for teachers and efforts of school district superintendents to improve mathematics achievement.

Conceptual Framework in PROM/SE: Coherence and Trajectories

By design, the PROM/SE project includes substantial opportunities for teachers professional development. In particular, that professional development is focused on building up teachers mathematical knowledge for teaching (see Ball & Bass, 2000), a construct that is clearly related to pedagogical content knowledge (Shulman, 1986). More specifically, we are working on what it means to help teachers develop mathematical knowledge for teaching that enables them to work toward curricular coherence, even in the context of overly-full state standards and textbooks.

The main conceptual organizer for focusing on coherence in both standards analysis/design and the mathematics professional development in PROM/SE is the notion of content trajectories. Others have discussed related ideas. Ball (1993) has urged that it is important for teachers to "keep an eye on the mathematical horizon."Ferrini-Mundy et al. (in preparation), in their work on mathematical knowledge for teaching at the secondary level, have argued that an important part of teachers' specialized knowledge is being able to understand "both the origins and extensions of core concepts and procedures knowing the basis for ideas in the domain, and understanding how those ideas grow and become more abstract or elaborated. A content trajectory may be characterized by a mathematical ordering or rationale as well as a sense of how ideas might be best organized to support student learning."

The related notion of learning progressions (Catley, Lehrer, & Reiser, 2005) has been highlighted as a focus in a recent call for proposals by the U.S. National Science Foundation. This call solicits proposals for Instructional Materials Development projects that aim to develop learning progressions that span grade bands and foster learning of key processes essential to one or more science or technology discipline ((NSF 05-612, (www.nsf.gov/pubs/2005/nsf05612/nsf05612.htm, Program Description, Background Information, Paragraph 2). The solicitation makes the point that the exemplars to be developed will be used a models to help guide both instructional design and teacher professional development.

The main discussion of learning progressions seems to be in the science education literature. Smith, Wiser, Anderson, and Krajcik (in press) describe them as: "descriptions of successively more sophisticated ways of reasoning within a content domain based on research syntheses and conceptual analyses organized around central concepts andprinciples of a discipline (i.e., its *big ideas*) and show how those big ideas are elaborated, inter-related, and transformed with instruction. They should also specify how those big ideas are enacted in specific practices that allow students to use them in meaningful ways, enactments we describe as *learning performances*." (p. 3).

This formulation contrasts somewhat with that inspired by the work of William Schmidt and his colleagues in the curriculum studies that were part of The Third International Mathematics and Science Study. Schmidt, Wang, and McKnight (2005) define content standards as coherent "if they are articulated over time as a sequence of topics and performances consistent with the logical and, if appropriate, hierarchical nature of the disciplinary content from which the subject-matter derives." (p. 528). This perspective suggests that content trajectories that derive from the nature of the mathematics might also be useful in instruction, and at a global level this view guides our efforts to define content trajectories in PROM/SE. However, in order to carry the notion of content trajectories into more fine-grained levels within specific content areas, we have found that it is important to also integrate knowledge that is available from research on student learning of school mathematics topics.

In the following sections are three examples of how this conceptual formulation of the idea of trajectory, together with research findings from the project, are influencing particular areas of practice. I discuss how the view of trajectories has been embedded into state standards developed in conjunction with PROM/SE staff; how the idea of trajectories is being implemented in the design of professional development for teachers; and how the concept is at the center of helping school superintendents in working toward coherence curriculum.

PROM/SE Findings About Student Achievement in Fractions: Implications for School District Superintendents

Because one of the major goals of PROM/SE is to help districts use data as a basis for decision-making, the project is producing a series of reports targeted at school superintendents, to help frame for them the issues that are emerging across the project from the baseline and ongoing data gathering. The topic of the first report is fractions (PROM/SE, 2006).

In the spring of 2004 PROM/SE conducted mathematics and science assessments with students in grades three through twelve across the partnership. Using a duplex design, nearly 300,000 students participated. Three basic instruments were used (for grades 3~5, 6-8, and 9-12) with some overlapping items across the grade spans. That analysis included examinations of student progress across the grades on key topic areas. In Figure 1 the overall results for the fractions area are shown.

Figure 1. PROM/SE Assessment Results in Fractions, By Grade (PROM/SE, 2006)

The analysis suggests that, if students had a stronger start in the third grade with foundational fraction concepts such as equivalence of fractions and meaning of fraction, then assuming that the kind of progress in their performance that is evident in these data in fourth and fifth grades (note that the assessment instrument for sixth grade is a different one) would still occur, they would be positioned much more solidly for the middle grades work that builds on fraction.

We have advised superintendents to undertake some examination of their curriculum and instruction around fractions in grade three as a starting point for moving toward the PROM/SE goals of greater coherence and higher expectations for all students. Specific advice offered to the superintendents in this report includes: find out what third grade teachers are doing in the area of fractions; examine your district curriculum and instructional materials in grade three to be certain that foundational concepts are being addressed; encourage the use of models and representations in the second and third grades to help students understand fractions. The report was only released a few weeks ago, but initial response has been quite positive.

Research About Student Learning of Fractions and Content Trajectories: Implications for Teacher Professional Development Practice

A survey of both K-8 standards in U.S. states and K-8 instructional materials reveals the multitude of meanings for fraction that are available. These include: part of a collection, part of a whole, point on a number line, measurement, ratio/rate, probability, division of two numbers, and abstract number. We have discussed with PROM/SE teachers how important it is, mathematically, to reach the last two meanings both fraction as the result of a division of two numbers, and fraction as a number. We then examined state standards to consider more carefully how teachers could find pathways through standards that highlight the trajectories for reaching these meanings. We discussed how to bridge from childrens understanding of fraction ideas toward these two meanings, how particular representations may lead to these meanings and the advantages and limitations of different fraction meanings for moving toward these two.

Part of the preparation for designing the teachers' professional development involved reviewing available literature[3] about fraction understanding. In a study of 3rd and 4th grade students understanding of fractions Mack (1995) looked at how students with limited prior knowledge of fraction symbols built on this informal knowledge, as well as their prior knowledge of whole numbers, to make meaning of fraction symbols and procedures during instruction. She found that students ability to relate symbolic representations for fractions to their informal knowledge is influenced by their prior knowledge of whole number symbolic representations. Students also overgeneralized the meanings of symbolic representations for whole numbers to fractions.

Gay and Aichele (1997) studied middle school students number sense related to percent, including their understanding the meaning of quantity expressed as a percent of a number, and their strategies for making comparisons about percents in both pictorial and abstract settings. Visual models proved supportive for students who had difficulty. Results suggest that analyzing a students knowledge of percent using only written test results can lead to incomplete or misleading conclusions. They also found that students successfully applied fractional relationships, estimation and mental computation to make comparisons, and used 50% and 100% as common reference points.

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3) Mustafa Demir developed a review of the fractions literature; this section is drawn from his internal PROM/SE project report.

In a comparative study of fourth graders, Moss and Case (1999) looked at the effects of introducing rational numbers through percent. Halving as a computational strategy was emphasized, and two-, three-, and one-place decimals were then introduced in a linear measurement context. Fraction notation was introduced last as an alternative means of representing decimals. In the experimental curriculum, children used percentage terminology to describe the fullness of different containers of water, answering such questions as Approximately what percentage of this beaker do you think is full? or Where will the liquid come to in this beaker when it is 25% full? For students in the control group, fractions were introduced first. The researchers found that students in the experimental group showed a deeper understanding of rational numbers than those in the control group, and made more frequent reference to proportional concepts in justifying their answers. No differences in conventional computation were apparent between the two groups.

In a study of four students, Mack (2000) examined the development of their understanding of multiplication of fractions. Results indicate that students continued to draw on their informal knowledge of partitioning when solving multiplication of fractions problems, several months after instruction ended. As students drew on their informal knowledge their understanding of units and the meaning of fractions deepened over time. Results suggest that building on informal knowledge of partitioning may be a viable way to develop students understanding of multiplication of fractions for both the short and long term.

Irwin (2001) examined the role of students everyday knowledge of decimals in supporting the development of their knowledge of decimals. A comparison of pretest and posttest results revealed that students who worked on contextual problems made significantly more progress in their knowledge of decimals than did those who worked on noncontextual problems. The students who solved contextualized problems built understanding of decimals by reflecting on their everyday knowledge related to the meaning of decimal numbers and the results of decimal calculations.

Our review of these and other studies led to a list of findings that were presented to the PROM/SE teacher participants in an early professional development workshop in the project. The list included the following:

- Number line at 4th grade is difficult
- Translations between various representations are hard for students

- Need a lengthy readiness period for fractions
- Fraction/ratio confusion
- It helps to encourage children to talk about their interpretation of fraction
- Greater emphasis is needed on division interpretation
- Need to recognize the limitations of the "part to whole" model
- Need to transition better from realm of counting numbers to rational numbers
- Knowing different meanings and interpretations strengthens understanding
- Students bring experience with part-to-whole interpretation, and fair shares
- Need variety of models for the "whole" (circles, rectangles, irregular shapes)
- In part-to-whole, not only do partitioning, but complete the whole

The research base that we have accumulated also has influenced the PROM/SE professional development design. In one of our initial sessions with teachers we asked them to do the following problem, in the way that a student in the grade level they teach might attempt it.

> Penny had a bag of marbles. She gave one-third of them to Rebecca and one-fourth of the remaining marbles to Aman. Penny then had 24 marbles left in her bag. How many marbles were in the bag to start with?

We then examined the teachers' responses and a mathematician in the project team discussed them with the participants, beginning with solutions from a teacher of kindergarten students and progressing up through the grades using the various solutions as illustrations of progressive growth in understanding.

As the professional development in PROM/SE continues, we are refining our efforts to help teachers understand the trajectories of various mathematical ideas. Figure 2 is a portion of a worksheet[4] we have used in activities with teachers this spring in an effort to highlight issues of trajectory relative to topics in number theory.

4) Gail Burrill, Elizabeth Jones, and Mary Bouck contributed to the development of this activity for teachers.

Instructional Materials Trajectory: Activity, PROM/SE Spring PD, 2006

Number Theory: Classifying Numbers, Factors,
Multiples, and Relationships

In this session you will examine the instructional materials used in your school/district and analyze the mathematics related to the development of number theory concepts: odd, even, prime, composite, factors, multiples, and their relationships.

1) In your team, respond to the following questions using and the chapters/units in each grade level set of instructional materials that address number theory concepts. Record your findings on the Curriculum Analysis Tool chart.

- When and how are terms such as factor, multiple, prime introduced and **defined**?
- Identify the **representations** students use to build understanding and express mathematical ideas. This might include physical objects, drawings, language, charts, graphs, equations, and symbols. When and how are **relationships** developed between even and odd, factor and multiples, and primes and composites? This includes when and how students come to understand **rules, properties and theorems** (e.g., Fundamental Theorem of Arithmetic)?
- When and how are **applications** of these ideas addressed? What **connections** are made toother areas of mathematics (e.g., factoring algebraically)?

3) Look across the grade levels to see how number theory concepts: odd, even, prime, composite, factors, multiples and relationships are developed.

- Identify any **gaps** you think exist within this set of instructional materials that could interfere with the development of the mathematics. Identify any **overlaps** that exist in the content trajectory for this mathematical area. Determine whether the overlaps are important for the development of understanding or whether are they primarily for review purposes.

Figure 2. Professional Development Activity to Develop Content Trajectory Understanding

Research About Student Learning of Fractions and Content Trajectories: Implications for State Policy

The PROM/SE focus on fractions was underway concurrently with the development of new grade level content expectations for K-8 mathematics in the state of Michigan. Mathematicians, mathematics education researchers, and teachers collaborated to produce a document that attempted to emphasize the notion of trajectories as an internal feature. Members of the PROM/SE leadership played central roles in this development process. In the area of fractions, we worked to blend the notion of coherence as described by Schmidt, Wang, and McKnight (2005) with elements of the learning progressions view as described by Smith, Wiser, Anderson, and Krajcik (in press).

Figure 2 includes an excerpt from the K-8 Michigan Grade Level Content Expectations (www.michigan.gov/documents/MathGLCE_140486_7.pdf), highlighted to reflect the trajectories that lead toward the number understanding of fraction and the part-whole understanding of fraction.

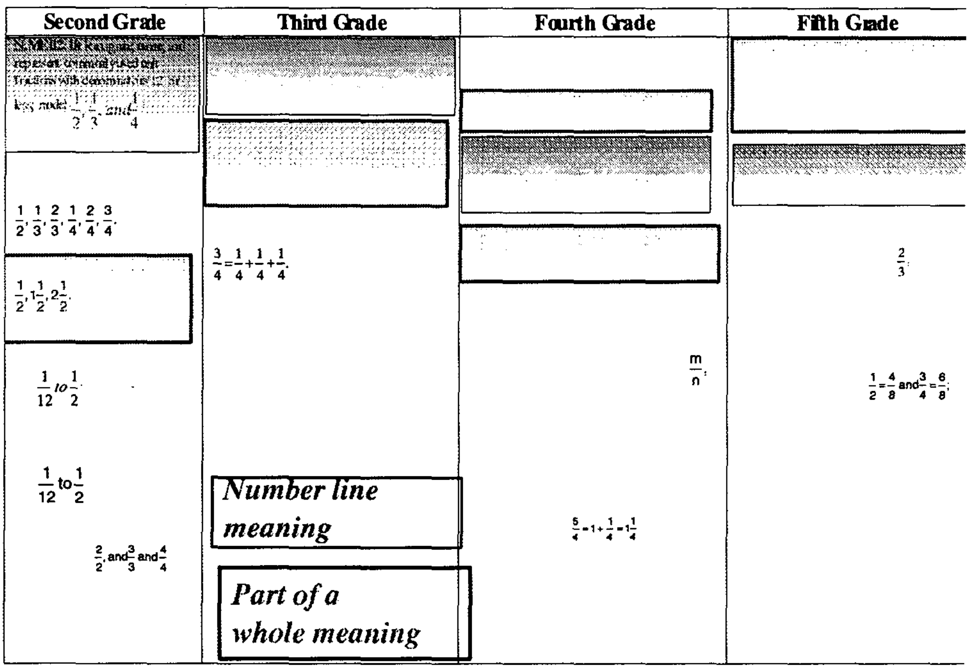

Figure 3. Michigan K-8 Grade Level Content Expectations, Fraction Trajectories Toward Number Line and Part-Whole Meanings of Fractions.

Issues and Conclusions

The research and practice dialectic is complex and interactive. It is not so clear that direct translation of research findings leads to improvements in educational policy and practice. Indeed, the framing provided by Silver (1990) that indicates that not only the findings of research have potential applicability to practice, but also the research methods and the theoretical constructs or perspectives that guide research all can be influential in practice. In the PROM/SE initiative all three aspects of research have been central in our efforts to improve both state policy and teacher professional development practice. In the example of how the ideas about coherence and trajectories have influenced our contributions to the development of state policy, we are drawing on the theoretical constructs and perspectives that come from the body of research about international achievement and curriculum initiated with TIMSS. In providing school district superintendents information about the performance of their students on fractions, we clearly are using the findings of research to attempt to influence practice. And, in creating professional development activities that are meant to support teachers' understanding of trajectories in particular areas, we have drawn on both the theoretical constructs and methods of research. All of this activity then feeds back into our ongoing research and efforts to inform policy and practice, in a cycle similar to the "cycle of knowledge production and improvement of practice" proposed in the RAND Mathematics Panel report (2003, p. 6).

A number of issues arise in the implementation of the cycle. I highlight three that have become especially salient in our work in PROM/SE. Both in our efforts to inform policy and standards development, and our efforts to introduce ideas such as coherence and trajectory to teachers, we encounter some resistance. These ideas are not necessarily familiar. In the policy work, as we have been part of the committees formulating standards and expectations, we find that there is strong reluctance to omit favorite and familiar topics from standards documents, even when it is difficult to make an argument that these topics are central or mathematically critical. In providing information to superintendents, we have found that they are not persuaded to take action simply on the basis of general data,but rather are interested in the specific data abut their own district. (Thus the recent fractions report actually provides a table with information specific to each district, coded so that only the district knows their placement.) And, in working with teachers on research-based ideas such as trajectory, we find that their expectations for what should happen in professional development sessions sometimes conflict with our goals. That is, they are interested in being able to see the immediate use of the activities and exercises they undertake.

Nonetheless, the PROM/SE project is committed to advancing the interactions among research, policy, and practice. In the process, we are learning about the enormous complexity of this work, and remain convinced that such an approach is crucial to the continuing improvement of mathematics teaching and learning.

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