韓國數學教育學會
<全國數學教育硏究大會 프로시딩>
제 36회. 2006. 5. 33-36

Korea Soc. Math. Ed.
Proceedings of National Meeting of Math. Ed.
The 36th. May. 2006. 33-36

On Generalized Mixed Quasi-variational-Like Inequalities

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This paper introduces a class of multivalued mixed quasi-variational-like inequalities and shows the existence of solutions to the class of quasi-variational-like inequalities in reflexive Banach spaces.

Ⅰ. Introduction and Preliminaries

The importance of applications of variational inequalities to many areas, for examples, optimization problems, differential equations, equilibrium problems in nonlinear analysis is known in many researches (see [1, 5, 10, 11]) and references therein). Parida and Sen [7] firstly posed variational-like inequalities and Aubin and Ekeland [1] also firstly introduced quasi-variational inequalities. Recently, Verma [9] introduced a class of monotone nonlinear variational inequalities and considered the existence of solutions. Very recently, Cho et al. [2], Fang et al. [4] and Huang et al. [6] generalized and improved the results of Verma [9] to a class of nonlinear quasi-variational-like inequalities.

This paper introduces a new class of generalized quasi-variational-like inequalities and, generalizes and improves the results of Cho et al. [2]. In the proof, some wrong part of the proof in [2] is corrected.

Let X be a real Banach space with dual space X^* and K a nonempty convex closed subset of X. Denote $\langle l, x\rangle = l(x)$, for all $l \in X^*$ and $x \in X$. Let $S, T : K \to 2^{X^*}$ be two multivalued mappings, $N : X^* \times X^* \to X^*$ and $g : K \to X^*$ be mappings.

In this paper, we consider the following generalized quasi-variational-like inequality;

For any $l \in X^*$, find $u \in K$ such that

$$\sup_{x \in S(u),\, y \in T(u)} \langle (g(u) + N(x, y)) - l,\ G(v) - G(u)\rangle + f(v) - f(u) \geq 0,$$

for all $v \in K$, where $G : K \to K$ is a mapping. (1, 1)

Definition 1.1. A mapping $S: K(\subset X) \to 2^{X^*}$ is said to be G-φ-p-monotone with respect to the first argument of a mapping $N: X^* \times X^* \to X^*$ if there exist a function $\varphi: [0, +\infty) \to [0, +\infty)$, a mapping $G: K \to K$ and a constant $p > 1$ such that

$$\langle N(x, \cdot) - N(y, \cdot), G(u) - G(v) \rangle \geq \varphi(\|G(u) - G(v)\|)\, \|G(u) - G(v)\|^p, \tag{1, 2}$$

for all $u, v \in K$, $x \in S(u)$ and $y \in S(v)$.

Definition 1.2. A mapping $T: K(\subset X) \to 2^{X^*}$ is said to be G-ψ-p-monotone with respect to the second argument of a mapping $N: X^* \times X^* \to X^*$ if there exist a function $\psi: [0, +\infty) \to [0, +\infty)$, a mapping $G: K \to K$ and a constant $p > 1$ such that

$$\langle N(\cdot, x) - N(\cdot, y), G(u) - G(v) \rangle \geq -\psi(\|G(u) - G(v)\|)\, \|G(u) - G(v)\|^p \tag{1, 3}$$

for all $u, v \in K$, $x \in T(u)$ and $y \in T(v)$.

Definition 1.3. A mapping $g: K(\subset X) \to X^*$ is said to be G-ϕ-p-relaxed Lipschitzian if there exist a function $\phi: [0, +\infty) \to [0, +\infty)$, a mapping $G: K \to K$ and a constant $p > 1$ such that

$$\langle g(v) - g(u), G(v) - G(u) \rangle \geq \phi(\|G(v) - G(u)\|)\, \|G(v) - G(u)\|^p,$$

for all $u, v \in K$. (1, 4)

II. Main Results

Now, we consider two kinds of variational inequalities, whose solution sets are the same.

Theorem 2.1. Let X be a reflexive Banach space, X^* be its dual and K be a nonempty convex closed subset of X, let $g: K \to X^*$ be a hemi-continuous mapping satisfying (1.4) and also let S and $T: K \to 2^{X^*}$ be lower semi-continuous multivalued mappings satisfying (1.2) and (1.3), respectively, where for functions $\varphi, \psi, \phi: [0, +\infty) \to [0, +\infty)$ satisfying $\varphi(t) + \phi(t) > \psi(t)$ for all $t > 0$, $\varphi + \phi - \psi$ is bounded in $[0, \delta]$ for some $\delta > 0$. In addition, suppose the $G: K \to K$ is an affine mapping, $f: K \to \mathbf{R} \cup \{+\infty\}$ is a proper convex functional and $N: X^* \times X^* \to X^*$ is continuous with respect to the weak* topology of X. Let a multivalued mapping $v \mapsto \{N(z, w) \in X^* : z \in S(v), w \in T(v)\}$ be lower hemi-continuous. Then for any $l \in X^*$, $u \in K$ is a solution of problem (1.1) if and only in $u \in K$ is a solution of the following problem:

Find $u \in K$ such that

$$\langle g(v)+N(z,\ w))-l,\ G(v)-G(u)\rangle\ +f(v)-f(u)$$
$$\geq (\varphi(\|G(v)-G(u)\|)-\psi(\|G(v)-G(u)\|)+\phi(\|G(v)-G(u)\|)\|G(v)-G(u)\|^{p} \qquad (2,\ 1)$$

for all $v \in K$, $z \in S(v)$ and $w \in T(v)$.

From Theorem 2.1, we have the following theorem, the main result of Cho et al. [2] as a corollary.

Corollary 2.2[2]. Let G be the identity mapping, $g \equiv 0$, $\phi \equiv 0$ and $N(x, y)=x-y$ for $x, y \in X^*$ in Theorem 2.1. Then for any $l \in X^*$, $u \in K$ is a solution of

$$\sup_{x \in S(u),\ y \in T(u)} \langle N(x,\ y)-l,\ v-u\rangle+f(v)-f(u) \geq 0 \text{ for all } v \in K$$

if and only in $u \in K$ is a solution of

$$\langle N(z,\ w)-l,\ v-u\rangle+f(v)-f(u) \geq (\varphi\|v-u\|-\psi(\|v-u\|)\|v-u\|^{p}$$

for all $v \in K$, $z \in S(v)$ and $w \in T(v)$.

Remark 2.1. In Theorem 2.1 in [2], a multivalued mapping $v \mapsto \{N(z, w) \in X^* : z \in S(v), w \in T(v)\}$ must have been defined instead of a wrong defined mapping $v \mapsto (N(S(v),\ T(v))$.

Fan-KKM Theorem 2.3[3]. Let K be a nonempty subset of a topological vector space X and $F : K \to 2^X$ be a KKM-mapping. If $F(x)$ is closed in Y for every x in K; and there exists at least a point $x_0 \in K$ such thet $F(x_0)$ is compact, then

$$\bigcap_{x \in K} F(x) \neq \emptyset.$$

Theorem 2.4. Let X be a real reflexive Banach space, X^* be its dual space and K be a nonempty bounded closed convex subset of X. Let S, T, g, N, G, φ, ψ and ϕ be the same as $f : K \to \mathbf{R} \bigcup \{+\infty\}$ is a proper convex lower semi-continuous. Then the problem (1,1) has a solution. Moreover if G is injective, then the solution is unique.

Remark 2.2. In the proof of the inequalities (2.6) and (2.7) of Theorem 2.4 in [2], the authors made mistakes by not using the definition of the supremum of the problem (1,1) at page 196.

Remark 2.3. Theorem 2.4 also improves and extends Theorem 2.4 of [2].

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