콘크리트 구조물의 시간 의존적 효과의 불확실성

Uncertainty of Time-Dependent Effects in Concrete Structures

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ABSTRACT

This paper is aimed at proposing the sampling method to reduce variance of statistical parameters in uncertainty analysis of concrete structures. The proposed method is a modification of Latin Hypercube sampling method. This uses specially modified tables of random permutations of rank number. Also, the Spearman coefficient is used to make modified tables. Numerical analysis is carried out to predict the uncertainty of axial shortening in prestressed concrete bridge. The numerical results show that the method is efficient for uncertainty analysis of complex structural system such as prestressed concrete bridges.

1. Introduction

Prestressed concrete bridge has complex structural system because it is constructed in stepwise, and its structural behavior is time-dependent due to creep and shrinkage of concrete. In this study a modified method of Latin Hypercube sampling is proposed by which the variance of statistical parameters of outputs can be reduced more[1]. The paper represents a basic theoretical reasoning sampling scheme. This method uses specially modified tables of random permutations of rank numbers, which form input samples for a simulation procedure. Finally, numerical analyses are carried out to show that proposed modification of Latin Hypercube sampling method can result in a significant decrease of variance in the estimates of commonly used statistical parameters.

2. Method for uncertainty analysis

The modified LHS method consists of two steps to obtain an $N \times K$ design matrix. The first step is dividing each input variable into N intervals with equal probability of $1/N$. The second step is the coupling of input variables with modified tables of random permutations of rank numbers. The general expression of a equation for analytical model is as follows.

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$$Y=f(X) \tag{1}$$

where, Y = output variable

$f(\cdot)$ = deterministic analytical model

X = the vector of input variables assumed to be random ones

$= [x_1, x_2, \cdots, x_n]^T$

Every input variable $x_k (k=1,2,\cdots,K)$ is described by its known CDF with the appropriate statistical parameters. The sample x_n of input variables, $n=1,2,\cdots,N$ (N being the number of sample equal to the number of simulations) is selected in the following way.

The representative parameter is used just once during the simulation procedure and so there are N observations on each of the K input variables. N observations on each of input variable x_k are associated with a sequence of integers (rank number of intervals) representing a random permutation of integers $1,2,\cdots,N$. They are ordered in the table of random permutations of rank numbers which have N rows and K columns. The rank numbers of intervals used in the n-th simulation are represented by the n-th row in the table. It means that this table forms the strategy for obtaining an input samples.

Tables used in LHS are commonly generated randomly. The possibility does exist that a certain statistical correlation among columns of the table is randomly introduced, which may have a significant influence on the results of simulation. It naturally affects the bias and variance of the estimates obtained. So, it is required that rank permutations are mutually independent. To diminish the dependence of the input variables, some adjustment are made to $N \times K$ design matrix.

Let **R** be an $N \times K$ matrix whose columns represent K permutations of integers 1, 2, $\cdots$, N. That is, matrix **R** is identical to the table of random permutations of rank numbers used in LHS schemes. Rank correlation among columns of this matrix is described by the rank correlation matrix **T**, where element T_{ij} (i,j= 1, 2,$\cdots$, K) are the Spearman coefficients among columns i and j of **R**. It is obvious that matrix **T** is symmetrical and in the case of uncorrelated column is equal to unit matrix **I**. Consider realizations of **R** for which matrix **T** is positive definite and let **S** be a lower triangular matrix such that

$$\mathbf{S} \times \mathbf{T} \times \mathbf{S}^T = \mathbf{I} \tag{2}$$

where

$$\mathbf{S} = \mathbf{Q}^{-1} \tag{3}$$

Because matrix **T** is positive definite, the Cholesky factorization scheme can be used to find the lower triangular matrix **Q**.

$$\mathbf{T} = \mathbf{Q} \times \mathbf{Q}^T \tag{4}$$

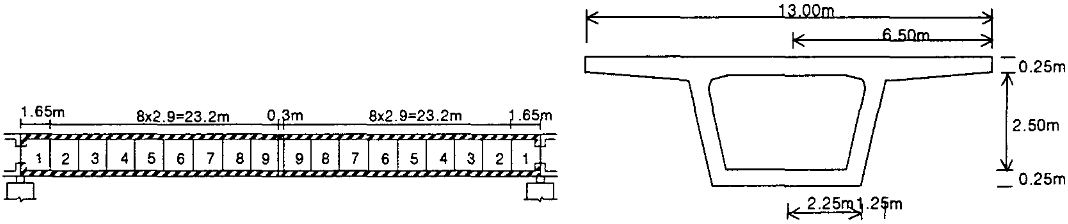
The following transformation results in an $N \times K$ matrix $\mathbf{R_B}$.

$$\mathbf{R_B} = \mathbf{R} \times \mathbf{S}^T \tag{5}$$

Statistical correlation among columns of this matrix is described by the rank correlation matrix $\mathbf{T_B}$. Matrix $\mathbf{T_B}$ should be close to **I**. That is, the difference between appropriate elements in matrix $\mathbf{T_B}$ and matrix **I** is lower than in the case of matrix **T** and matrix **I**. The values in each column of input matrix **R** can be now arranged so that they will have the same ordering as the corresponding column of matrix $\mathbf{R_B}$. As a result, the rank correlation matrix **T** equals $\mathbf{T_B}$ and the rank correlation among columns of **R** and also among columns of the table of random permutations of rank numbers is reduced.

3. Numerical analysis

Particular attentions have given to the uncertainty problem of creep and shrinkage[1,2]. To study the effectiveness of proposed method, probabilistic analysis to predict axial shortening of prestressed concrete box girders are performed and the statistical parameters of outputs are estimated. Span and cross section geometry of girder for numerical example are shown in Fig. 1. Estimates are compared for 10, 20 and 30 simulations. Sampling is repeated 10 sets for each number of simulations. In the case of LHS the tables are randomly generated in every run. In the case of modified LHS the tables are rearranged and statistical correlation of their columns is diminished. Intervals have the same probability $1/N$ and representative parameters are taken at the centroid of intervals.



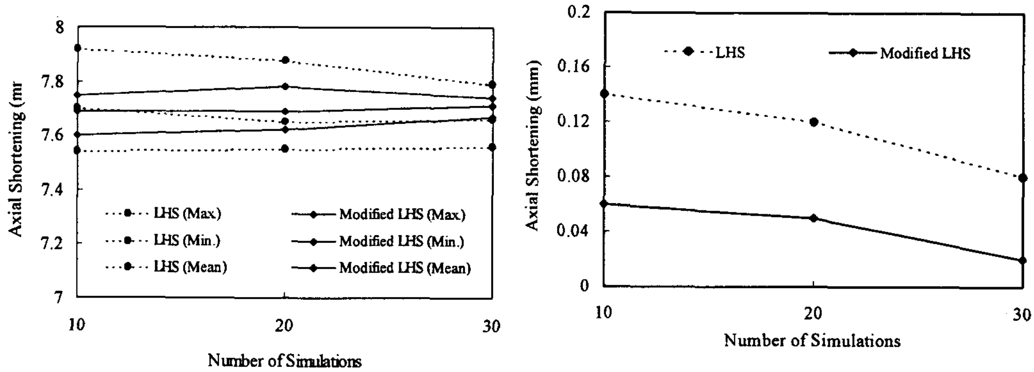
(a) Span segment geometry (b) Typical cross section

Fig. 1 Span and cross section geometry

Statistical properties of the axial shortening of prestressed concrete box girder at 10,000days after construction are presented in Fig. 2 and Fig. 3. The results obtained by modified LHS are plotted as a full line and by LHS as a dashed line in the figures. Each figure consists of part (a) and part (b). In each figure, part (a) gives 3 plots – (1) mean value of an appropriate statistical parameter obtained from 10 sets, (2) minimum value of an appropriate statistical parameter obtained from 10 sets, and (3) maximum value of an appropriate statistical parameter obtained from 10 sets. On the horizontal axis the numbers of simulations are plotted. On the vertical axis the value of appropriate parameter Z are plotted. Statistics of mean values are shown in Fig. 2. Mean value in case of modified LHS is almost same as that in case of LHS. However, the difference between maximum value and minimum value in case of modified LHS is much smaller than that in case of LHS. Also, standard deviation in case of modified LHS is much smaller than that in case of LHS.

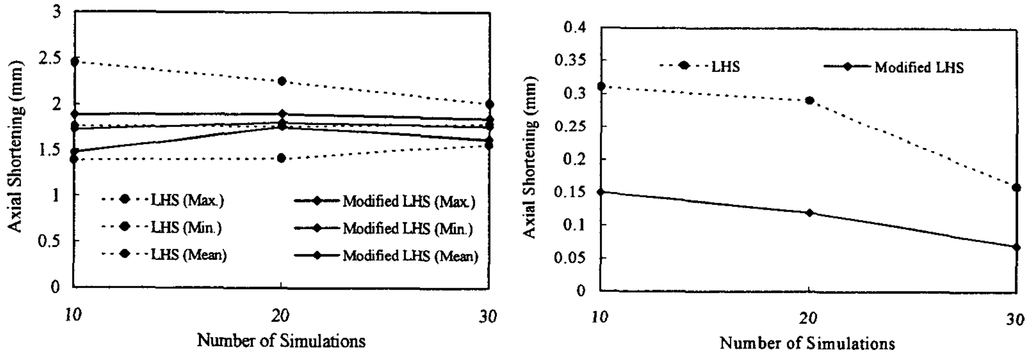
Statistics of mean values are shown in Fig. 2. Mean value in case of modified LHS is almost same as that in case of LHS. However, the difference between maximum value and minimum value in case of modified LHS is much smaller than that in case of LHS. Also, standard deviation in case of modified LHS is much smaller than that in case of LHS.

Statistics of standard deviations and statistics of coefficient of variation are shown in Fig. 3. Statistical characteristics of Fig. 3 are similar to those of Fig. 2. Mean value in case of modified LHS is almost same as that in case of LHS. The difference between maximum value and minimum value in case of modified LHS is much smaller than that in case of LHS. Standard deviation in case of modified LHS is much smaller than that in case of LHS.



(a) Mean value, maximum value and minimum value (b) Standard deviation

Fig. 2 Statistics of mean values



(a) Mean value, maximum value and minimum value (b) Standard deviation

Fig. 3 Statistics of standard deviations

4. Conclusions

An effective probabilistic analysis method is proposed for uncertainty analysis of concrete structures. The results of numerical analysis show that the difference between maximum and minimum value for mean values, standard deviation and coefficient of variation by proposed method is much smaller than that by conventional LHS method.

References

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2. Li, C. Q. and Melchers, R. E., "Reliability analysis of creep and shrinkage effects." *Journal of Structural Engineering*, ASCE, Vol. 118, No. 5, 1992, pp.1532-1548.