

# **LES of Geophysical Turbulence and its Application**

**노 의 근**  
**(연세대 대기과학과)**



# **LES of Geophysical Turbulence and its Application**

*Yign Noh*

*Dept. Atmospheric Sciences, Yonsei  
Univ.  
Seoul, Korea*

## **1. Why LES?**

## **2. Application to Oceanography**

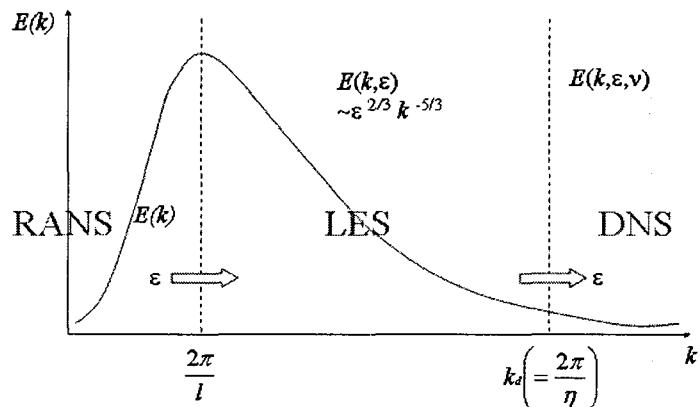
- 2.1 the ocean mixed layer model
- 2.2 the ocean mixed layer under the heat flux
- 2.3 particle settling in the ocean mixed layer
- 2.4 open-ocean deep convection

## **3. Application to Meteorology**

- 2.1 the PBL model development
- 2.2 heterogeneous effects in the PBL
- 2.3 a cumulus cloud

## **4. Further Application at RIAM**

## What is LES?



- simulate most of turbulent eddies
- parameterized turbulent eddies in the inertial subrange

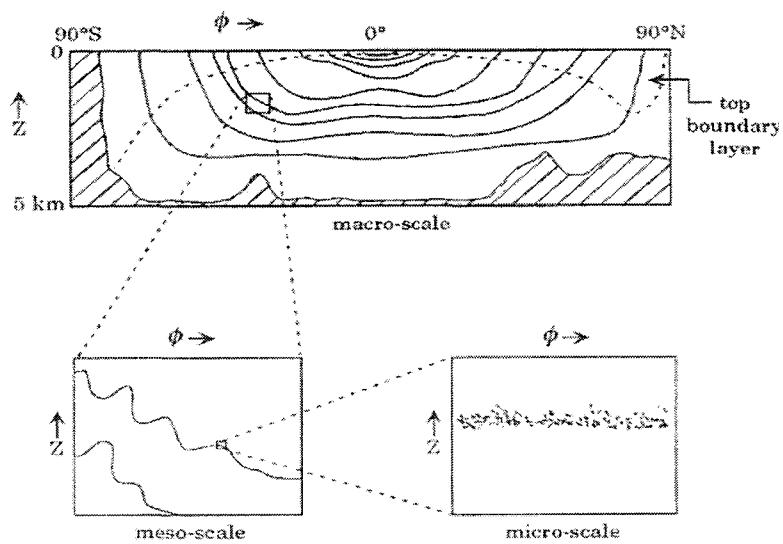
## We can use LES

1. to understand the fundamental physics of turbulence  
ex) cascade process, turbulent mixing, particle dispersion
2. to investigate the 3-dimensional structure of turbulence  
ex) coherent structure, vortices, plumes, Langmuir circulation, horizontal inhomogeneity
3. to investigate the motion of suspended particles in turbulent eddies simulated by LES
4. to provide the 'pseudo' observational data
  - obtained under the ideal condition
  - provide the whole spectrum of information

## Parameterization of SGS Turbulence in the Climate Model

- Grid sizes of the climate model (*even at Earth Simulator*) are still much larger than the eddy size of turbulence
  - ~ 10-100 km (horizontal)
  - ~ 10-100 m (vertical)
- ⇒ We have to parameterize the turbulence within the grid.

ex) OGCM - ocean mixed layer, ocean deep convection,  
meso-scale eddies,  
AGCM – atmospheric boundary layer, cumulus convection,  
cloud microphysics,  
Biogeochemical Model  
- particle flux and biogeochemical process in the ocean  
- aerosol and chemical process in the atmosphere

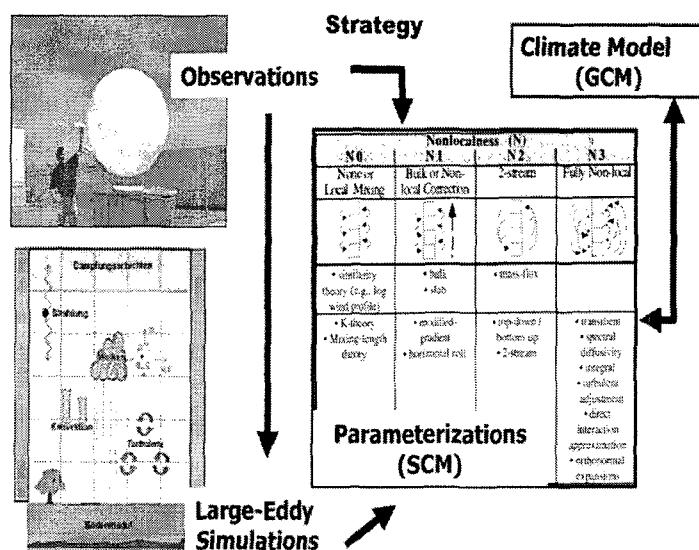


# Verification of the SGS Parameterization (e.g., Ocean Mixed Layer)

1. Comparison with the observation data
2. Improvement of the OGCM results with a new mixed layer model
3. Analysis of more realistic turbulence simulation results, i.e., LES
  - One can examine not only the performance of the model, but also the hypotheses used in the model.
  - One can examine the model under the ideal condition.



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# Parallel Large eddy simulation Model (PALM)

( Univ. Hannover, Germany/Yonsei Univ., Korea)

- equation

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\varepsilon_{ijk} f_{ijk} \bar{u}_k + g \frac{\bar{\theta} - \theta_0}{\theta_0} \delta_{ij} - \frac{1}{\rho_0} \frac{\partial p}{\partial x_j} - \frac{\partial}{\partial x_j} \bar{u}_i \bar{u}_j$$

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0$$

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{u}_i \frac{\partial \bar{\theta}}{\partial x_i} = - \frac{\partial}{\partial x_j} \bar{u}'_j \bar{\theta}'$$

- SGS parameterization (Deardorff scheme)

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = -(\bar{u}'_i \bar{u}'_j) \frac{\partial \bar{u}_i}{\partial x_j} + g \frac{\bar{u}'_j}{\theta_0} \overline{\left( e' + \frac{p'}{\rho_0} \right)} - \varepsilon$$

$$\bar{u}'_i \bar{u}'_j = K_m \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \frac{2}{3} e \delta_{ij}$$

$$\bar{u}'_i \bar{\theta}' = -K_h \frac{\partial \bar{\theta}}{\partial x_i} \quad l = \Delta s = (\Delta x \Delta y \Delta z)^{\frac{1}{3}}$$

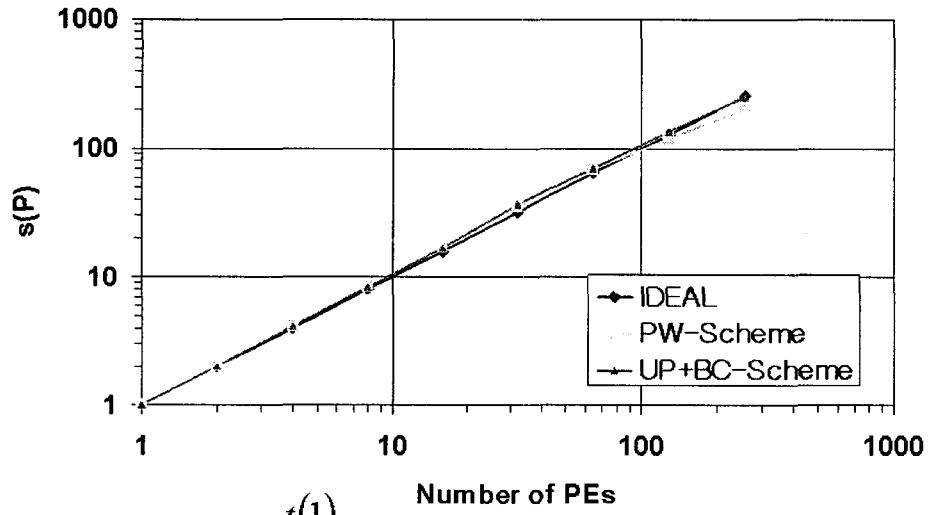
$$\overline{\left( e' + \frac{p'}{\rho_0} \right)} = -2K_m \frac{\partial \bar{e}}{\partial x_j} \quad l = \min \left[ \Delta s, 0.76 \sqrt{e} \left( \frac{g}{\theta} \frac{\partial \bar{\theta}}{\partial z} \right)^{-\frac{1}{2}} \right]$$

$$K_m = -0.10l \sqrt{e} \quad \varepsilon = C \frac{e^{-2/3}}{l}$$

$$K_h = \left( 1 + \frac{2l}{\Delta s} \right) K_m \quad C = 0.19 + 0.51 \frac{l}{\Delta s}$$

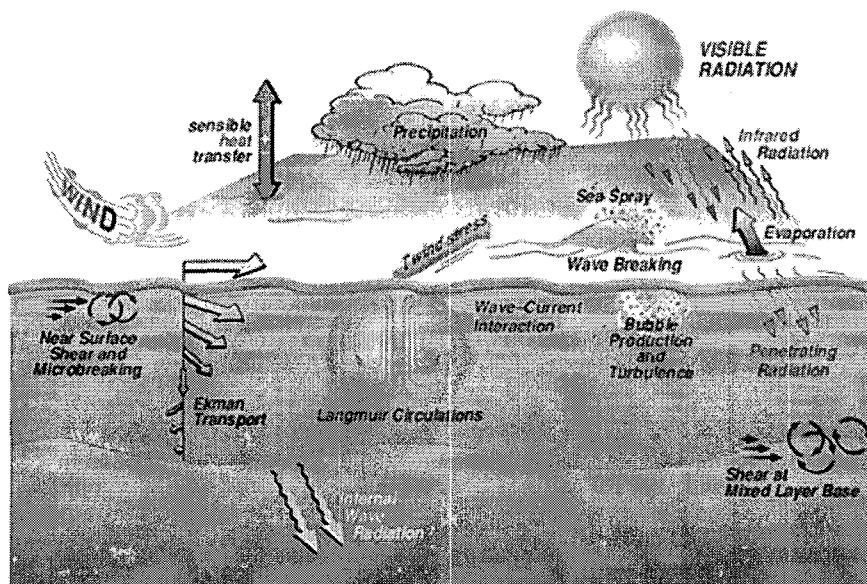
## Scalability and performance

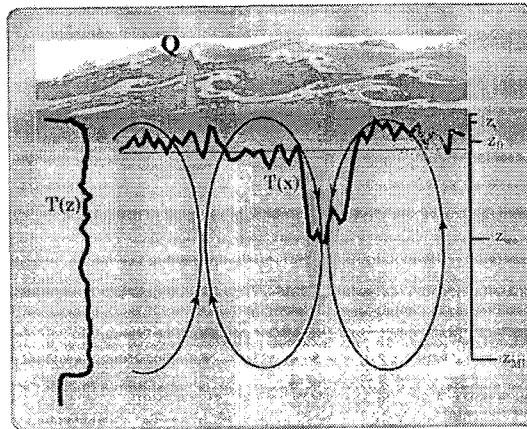
✓ Results for SGI/Cray-T3E (160\*160\*64 gridpoints)



$$\text{speedup: } s(P) = \frac{t(1)}{t(P)}$$

## 2.1 Ocean Mixed Layer





Sketch of processes relevant for near-surface temperature fine structure

*What are the roles of wave breaking and Langmuir circulation in the turbulent structure in the ocean mixed layer?*

## LES of the Ocean Mixed Layer

- Parallel Large Eddy Simulation Model (PALM; Hannover/Yonsei)
- Inclusion of the effects of Langmuir circulation and wave breaking
- Noh et al. (JPO 2004)

### \* Equation

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + g \frac{\rho - \rho_0}{\rho_0} \delta_{ij} - \frac{\partial}{\partial x_j} \overline{u_i u_j} - \varepsilon_{ijk} f_j (u_k + v_{sk}) \\ + \varepsilon_{ijk} v_j w_k + F(k, z, u_*)$$

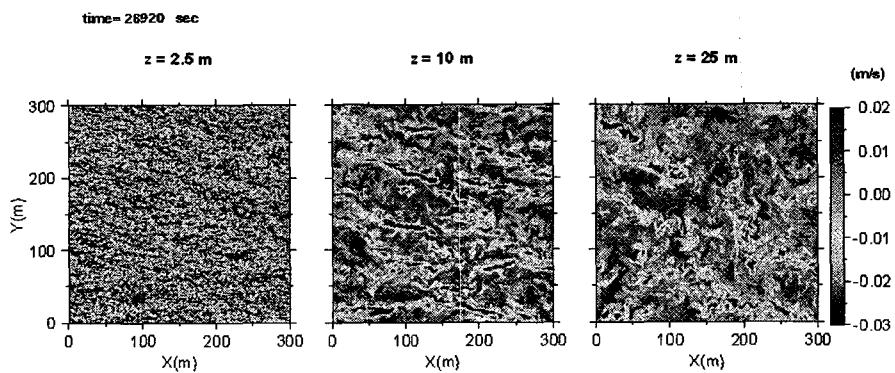
$$\# F(k, z; u_*) = \frac{\alpha u_*}{\tau} G(x, y, t) \delta(k - 2\pi/z_0) \delta(z) \delta(1 - \delta_{ij}),$$

$G(\mu; \sigma)$  = Gaussian random function with mean  $\mu$  and variation  $\sigma^2$

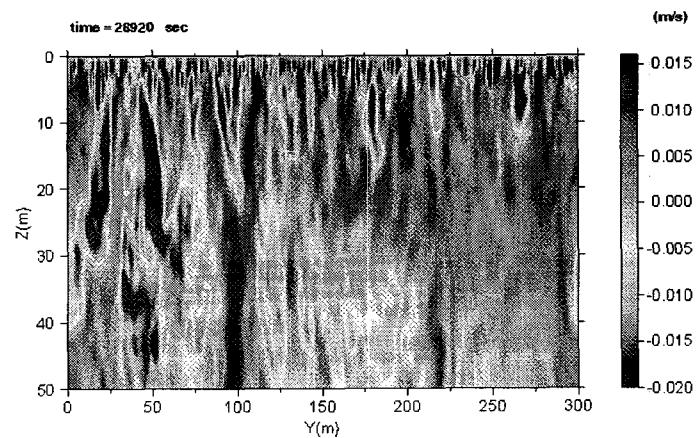
$z_0$  = length scale of eddy at  $z = 0$  (= 1.25 m)

$\diamond_s$  = Stocks velocity

$$= \left( \pi \frac{h}{\lambda} \right)^2 \sqrt{\frac{g \lambda}{2\pi}} \exp\left(-\frac{4\pi z}{\lambda}\right) \quad (h = \text{wave height}, \lambda = \text{wave length})$$

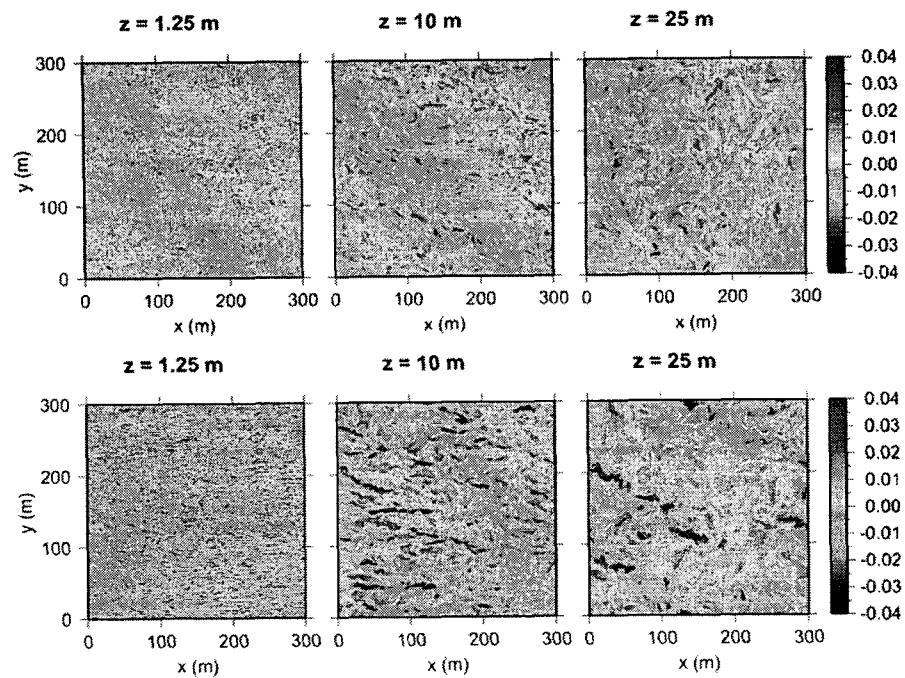


SBH\_homo; w (H= 1.0,  $\alpha = 0.8$ ,  $u_r = 0.01$ ,  $Q_0 = 0$ )



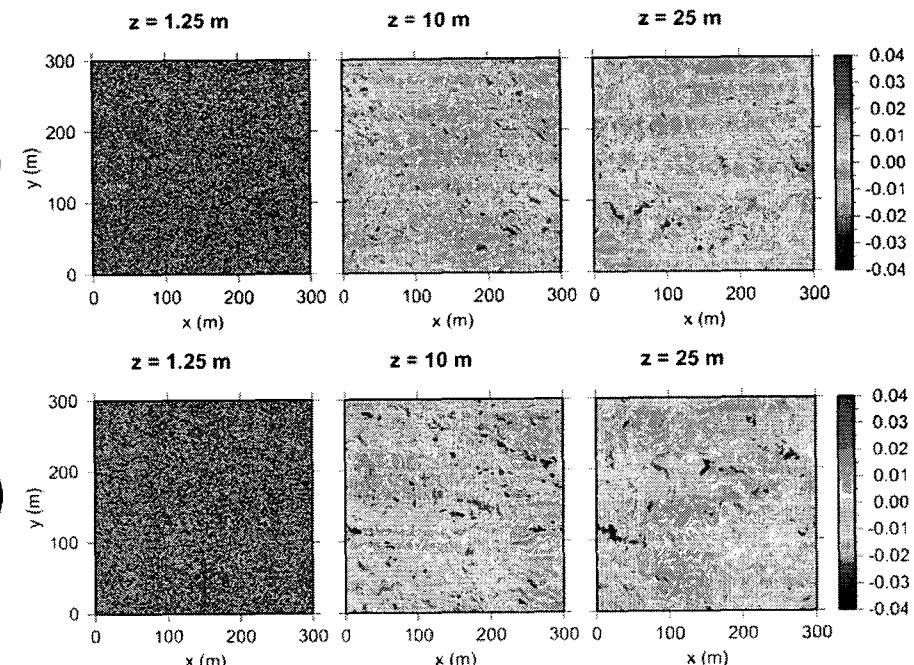
SBH\_homo; w (H= 1.0,  $\alpha = 0.8$ ,  $u_r = 0.01$ ,  $Q_0 = 0$ )

**EXP\_O**

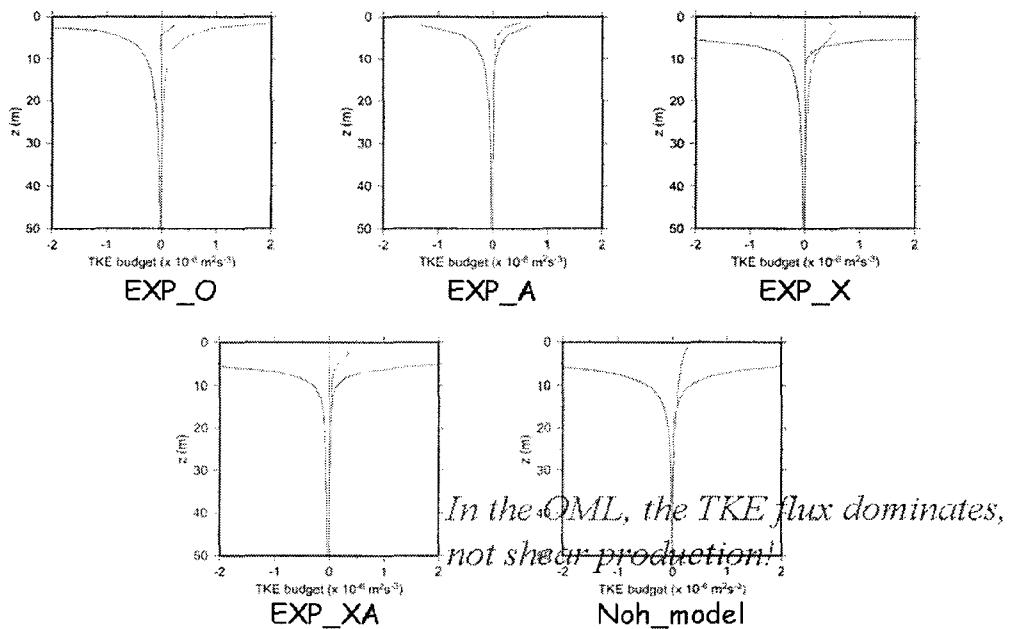
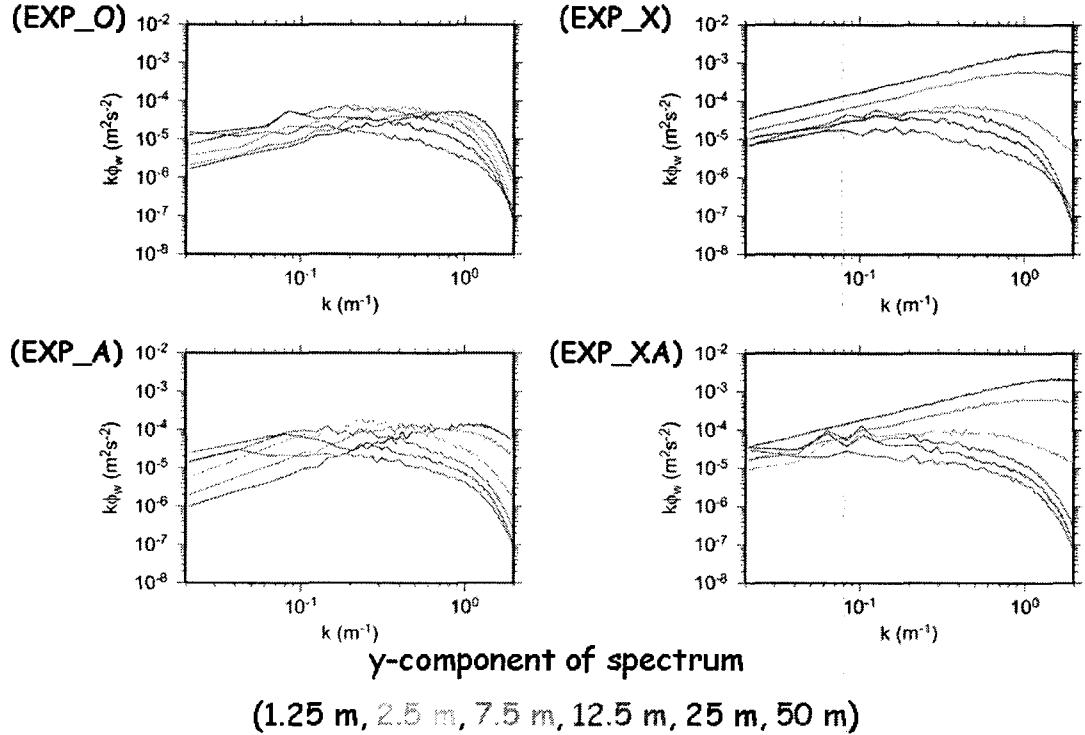


Horizontal cross-section of vertical velocity( $W$ ) ( $\text{m/s}$ );  
EXP\_O (no WB & LC), EXP\_A (LC only)

**EXP\_X**

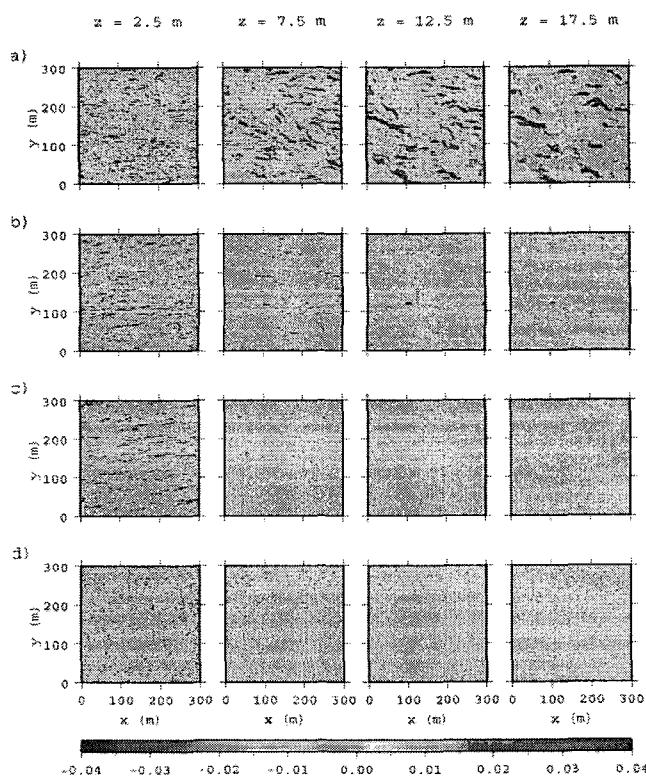
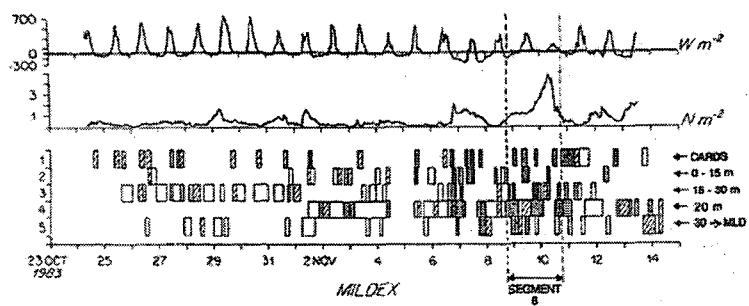


Horizontal cross-section of vertical velocity( $W$ ) ( $\text{m/s}$ );  
EXP\_X (WB only), EXP\_XA (both WB & LC)

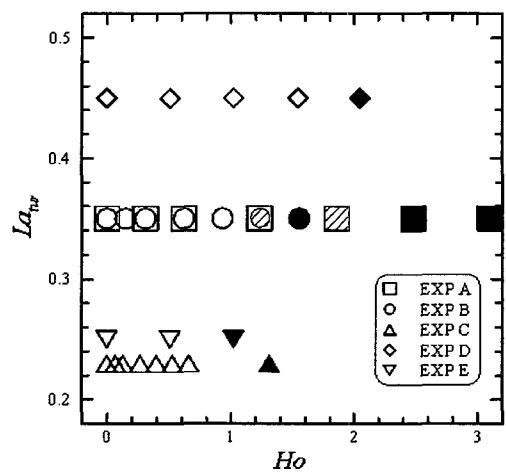


Fig; TKE budget (TKE flux, shear production, dissipation)

## 2.2 Ocean Mixed Layer under the Surface Heating

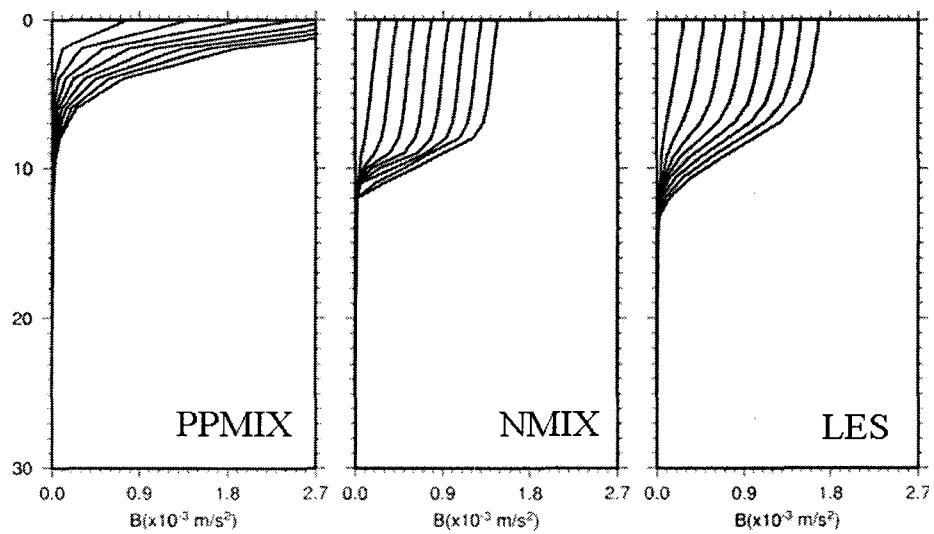


Destruction of LC  
with increasing  
heating

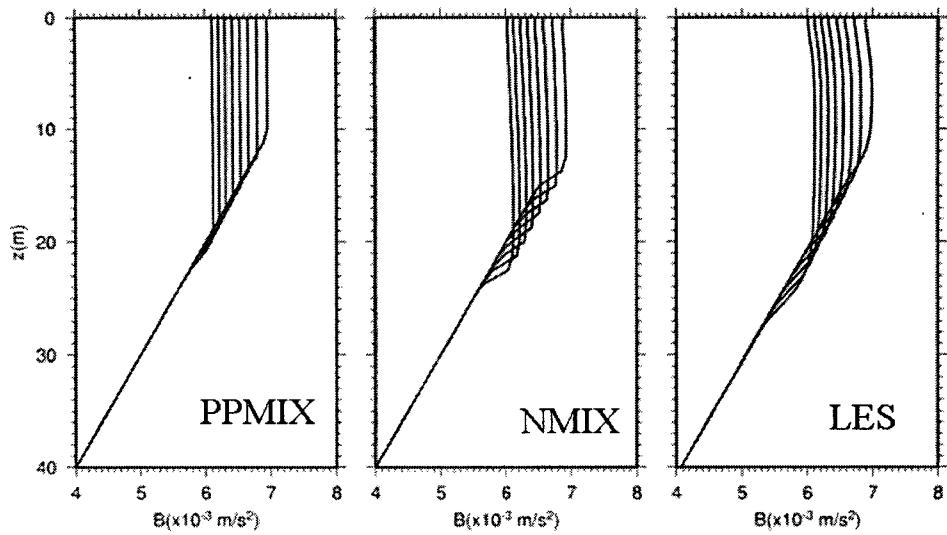


A regime diagram for the breakdown of LC

$$\Rightarrow Ho_c (= 2B_0/kU_s u_*^2) \sim 1 - 2$$



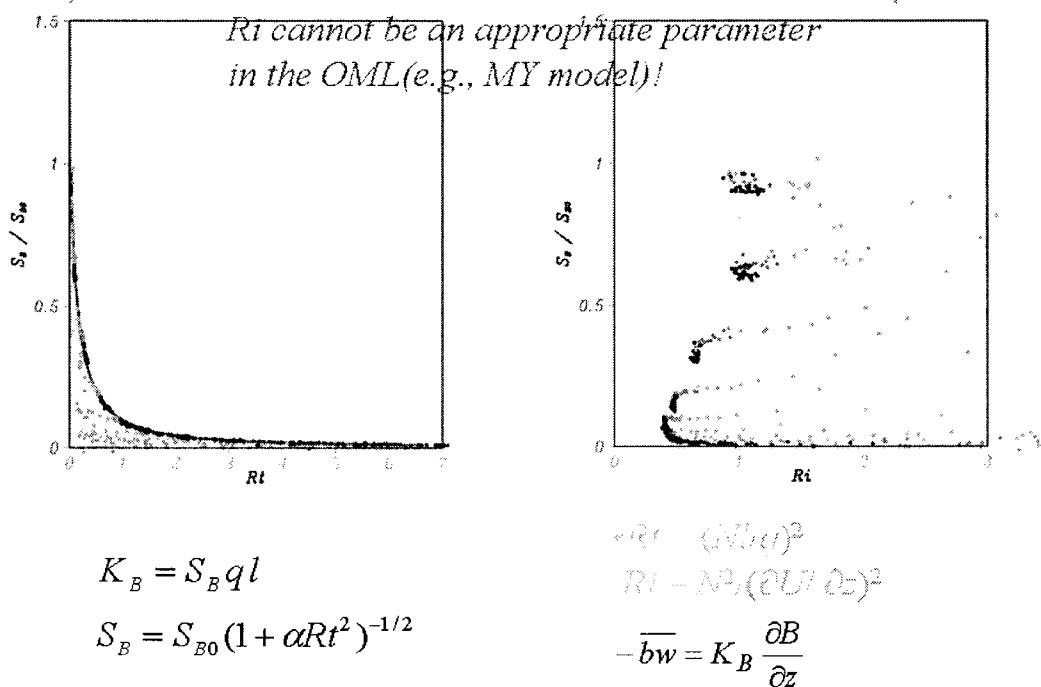
Evolution of temperature profiles under the heating

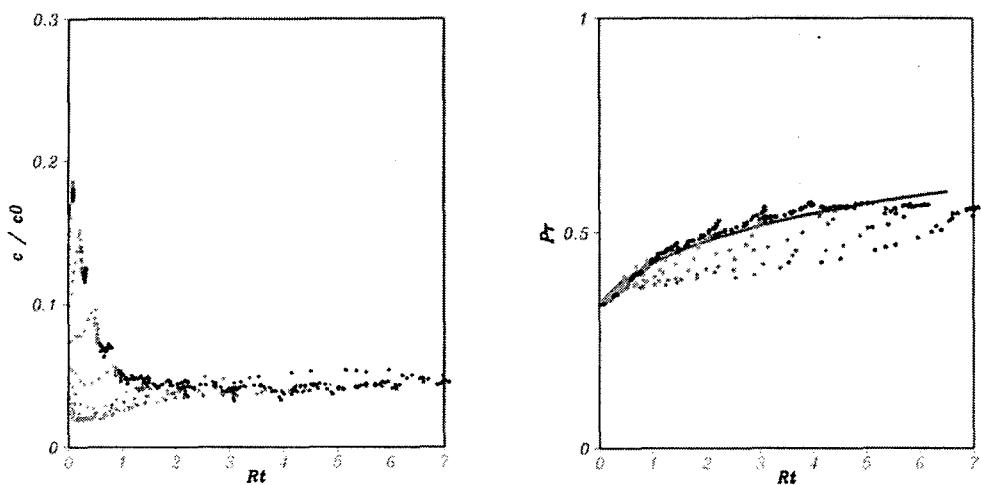


### Evolution of temperature profiles under the cooling

- PPMIX - convective adjustment only
- NMIX- convective adjustment + entrainment

### Effects of Stratification on $K_h$



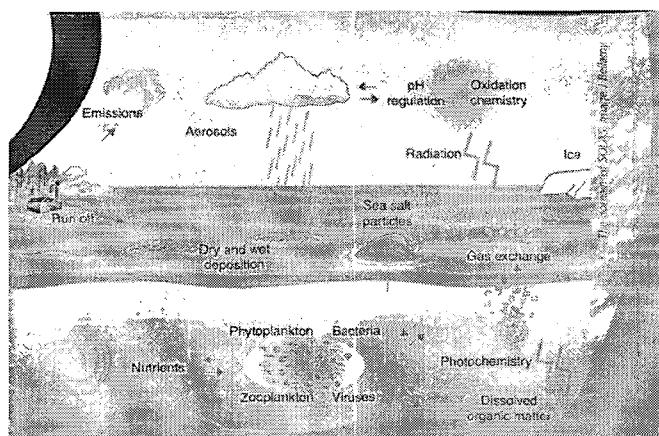


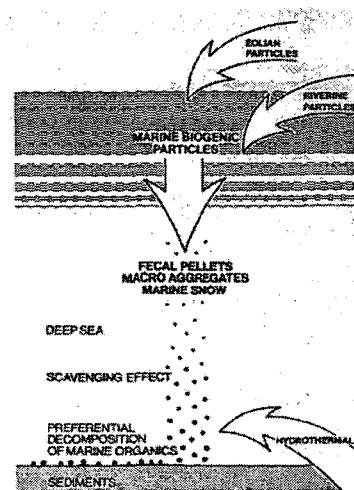
$$Pr = Pr_0 (1 + \alpha_p Rt)^{1/2} \quad \alpha_p \approx 0.5$$

### Effects of Stratification on $\varepsilon$ and $Pr$

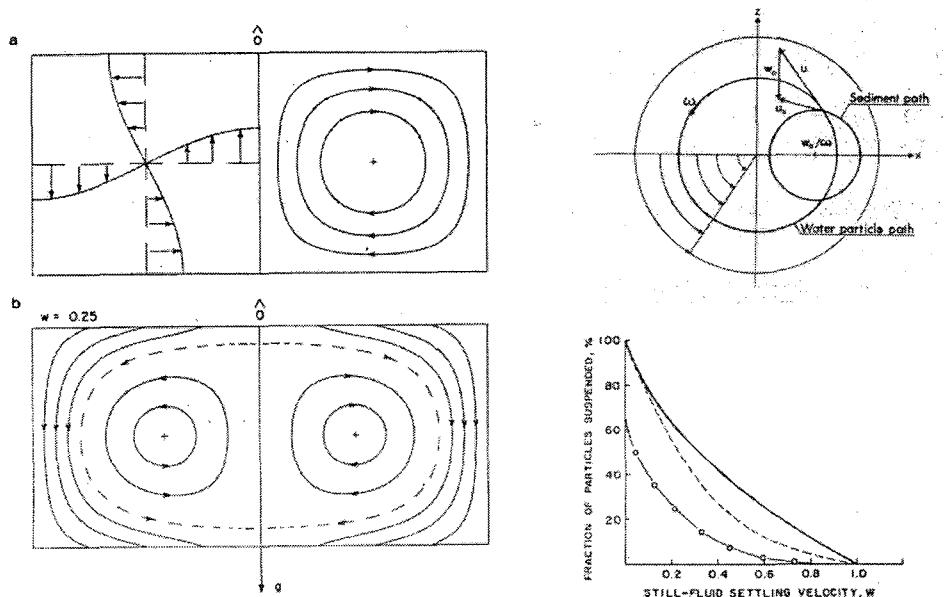
## 2.4 Particle Settling in the Ocean Mixed Layer

(Noh et al. *JFM*, submitted)





biological pump by particle flux



Stommel (1949), Maxey and Riley (1986);

- Particles can be suspended indefinitely in the presence of large vortices

## ♦ Motion of a Particle

$$\frac{dV_i}{dt} = \frac{1}{\tau_p} (u_i - V_i) + g' \delta_{i3}$$

$$\tau_p = \frac{2\rho_p a^2}{9\rho_f v} : \text{responding time scale (Re} \ll 1)$$

$$g' = \frac{\rho_p - \rho_f}{\rho_p} g : \text{reduced gravity}$$

$V_i$ : Lagrangian velocity of a particle

$u_i$ : Eulerian velocity of a fluid

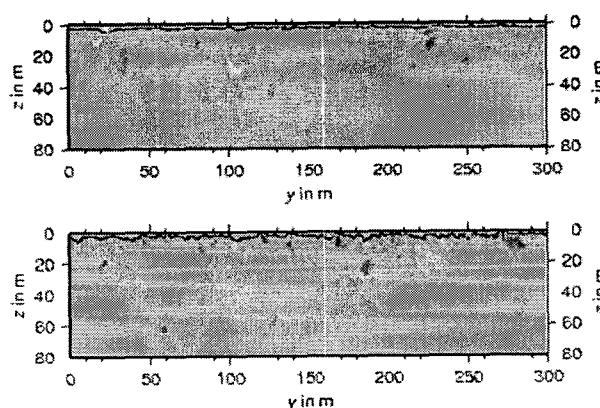
$a$ : the radius of a particle

$\rho_p$ : density of a particle

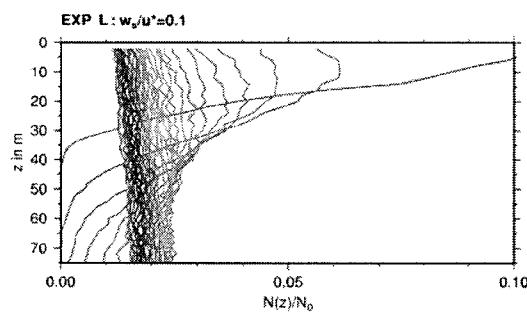
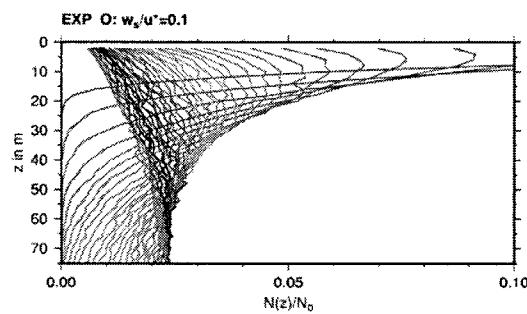
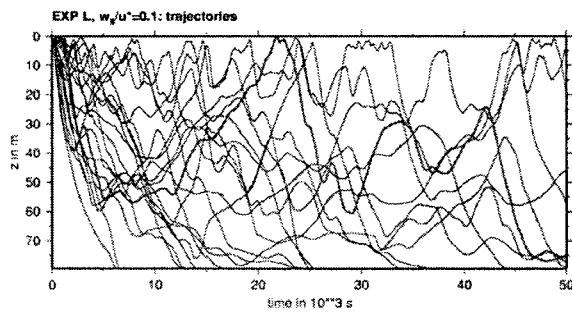
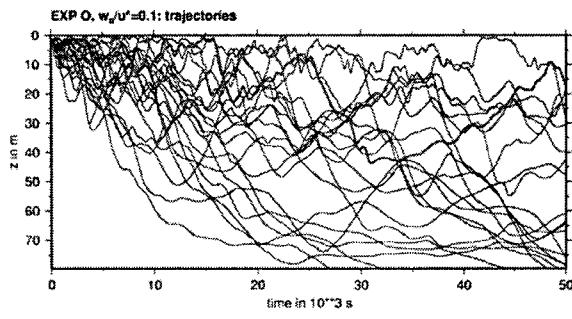
$\rho_f$ : density of a fluid

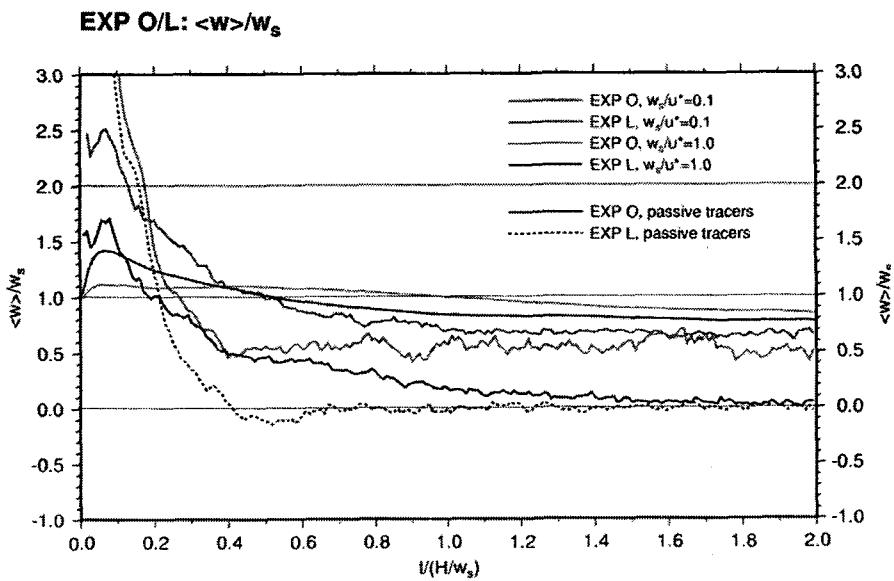
In the still fluid ( $u_i = 0$ ),

$$W = w_s (= \tau_p g')$$



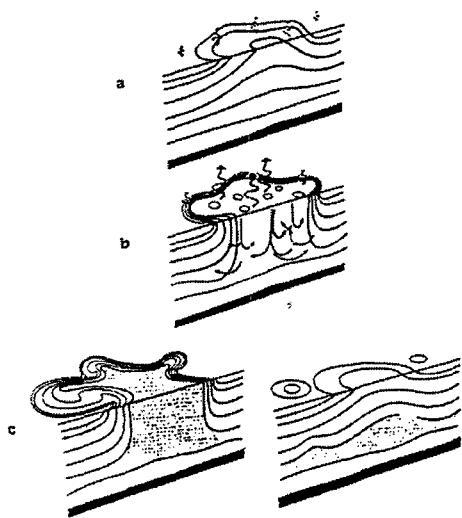
(a) EXP O, (b) EXP L (Langmuir circulation)

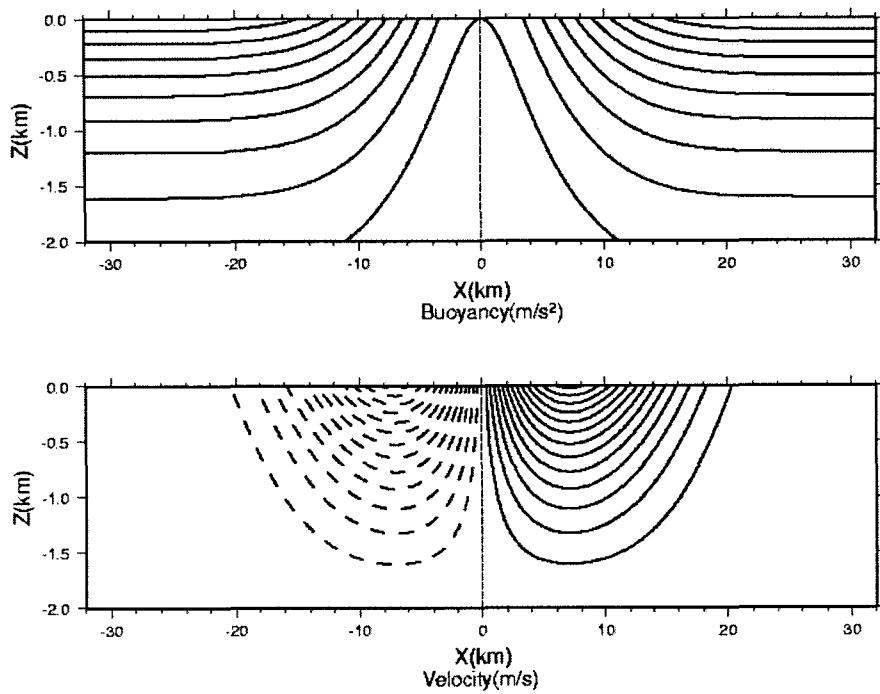




## 2.3 Open-Ocean Deep Convection

(Noh et al. *JPO* 2003)





Preconditioning for the onset of the open-ocean Deep Convection

### **Experiment**

- preconditioning

$$b = Ae^{-\beta z} \left[ 1 - Re^{-(r/l)^2} \right]$$

$$u_\theta = 2 \frac{AR}{f\beta l^2} r e^{(r/l)^2} \left( e^{-\beta z} - e^{-\beta H} \right)$$

$$\beta = 10^{-3} \text{ m}^{-1} \quad l = 10^4 \text{ m} \quad A, \beta \rightarrow \text{background stratification}$$

❖ L = eddy size

R = intensity of eddy

- surface buoyancy flux;

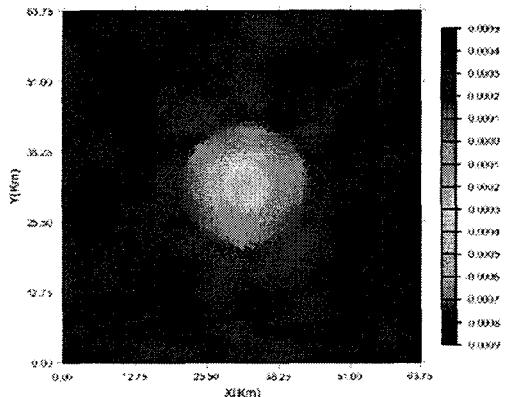
$$B = 10^{-7} \text{ m}^2 \text{s}^{-3}$$

- dimensions

$$L = 64 \text{ km}, \quad H = 2 \text{ km}$$

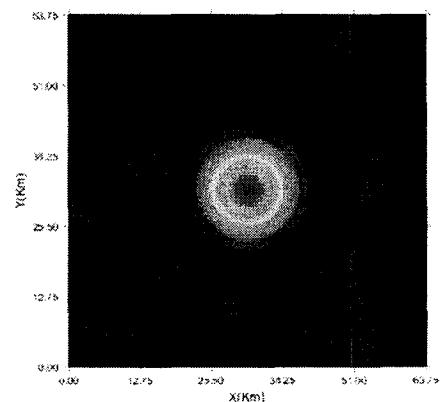
$$\Delta x = 250 \text{ m}, \quad \Delta z = 100 \text{ m} \quad (256 \times 256 \times 20)$$

$$A = 0.3 \times 10^{-4} \text{ ms}^{-2}$$



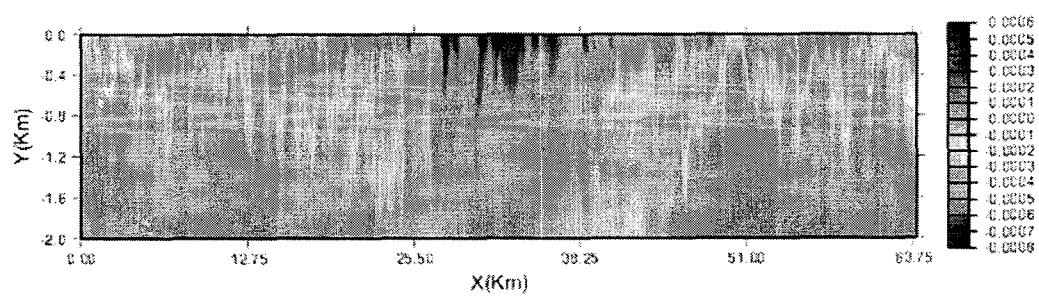
distributed convection

$$A = 6.7 \times 10^{-4} \text{ ms}^{-2}$$

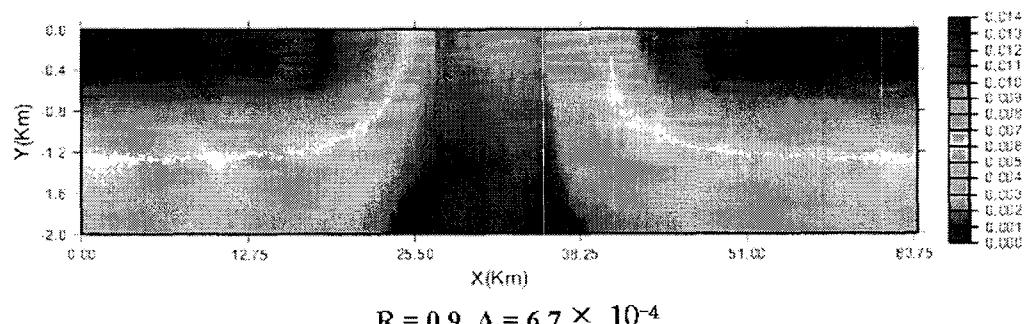


localized convection  
- generation of baroclinic eddies

Horizontal cross section of potential temperature

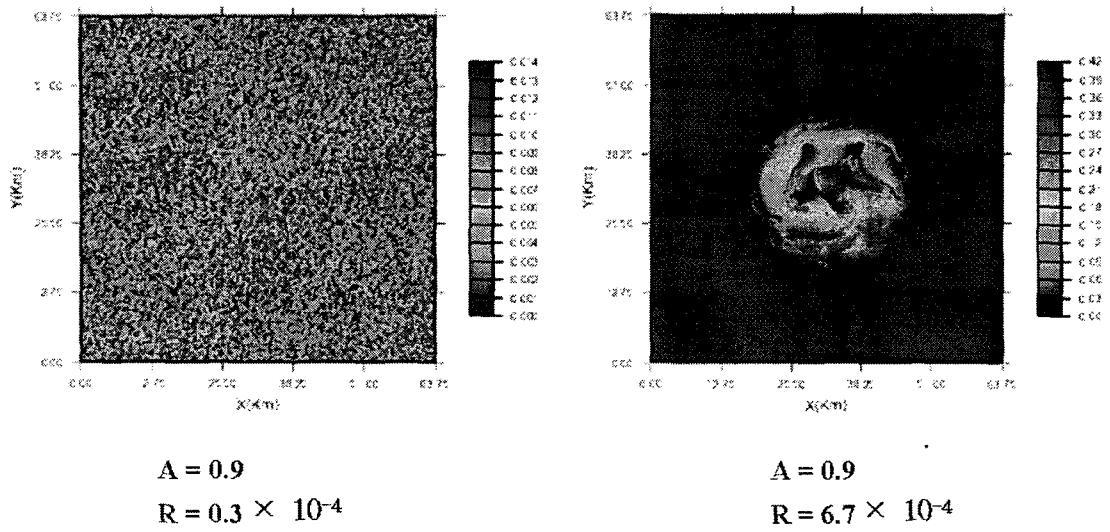


$$R = 0.9, A = 0.3 \times 10^{-4}$$

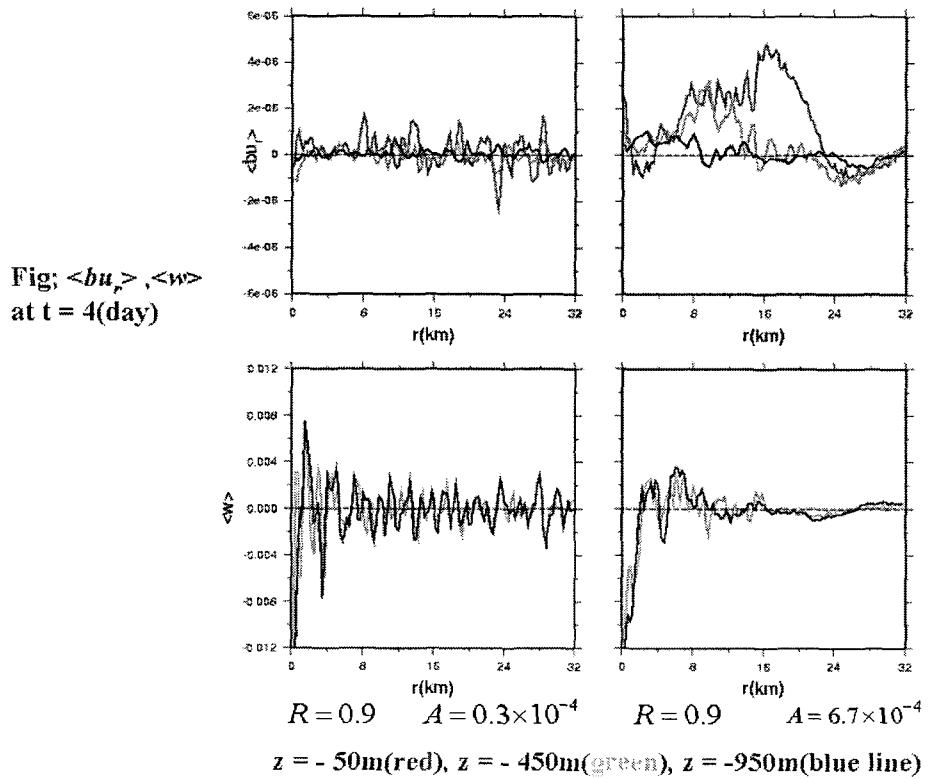


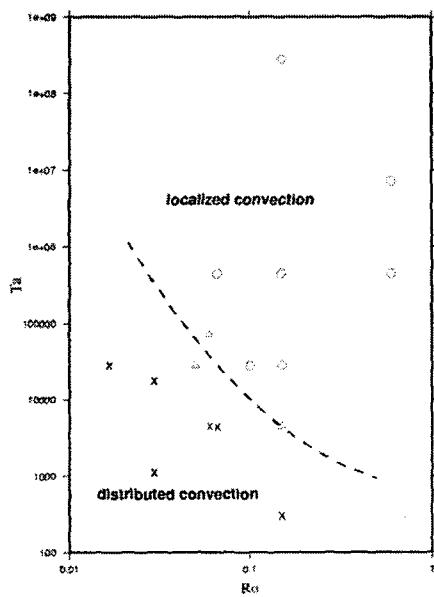
$$R = 0.9, A = 6.7 \times 10^{-4}$$

**Fig.** Vertical cross-section of potential temperature ( $\theta$ -300K)



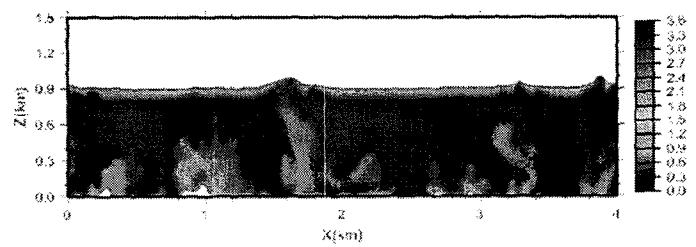
**Fig.** Horizontal cross section of  $U^2 + V^2$



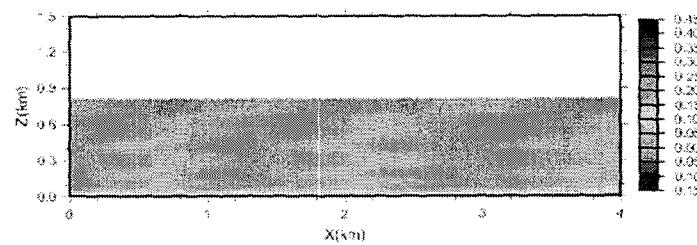


A regime diagram of convection pattern

### 3.1 PBL Modeling (Noh et al. *BLM* 2003)



convective boundary layer



shear-driven boundary layer

## Troen & Mahrt Model (BLM, 1986)

✓ K-profile model

(KPP model for the ocean mixed layer)

✓ Non-local mixing

$$-\overline{w' u'} = K_m \left( \frac{\partial u}{\partial z} - \gamma_m \right) \quad -\overline{w' \theta'} = K_h \left( \frac{\partial \theta}{\partial z} - \gamma_h \right)$$

$$K_m = w_s h k \frac{z}{h} \left( 1 - \frac{z}{h} \right)^2, \quad z > \varepsilon h \quad K_h = \text{Pr}^{-1} \times K_m$$

$$w_s = \left( u_*^3 + 7 \varepsilon k w_*^3 \right)^{1/3} \quad \Rightarrow \text{ by matching } K_m = u_* k z \Phi_m^{-1}(z/L)$$

at  $z < \varepsilon h$  (surface layer). ( $\varepsilon=0.1$ )

$\gamma_m = 0 \quad \Rightarrow$  no non-local mixing for momentum

$$\gamma_h = C \frac{\overline{w' \theta'_0}}{w_s h}$$

✓ Entrainment is given by a critical  $Ri$

$$h = Ri \frac{T_0 |\tilde{u}(h)|^2}{g(\theta_v(h) - \theta_T)}$$

$$\theta_s = \theta(z_1) + \theta_T \quad \theta_T = C \frac{\overline{w' \theta'_0}}{w_s}, \quad (C \approx 6.5)$$

✓  $\text{Pr} = \left[ \frac{\Phi_h}{\Phi_m} \left( \frac{z}{h} \right) + k \frac{z}{L} C \right]^{-1}$

( $L$  : Monin-Obukhov length scale  $= \frac{(\tau/\rho)^{3/2}}{k(g/\theta_0)(H_0/\rho c_p)}$  )

❖ if unstable

$$\phi_m = (1 - 16z/L)^{-1/4}$$

$$\phi_h = (1 - 16z/L)^{-1/2}$$

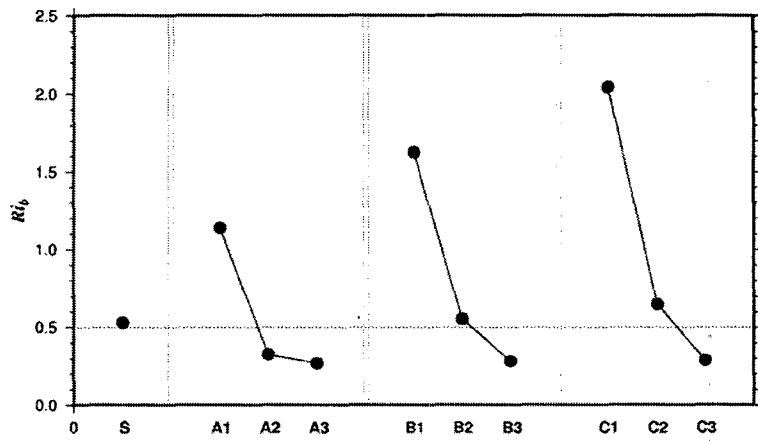


Fig.10; Variation of the bulk Richardson number at the inversion layer

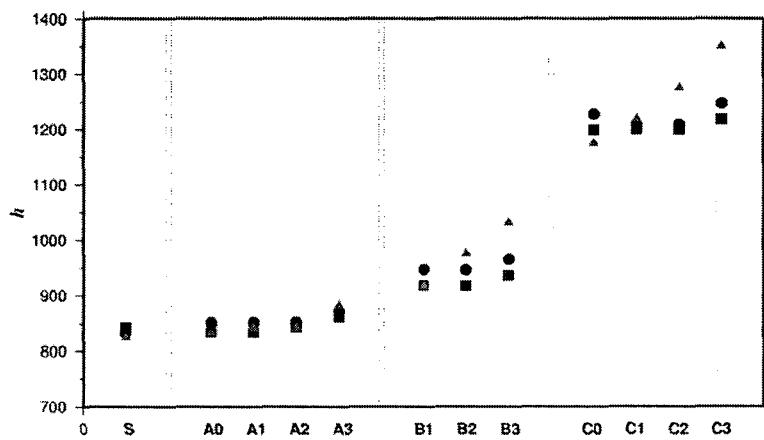
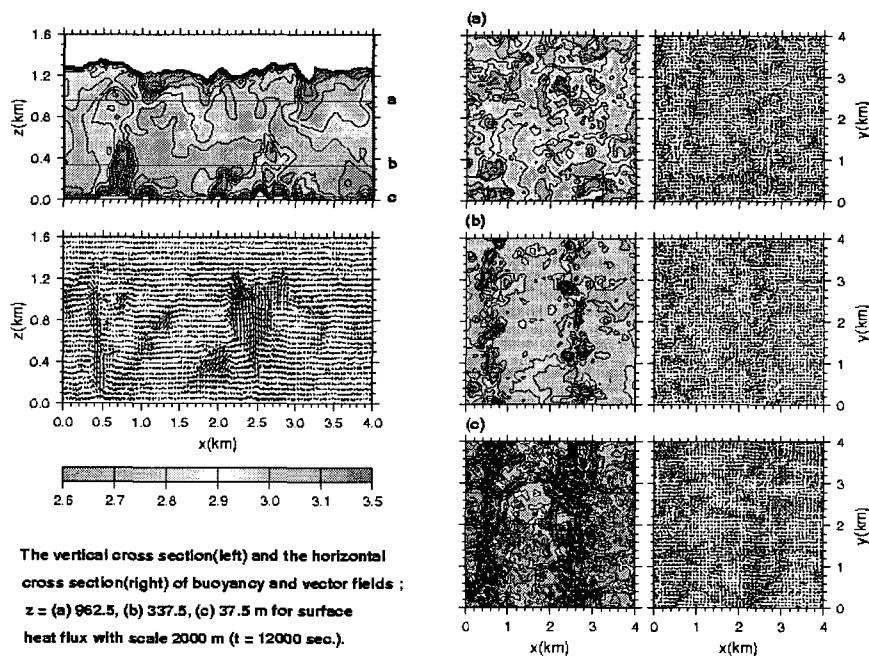
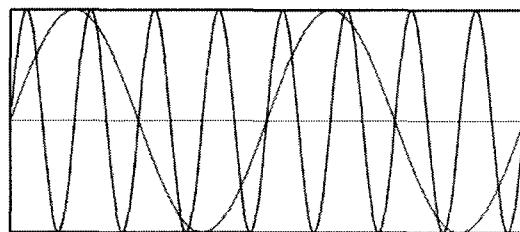


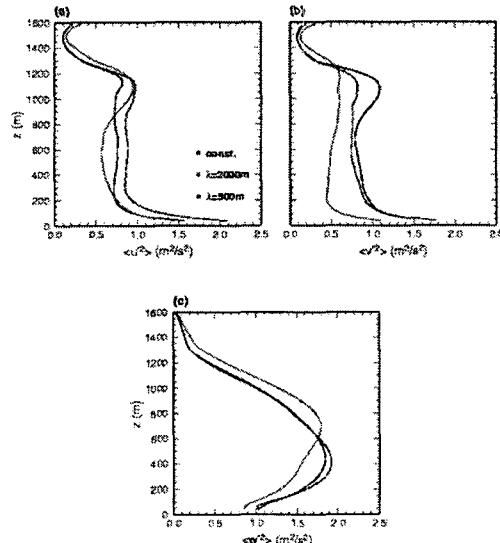
Fig.9; The comparison of the PBL height prediction at  $t = t_0 + 6000$  s  
 $h_{PBL}$ (blue square),  $h_{LES}$ (black circle),  $h_{TM}$ (red triangle)

### 3.2 Heterogeneous Convection

(Kim et al., *BLM* 2004)

	Surface heat flux (K m/s)	Etc.
EXP1	$Q_0 + A \sin(2\pi x/\lambda)$	$Q_0 = A = 0.2$ $\lambda = 2000 \text{ m}$
EXP2	$Q_0 + A \sin(2\pi x/\lambda)$	$Q_0 = A = 0.2$ $\lambda = 500 \text{ m}$



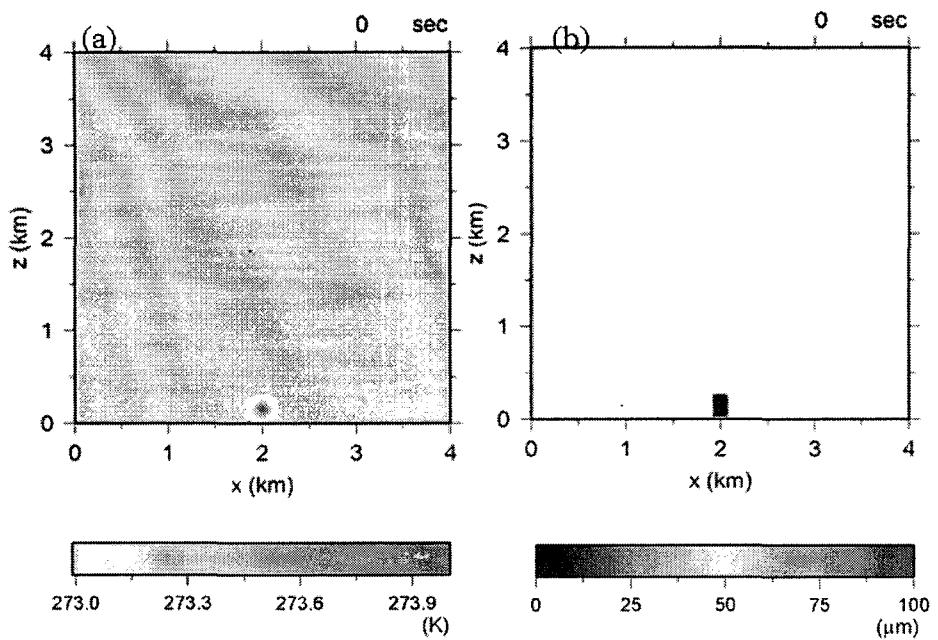


Horizontally averaged vertical profiles of (a)  $u$ -variance, (b)  $v$ -variance and (c)  $w$ -variance, for heat fluxes from an inhomogeneous surface with scale 2000 m (EXP2 ; red), 500 m (EXP6 ; blue), and from homogeneous surface (EXP0 ; black) ( $t \approx 12000$  sec.).

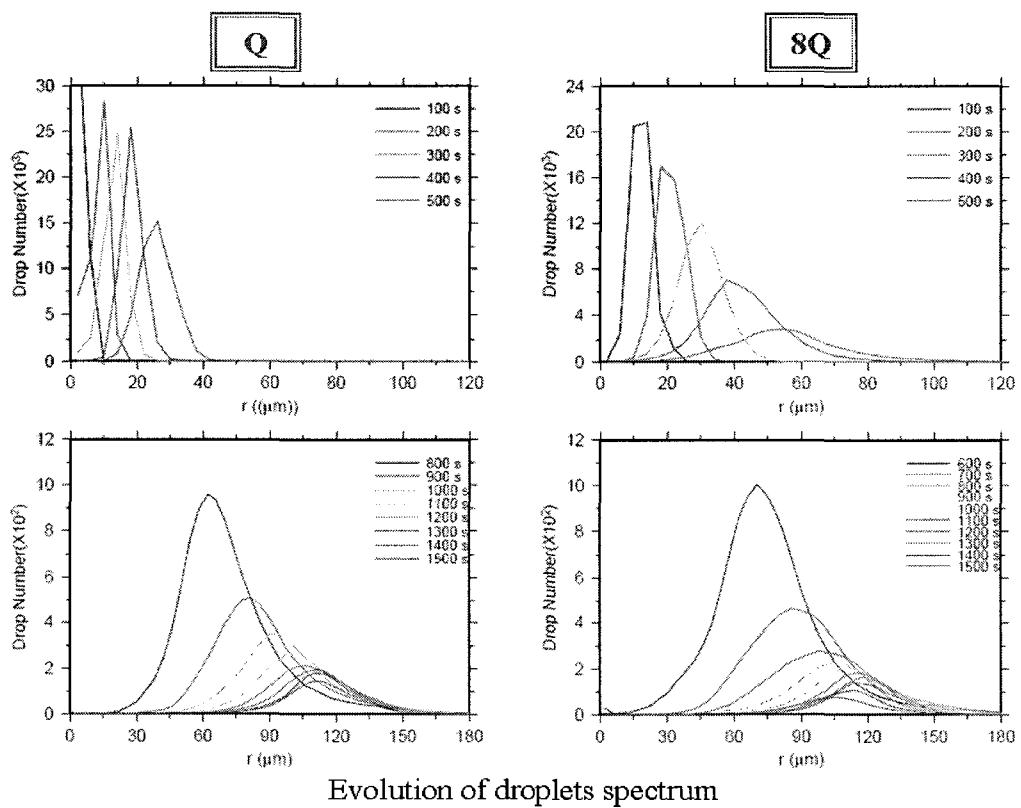
### 3.3 LES of a Cumulus Cloud with Lagrangian Droplets

#### Cloud Microphysics with Lagrangian Droplets

- $N = \sum_p N_p$  ( $N_p$  = number of droplets within a grid)
- $\frac{dr_i}{dt} = P + Q$ 
  - P : condensation - evaporation
  - Q : collision
  - \* Every individual droplet has its own  $r_i$
- $q_t = A_p \frac{1}{\rho} \left[ \frac{\rho_L}{\Delta V} \sum_{i=1}^{N_p} \frac{4}{3} \pi r_i^3 \right]$ 
  - $A_p$  = weighting factor
  - =  $\frac{\text{mass of real droplets per unit volume}}{\text{mass of simulated droplet per unit volume}}$
- During one time step
  - [1] condensation-evaporation
  - [2] collision
  - [3] advection



Evolutions of (a) potential temperature and (b) droplets position with radius



A new Lagrangian LES cloud model can provide information automatically

- Convection from cloud water to rain.
- Effects of turbulence on condensation and collision.
- Mixing process (entrainment, internal mixing, etc)
- Evolution of the droplet spectrum.
- Spatial distribution of the droplet concentration.
- Time history of droplets.
- Interaction between droplets and turbulence

## Conclusion

- LES is a highly effective tool to understand geophysical turbulence process and to develop its parameterization
- Much work is required, however, for more realistic simulation of turbulence by LES (e.g. SGS parameterization)