모드 유연도 및 정규화된 모드차를 이용한 모드형상 전개

Use of Modal Flexibility and Normalized Modal Difference(NMD) for Mode Shape Expansion

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ABSTRACT

In this paper, two possible ways for mode shape expansion are proposed and opened for discussion for future use. The first method minimizes the modal flexibility error between the experimental and analytical mode shapes corresponding to the measured DOFs to find the multiplication matrix which can be treated as the least-squares minimization problem. In the second method, Normalized Modal Difference (NMD) is used to calculate multiplication matrix using the analytical DOFs corresponding to measured DOfs. This matrix is then used to expand the measured mode shape to unmeasured DOFs. A simulated simply supported beam is used to demonstrate the performance of the methods. These methods are then compared with two most promising existing methods namely Kidder dynamic expansion and Modal expansion methods. It is observed that the performance of the modal flexibility method is comparable with existing methods. NMD also have the potential to expand the mode shapes though it is seen more sensitive to the distribution of error between FEM and actual test data.

***Keywords**: modal expansion, modal flexibility, NMD, least square minimization, Kidder dynamic expansion*

1. INTRODUCTION

The coordinate incompleteness of the experimental model may be due to physical and financial constraints. These reasons include;(i)difficulties in the measurement of rotations DOFs (ii) physically inaccessibility of the coordinates (iii)the size of the FE model due to the fact that measuring many coordinates according to FE model is expensive and time consuming. However, in most of the cases it is necessary to know the measurement at all DOFs of the structure under consideration. The potential cases may include; (i) for correlation of test and analysis results (ii) for finite element model updating and damage detections using system matrices (iii) to visualize the mode shapes obtained from

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experimental modal analysis effectively. The coordinate incompatibility can also be overcome by model reduction, but model reduction has disadvantages. The connectivities of the original model are lost, and extra inaccuracies are introduced, especially as the experimental coordinates are often not the best points to choose as masters. Therefore, experimental mode shape expansion is considered the logical alternative.

Existing mode shape expansion methods can be divided into four broad categories. The first approach involves the interpolation of the measured DOFs to those of the full model [1]. These methods use the FE model geometry to infer the mode shape at unmeasured locations and are very sensitive to spatial discontinuities and are mainly used for plate-like structures such as aircraft wings [2]. The second approach uses the FE model properties, such as mass and stiffness, to obtain a closed form solution of the mode shapes at unmeasured DOFs. These methods include the Guyan static expansion [3], which assumes that the inertial forces at the unmeasured DOFs are negligible, and the Kidder dynamic expansion [4] which uses the full dynamic equations to infer the mode shapes at the unmeasured DOFs. The third approach is presented in [5,6] and based on the assumption that the measured mode shapes can be expressed as a linear combination of the analytical ones. Another expansion method using the analytical mode shapes and the MAC matrix has been suggested by Lieven and Ewins [7]. The validity and performance of these expansion techniques are highlighted in [7,8]. A systematic study of the second and third approach explained above is carried out in [2] to define the validity boundaries of the methods. It is concluded that the quality of the expanded mode shapes are case-dependent. To account for uncertainties in the measurements and in the prediction, new expansion techniques based on least squares minimization techniques with quadratic inequality constraints (LSQI) are proposed in [9]. The behavior of modal expansion methods towards imprecise data is studied in [10].

In this paper, two possible ways for mode shape expansion are proposed and opened for discussion for future use. The first method minimizes the modal flexibility error between the experimental and analytical mode shapes corresponding to the measured DOFs to find the multiplication matrix which can be treated as the least squares minimization problem. In the second method, Normalized Modal Difference (NMD) is used to calculate multiplication matrix using the analytical DOFs corresponding to measured DOFs. This matrix is then used to expand the measured mode shape to unmeasured DOFs.

2. DESCRIPTION OF TWO EXISTING METHODS

2.1 Kidder Dynamic Expansion

This method is based on the eigenvalue equation. Partitioning the mass and stiffness matrix from the finite element model into measured a and unmeasured o coordinates and substituting the measured natural frequency and mode shape;

$$\left(\begin{bmatrix} K_{aa} & K_{ao} \\ K_{oa} & K_{oo} \end{bmatrix} - \bar{\omega}_j^2 \begin{bmatrix} M_{aa} & M_{ao} \\ M_{oa} & M_{oo} \end{bmatrix}\right) \begin{Bmatrix} \bar{\phi}_{aj} \\ \phi_{oj} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

where, $\bar{\omega}_j$ and $\bar{\phi}_{aj}$ represents jth measured natural frequency and corresponding mode shape at the measured coordinates and ϕ_{oj} represents the estimated mode shape at unmeasured DOFs. The estimates of the unmeasured DOFs can be obtained using Eq.(1)

which gives Eqs.(2) and (3):

$$\left(\left[K_{oa}\right]-\bar{\omega}_j^2\left[M_{oa}\right]\right)\left\{\bar{\phi}_{aj}\right\}+\left(\left[K_{oo}\right]-\bar{\omega}_j^2\left[M_{oo}\right]\right)\left\{\phi_{oj}\right\}=\{0\} \tag{2}$$

$$\left\{\phi_{oj}\right\}=-\left(\left[K_{oo}\right]-\bar{\omega}_j^2\left[M_{oo}\right]\right)^{-1}\left(\left[K_{oa}\right]-\bar{\omega}_j^2\left[M_{oa}\right]\right)\left\{\bar{\phi}_{aj}\right\} \tag{3}$$

2.2 Modal Expansion method

In this method, the measured modes are assumed to be a linear combination of the analytical modes and transformation A_{pp} is defined by

$$\left[\bar{\phi}_{ap}\right]=\left[\phi_{ap}\right]\left[A_{pp}\right] \tag{4}$$

where p is the number of modes considered, $\bar{\phi}_{ap}$ is the measured mode shape vector and ϕ_{ap} is the analytical mode shape corresponding to measurement DOFs. Applying pseudo inverse to Eq.(4) gives the transformation as ;

$$\left[A_{pp}\right]=\left[\phi_{ap}\right]^{+}\left[\bar{\phi}_{ap}\right] \tag{5}$$

This transformation is then used to expand the measured mode shape to unmeasured DOFs according to Eq.(6).

$$\left[\bar{\phi}_{op}\right]=\left[\phi_{op}\right]\left[A_{pp}\right] \tag{6}$$

3. FORMULATION OF TWO PROPOSED METHODS

3.1 Modal Flexibility Method

The modal flexibility is the accumulation of the contribution from all available mode shapes and corresponding natural frequencies. The modal flexibility matrix $[G]_{n\times n}$ is defined as [11]

$$[G]=[\phi]\left[\Lambda^{-1}\right][\phi]^T \tag{7}$$

in which $[\phi]$ is the mass normalized mode shape matrix and Λ is the matrix of eigenvalue. The purpose of the method is to identify the multiplication matrix A_{pp} to minimize the Frobenius Norm of differenece between experimental and analytical modal flexibility at coordinates corresponding to measured dofs in least square sense such that the experimental eigenvalue equals the analytical eigenvalue at each mode considered.

$$\min_{A_{pp}} \left\|\bar{\phi}_{ap}\bar{\Lambda}_{pp}^{-1}\bar{\phi}_{ap}^{T}-(\phi_{ap}A_{pp})\Lambda_{pp}^{-1}(\phi_{ap}A_{pp})^T\right\|_F \quad \text{Such that} \quad \bar{\Lambda}_{pp}=\Lambda_{pp} \tag{8}$$

When substituting the so called constraint into the objective function, the problem is cast into unconstrained form which can be easily solved to get the transformation matrix A_{pp}. In this work, the multivariate function *fminsearch* with the optimization Toolbox of MATLAB [12] was used. This transformation is then used to expand the measured mode shape to unmeasured DOFs according to Eq.(6).

3.2 NMD method

Normalized Modal Difference (NMD) is the comparison technique between the mode shapes obtained from experimental and analytical modal analysis proposed by Waters [14]. Physically, the NMD represents the error fraction on average by which each DOF differs between the two modes. So, this error fraction obtained from the measurement and corresponding analytical DOFs can be used to estimate the mode shape at unmeasured DOFs with the help of corresponding analytical mode shape. The NMD between experimental $\{\bar{\phi}\}$ and analytical $\{\phi\}$ mode shape is defined as;

$$NMD\left(\{\bar{\phi}\},\{\phi\}\right)=\frac{\left\|\{\bar{\phi}\}-\gamma\{\phi\}\right\|_2}{\left\|\gamma\{\phi\}\right\|_2};\ Modal\ Scale\ Factor(\gamma)=\frac{\left|\{\bar{\phi}\}^T\{\phi\}\right|}{\{\phi\}^T\{\phi\}} \tag{9}$$

Then, Eq.(10) is used to expand the measured mode shape at unmeasured DOFs.

$$\left[\bar{\phi}_{op}\right]=\left[\phi_{op}\right][C_1];\ C_1=diag\left(1-diag(NMD)\right) \tag{10}$$

4. PERFORMANCE METRICS

Three performance metrics are defined in this work to see the accuracy of different methods. The orthogonality properties of eigenvectors, as inferred in Modal Assurance Criteria (MAC) can be used as a performance metric. Modal Assurance Criterion (MAC) between known full eigenvector ϕ_i and expanded counterpart ϕ_j is defined as;

$$MAC_{ij}=\frac{\left|\phi_i^T\phi_j\right|^2}{\left(\phi_i^T\phi_i\right)\left(\phi_j^T\phi_j\right)} \tag{11}$$

The second and third performance metrics to compare the expanded and known exact measured mode shape is given by Eqs (12).

$$Error_1(\%)=\left|\frac{\left\|[\phi]_{exact}\right\|-\left\|[\phi]_{expanded}\right\|}{\left\|[\phi]_{exact}\right\|}\right|*100\ ;\ \ Error_2(\%)=\left|\frac{\left\|[\phi]_{exact}-[\phi]_{expanded}\right\|}{\left\|[\phi]_{exact}\right\|}\right|*100 \tag{12}$$

$Error_1$ gives a global appreciation of the error between the two mode shapes while $Error_2$ is more sensitive with the localized error.

5. CASE STUDY

A simulated simply supported beam as shown in Figure 1 is used to demonstrate the performance of the proposed methods. The simulated simply supported beam of 6m length is equally divided into 15 two dimensional beam elements; thus yielding 48 DOFs of which three are grounded. The density and modulus of elasticity of material of beam are 2500Kg/m^3 and 3.2E+10 N/m^2 respectively. Similarly, area of cross section and moment of inertia of simulated beam are 0.05 m^2 and 1.66E−04 m^4 respectively.

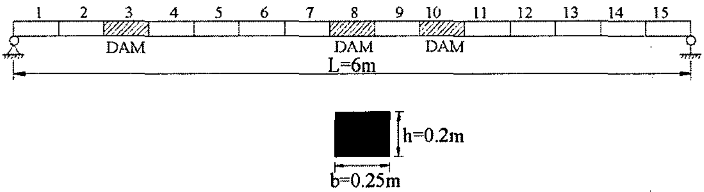

Figure 1 A Simulated Simply Supported Beam

Modal analysis is carried out using Matlab Based Modal Analysis Tool MBMAT [15] to get the FE frequency and mode shapes. All mode shapes have been normalized to the mass matrix. To get assumed experimental modal parameters, several damage are introduced by reducing the stiffness of assumed elements as shown in Figure 1. Modulus of elasticity of elements 3, 8 and 10 are reduced by 20%, 50% and 30% respectively. The modal analysis is again carried out in this damaged beam to get the assumed experimental modal parameters. The vertical DOFs are assumed as measured ones and hence mode shape vectors of damaged case corresponding to vertical DOFs are used for modal expansion and remaining DOFs are used to check the result of different expansion methods. At first, the expansion is carried out by flexibility method. The *fminsearch* function of the optimization Toolbox of MATLAB was used for minimization which predicted the value of matrix A_{pp} which is used to obtain expanded mode shapes from remaining DOFs of the analytical model. Similarly, for the NMD method. Modal Scale Factor is calculated and NMD value is predicted between the measured DOFs and corresponding analytical counterparts. Then, mode shape expansion is carried out.

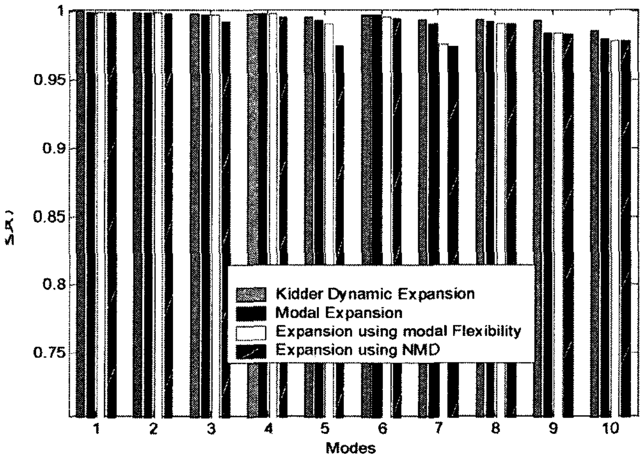

Figure 2 MAC values between the actual and expanded mode shapes

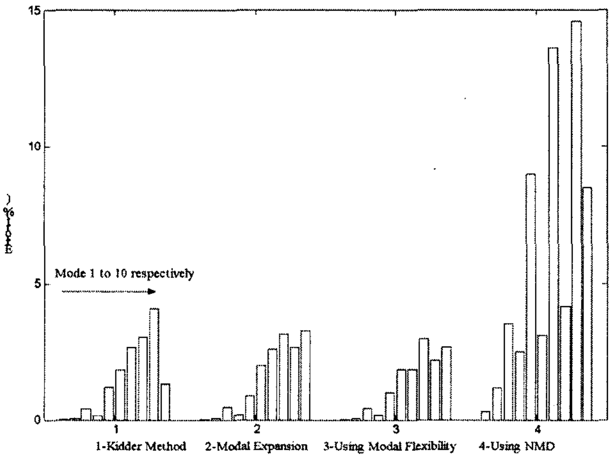

Figure 3 Norm errors for different expansion methods

To compare the results, modal expansion is carried out using Kidder dynamic expansion and Modal expansion method. The MAC value between the actual and expanded mode shapes for all four methods are plotted in Figure 2. It is observed that the performance of kidder method is the best and except for 5th and 7th mode, the MAC values from all methods are good.

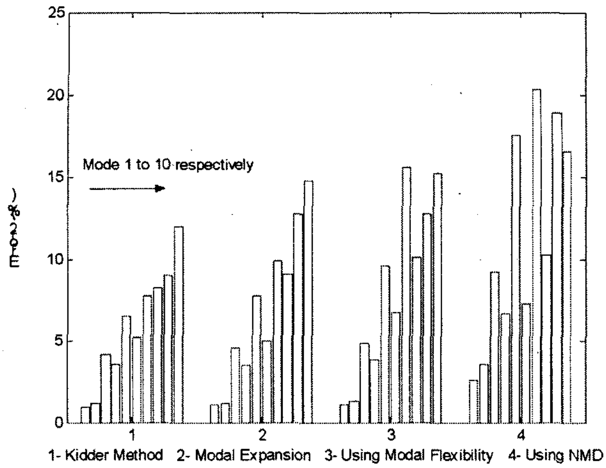

Figure 4 Norm of eigenvector differences

For the first four modes, the initial correlation before expansion is good. It is seen that the expanded result of these four modes from all methods are good. The similar trend can be observed for higher modes. The other performance metrics $Error_1$ and $Error_2$ defined in Eq.(12) are plotted in Figures 3 and 4 respectively. $Error_1$ gives a global appreciation of the error between the two mode shapes and $Error_2$ is more sensitive with the localized error. But one must take care with the definition of $Error_2$ which highlights error in small modal displacements. Figure 3 and 4 clearly shows that the performance of modal flexibility method is comparable with those of the existing methods. NMD method also has the potential to expand the mode shapes although it is seen more sensitive to the distribution of error between FEM and actual test data. It is because, for the well correlated modes with higher value of initial MAC, the error in expanded mode shape is less which can be observed from above figures.

6. CONCLUSION

Two possible ways for modal expansion were proposed. The first method minimizes the modal flexibility error between the experimental and analytical mode shapes corresponding to the measured DOFs to find the multiplication matrix which can be treated as the least-squares minimization problem. In the second method, NMD was used to calculate multiplication matrix using the analytical DOFs corresponding to measured DOfs. This matrix was then used to expand the measured mode shape to unmeasured DOFs. A simulated simply supported beam was used to demonstrate the performance of the methods. These two methods were compared with Kidder dynamic expansion and Modal expansion methods. It is observed that the performance of the modal flexibility method is comparable with the existing methods. NMD also have the potential to expand the mode shapes though it is seen more sensitive to the distribution of error between FEM and actual test data.

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REFERENCES

1. E.J. Williams and J.S. Green, "A special curve-fitting techniques for estimating rotational degree of freedom," *Proc. IMAC*, 1990, pp. 376-381.
2. M. Imregun and D.J. Ewins, "An investigation into mode shape expansion techniques," *Proc. IMAC*, Vol. 1, 1993, pp. 168-175.
3. R.J. Guyan, "Reduction of stiffness and mass matrices," *AIAA Journal*, 1965,Vol. 3(2).
4. R.L. Kidder, "Reduction of structural frequency equations" , *AIAA Journal*, 1973,Vol. 11(6).
5. J.C. O' Callahan, I.W. Lieu, P. Avitable and R. Madden, "An efficient method of determining rotational degrees of freedom from analytical and experimental modal data" *Proc. IMAC*, 1986, pp. 50-58.
6. J. Lipkins and U. Vandeurzan, "The use of smoothing techniques for structural modification applications," *Proc. ISMA*, 1987, KU Leuven.
7. N.A.J. Lieven and D.J. Ewins, "Expansion of modal data for correlation," *Proc. IMAC*, 1990, pp. 605-609.
8. H. Gysin, "Comparison of expansion methods for FE modelling and error localisation," *Proc IMAC*, 1990, pp. 195-204.
9. M. L.West, A. Kissil and M. Milman, "Evaluation of mode shape expansion techniques on the micro-precision interferometer truss," *Proc IMAC*, 1994.
10. T.Tison, B.Lallemand,G.Plessis and P.Level, "Behavior of modal expansion methods towards imprecise data," *Proc. IMAC*, 2000, pp. 33-39.
11. A. Hoyos and A.E. Aktan, "Regional identification of civil engineered structures based on impact induced transient responses," *Research Report 87-1, Louisiana state university,Baton Rouge*, 1987.
12. Matlab, Users Manual Version 5.2, The Math Works,Inc.,1998.
13. J.A. Nelder and R. Mead, "A simplex method for function minimization," *Computer Journal*, vol.7, 1965, pp. 308-313.
14. T.P. Waters, "Finite element model updating using measured frequency response functions," Ph.D. Thesis, Department of Aerospace Engineering, University of Bristol,U.K., 1995.
15. MBMAT, "Matlab Based Modal Analysis Tool", Fuzhou University, P.R. China